Complete stability region of PD controllers for TCP/AQM networks

Adrian Puerto-Piña and Daniel Melchor-Aguilar

Abstract— This paper addresses the stabilization problem of delay models of Transmission Control Protocol/Active Queue Management (TCP/AQM) by using a Proportional- Derivative (PD) controller as AQM strategy. The complete set of PD controllers that exponentially stabilizes the linearization is determined in counterpart with the existing works in the literature which only give an estimate of it. Additionally, a simple procedure for determining a non-fragile PD controller that admits controller coefficient perturbations is provided.

I. INTRODUCTION

One of the major problems in communication networks is congestion. To address the congestion problem Active Queue Management (AQM) scheme is recommended. The AQM strategy aims to minimize the risk of congestion by regulating the average queue size at the routers. Since the mathematical model for approximately describing the behavior of congested routers in TCP/AQM networks was introduced in [4] several feedback control approaches as, for instance, proportional (P) [4], [8], proportional-integral (PI) [4], [7], proportional-derivative (PD) [1], [6], [13], and $\mathcal{H}^{\infty}$ [11] controllers have been proposed as AQM strategies.

Among these works, it is interesting to note the state-space feedback formulation proposed in [6] of the TCP/AQM control problem. It is there shown that a PD-type control structure in terms of the queue length is the natural state feedback control to fully support TCP dynamics. The capabilities of the PD AQM control on regulating the queue length under different network scenarios as well as comparisons with other AQM strategies have been illustrated by simulations in [1] and [13].

Although the existing designs of PD AQM controllers can give satisfactory results they are only based on sufficient conditions for guaranteeing the closed-loop stability of the linearization of the models, whereas the controller gains are derived by some heuristic rules [13], the minimization of a linear quadratic cost function [6], and in terms of linear matrix inequalities [1]. As a consequence, these designs do *not provide* the set of *all stabilizing PD gain values*. This fact motivates us for searching a complete characterization of the set of all PD controllers that exponentially stabilizes the linearization of a simplified version of the model considered in [1] and [6].

In developing the analysis, we noticed that the closed-loop system under a PD AQM controller is a delay system of the *neutral type* but, however, the designs in [1] and [6] are based on a closed-loop delay system of the *retarded type* which is

considered equivalent to the neutral one. We revise the results in [1] and [6] and show that the neutral and retarded type closed-loop delay systems are not equivalent in general but only for some particular initial conditions. This result, that to the best of our knowledge it has not been reported in the literature, provides a formal justification for designing a PD AQM controller based on a retarded type closed-loop delay system instead of the corresponding neutral type one for which the stability analysis is known to be more complicated.

One of the main advantages of knowing the set of all stabilizing controllers is that allows us to perform an appropriate comparison of robustness and fragility issues of various stable designs, see [7] for the complete characterization and [14] for comparisons on robustness and fragility of PI AQM stabilizing controllers. Here, using the complete characterization of all stabilizing PD controllers, we present a simple methodology to examine the fragility of a given PD stabilizing controller and propose an algorithm for determining a non-fragile one. To have a non-fragile controller is very important for the practical application of the designs since it is required to maintain the closed-loop stability by perturbations in the controller coefficients naturally arising from round-off errors during implementation and possible tuning around a nominal design for getting a desired closed-loop performance, see [5].

The remaining part of the paper is organized as follows. Section II presents the TCP/AQM model under study and the PD control as AQM strategy. The justification for considering a closed-loop delay system of the retarded type instead of the closed-loop delay system of the neutral type is given in section III. The complete characterization of PD stabilizing controllers for the linearization is provided in section IV. A numerical example illustrating that the PD designs by the approaches in [6], [1], and the classic methodology of Ziegler-Nichols [10] belong to the complete stabilizing region is also given. Section V presents the fragility analysis, where an algorithm for computing a non-fragile PD controller is provided. Fragility comparison of some stabilizing controllers is performed by means of a numerical example. Concluding remarks end this paper.

II. MATHEMATICAL MODEL AND PD CONTROL

We consider the dynamic fluid-flow model introduced in [4] for describing the behavior of TCP/AQM networks. Such a model, relating the average value of key network variables of n homogeneous TCP-controlled sources and a single congested router, is described by the following coupled non-

The authors are with Division of Applied Mathematics, IPICYT, San Luis Potosí, 78216, México {adrian.puerto,dmelchor}@ipicyt.edu.mx

linear differential equations including time-varying delays:

$$\begin{cases} \dot{w}(t) = \frac{1}{\tau(t)} - \frac{w(t)w(t-\tau(t))}{2\tau(t-\tau(t))}p(t-\tau(t)), \\ \dot{q}(t) = \frac{n(t)}{\tau(t)}w(t) - c(t), \end{cases} \quad (1)$$

where $w(t)$ denotes the average of TCP windows size (packets), $q(t)$ is the average queue length (packets), $\tau(t) = \frac{q(t)}{c} + \tau_p$ is the round-trip time (secs) with τ_p representing the propagation delay, $c(t)$ is the link capacity (packets/sec), $n(t)$ is the number of TCP sessions and $p(\cdot)$ is the probability of a packet marking which represents the AQM control strategy.

Following similar arguments to the one proposed by [4], we assume that the number of TCP sessions, the round-trip time delay and the link capacity are constant, i.e., $n(t) = n, \tau(t) = \tau$ and $c(t) = c$. Then, the model (1) is approximated by the following system:

$$\begin{cases} \dot{w}(t) = \frac{1}{\tau} - \frac{w(t)w(t-\tau)}{2\tau}p(t-\tau), \\ \dot{q}(t) = \frac{n}{\tau}w(t) - c, \end{cases} \quad (2)$$

whose unique equilibrium point is given by

$$(w_e, p_e) = \left(\frac{c\tau}{n}, \frac{2n^2}{(c\tau)^2} \right).$$

In [3] and [8] was shown that if $w_e \gg 1$ then the local behavior of the model (2) around the equilibrium can be approximated by the local behavior of

$$\begin{cases} \dot{w}(t) = \frac{1}{\tau} - \frac{w^2(t)}{2\tau}p(t-\tau), \\ \dot{q}(t) = \frac{n}{\tau}w(t) - c. \end{cases} \quad (3)$$

Although the condition $w_e \gg 1$ imposes a restriction on the network parameters for considering (3) as a good approximation of (2), it is satisfied for typical range of parameters arising in practice as argued in [3] and [8].

We here consider the simplified model (3) and a PD AQM controller of the form

$$p(t) = K_p q(t) + K_d \dot{q}(t). \quad (4)$$

In [6] was shown that a PD control of the form (4) is needed in order to fully support the TCP dynamics. Roughly speaking, the main reasoning behind this is that the windows size $w(t)$ and queue length $q(t)$ are the state variables of the system (3) and, therefore, they need to be involved in a state feedback for completely controlling the dynamics. Now, since the second equation of (3) expresses the queue dynamic as a function of the windows size then it appears that $\dot{q}(t)$ may be used instead of $w(t)$ thus leading to a PD-type control structure. On the other hand, the use of a PD control overcomes the implementation restriction of having a measure or estimation of $w(t)$ which is not accessible at the router's side in real networks, see [1] for discussions.

III. TRANSFORMATION FROM NEUTRAL TO RETARDED CLOSED-LOOP SYSTEMS

The closed-loop system (3)-(4) is

$$\begin{cases} \dot{w}(t) = \frac{1}{\tau} - \frac{w^2(t)}{2\tau}[K_p q(t-\tau) + K_d \dot{q}(t-\tau)], \\ \dot{q}(t) = \frac{n}{\tau}w(t) - c. \end{cases} \quad (5)$$

Clearly, the delay system (5) is of *neutral type* as involves the time derivative of past values of $q(t)$, see [16] for discussions when a PD-type feedback is applied to a system in the presence of delays in the input signal.

By following the approaches presented in [1] and [6], let us differentiate the second equation of (3) and substitute the right-hand sides of the first and second equations of (3)

$$\ddot{q}(t) = \frac{n}{\tau^2} - \frac{1}{2n}(\dot{q}(t) + c)^2 p(t-\tau). \quad (6)$$

The closed-loop system (6)-(4) is

$$\ddot{q}(t) = \frac{n}{\tau^2} - \frac{1}{2n}(\dot{q}(t) + c)^2 (K_p q(t-\tau) + K_d \dot{q}(t-\tau)), \quad (7)$$

a delay system of the *retarded type* which is considered equivalent to the neutral system (5) in [1] and [6] for designing the gain values.

Formally speaking, the process of converting the coupled dynamics for $w(t)$ and $q(t)$ in (3) to a single dynamic for $q(t)$ in (6) represents a special system transformation that, under a PD type controller, is only valid for some particular initial functions.

More precisely, when transforming (3) in (6) the second equation of (3)

$$\dot{q}(t) = \frac{n}{\tau}w(t) - c,$$

is used. Nevertheless, this equation holds only for $t \geq 0$ and since the PD control (4) involves $\dot{q}(t)$ then it is required that

$$\dot{q}(t-\tau) = \frac{n}{\tau}w(t-\tau) - c,$$

holds for $t \in [0, \tau]$. Evidently, the above equation holds only if a restriction on the initial conditions is imposed.

Lemma 1: Consider the neutral delay system (5) and the retarded delay system

$$\begin{cases} \dot{w}(t) = \frac{1}{\tau} - \frac{w^2(t)}{2\tau}\left[K_p q(t-\tau) + K_d \left(\frac{nw(t-\tau)}{\tau} - c\right)\right], \\ \dot{q}(t) = \frac{n}{\tau}w(t) - c. \end{cases} \quad (8)$$

For initial conditions

$$w(t) = \varphi_w(t), q(t) = \varphi_q(t), t \in [-\tau, 0], \quad (9)$$

satisfying

$$\dot{\varphi}_q(t) = \frac{n}{\tau}\varphi_w(t) - c, \quad t \in [-\tau, 0], \quad (10)$$

we have that the corresponding solutions of both systems (5) and (8) coincide. Here, we assume that $\varphi_w \in \mathcal{C}([-\tau, 0], \mathbb{R})$, the space of continuous functions mapping the interval $[-\tau, 0]$ to $\mathbb{R}$, and $\varphi_q \in \mathcal{C}^1([-\tau, 0], \mathbb{R})$, the space of the continuously differentiable functions mapping the interval $[-\tau, 0]$ to $\mathbb{R}$.

Proof: Let $w(t, \varphi_w, \varphi_q)$ and $q(t, \varphi_w, \varphi_q)$ be the solutions of the neutral delay system (5) for the initial conditions (9) satisfying (10). From the second equation of (5) and the restriction on the initial conditions (10) follow that

$$\dot{q}(t-\tau, \varphi_w, \varphi_q) = \frac{n}{\tau}w(t-\tau, \varphi_w, \varphi_q) - c \quad (11)$$

holds for all $t \geq 0$. Then, a direct substitution of (11) in the first equation of system (5) shows that $w(t, \varphi_w, \varphi_q)$ and $q(t, \varphi_w, \varphi_q)$ satisfy the system (8) for all $t \geq 0$.

Conversely, if $w(t, \varphi_w, \varphi_q)$ and $q(t, \varphi_w, \varphi_q)$ are the solutions of the retarded delay system (8) for the initial conditions (9) satisfying (10) then the equation (11) holds for all $t \geq 0$. By substituting (11) in the first equation of (8) ones arrives at the result that these solutions $w(t, \varphi_w, \varphi_q)$ and $q(t, \varphi_w, \varphi_q)$ also satisfy the neutral delay system (5). ∎

Remark 1: For the initial functions (9) satisfying (10) the corresponding solution $q(t, \varphi_w, \varphi_q)$ of the retarded delay system (8) also satisfies the equation (7) and vice versa. Hence, under the restriction (11) on the initial conditions (9), the neutral delay system (5) and the retarded one (7) are equivalent as proposed in [1] and [6] but not justified.

Based on the Lemma 1 and Remark 1 we now proceed to develop the local stability analysis around the equilibrium of the closed-loop retarded delay system (8).

The unique equilibrium point of (8) is given by

$$(w_e, q_e) = \left(\frac{c\tau}{n}, \frac{2n^2}{K_p(\tau c)^2} \right).$$

The linearization around the equilibrium (w_e, q_e) is

$$\dot{\xi}(t) = A\xi(t) + B\xi(t - \tau), \quad (12)$$

where

$$\xi(t) = \left(\begin{array}{c} \bar{w}(t) \\ \bar{q}(t) \end{array} \right), A = \left(\begin{array}{cc} -\frac{2n}{c^2\tau} & 0 \\ \frac{n}{\tau} & 0 \end{array} \right),$$

$$B = \left(\begin{array}{cc} -\frac{c^2}{2n}K_d & -\frac{\tau c^2}{2n}K_p \\ 0 & 0 \end{array} \right),$$

$\bar{w}(t) = w(t) - w_e$ and $\bar{q}(t) = q(t) - q_e$.

Remark 2: As it was stated in [7] for the case of PI AQM controllers, it is not possible to directly investigate the stability of (12) for the delay-free case ($\tau = 0$) since the matrices A and B depend explicitly on the parameter $1/\tau$. From this fact follows that the approach developed in [12], which is first based on computing the set of PD stabilizing controllers for the delay-free case cannot be directly applied. In the sequel, we address the problem by following the ideas in [8] and [7], where the D-decomposition method, introduced by Neimark in [9] is used and exploited for the case of P and PI AQM controllers.

IV. COMPLETE CHARACTERIZATION OF PD STABILIZING CONTROLLERS

It is well known that the system (12) is exponentially stable if and only if its characteristic function (quasipolynomial)

$$f(s) = s^2 + \left(\frac{2n}{c\tau^2} \right) s + \left(\frac{K_d c^2}{2n} \right) s e^{-\tau s} + \left(\frac{K_p c^2}{2n} \right) e^{-\tau s}$$

has no zeros with non-negative real parts, see, e.g. [2].

The following result provides the complete characterization of the controller's gains (K_p, K_d) for which (12) is exponentially stable.

Proposition 2: Given the network parameters (n, τ, c), the system (12) is exponentially stable if and only if the controller's gains (K_p, K_d) belong to the stability region Γ, plotted in Fig. 1, whose boundary in the controller's gains space (K_p, K_d) is defined by

$$\partial\Gamma = \{ (K_p(\omega), K_d(\omega)) : \omega \in (0, \bar{\omega}) \}$$
$$\cup \{ (K_p, K_d) : K_d \in [K_d(0), K_d(\bar{\omega})] \text{ and } K_p = 0 \}, \quad (13)$$

where

$$K_p(\omega) = \frac{2n}{c^2} \left[\omega^2 \cos(\tau\omega) + \frac{2n\omega}{c\tau^2} \sin(\tau\omega) \right], \quad (14)$$

$$K_d(\omega) = \frac{2n}{c^2} \left[\omega \sin(\tau\omega) - \frac{2n}{c\tau^2} \cos(\tau\omega) \right], \quad (15)$$

and $\bar{\omega}$ is the solution of

$$\tan(\tau\omega) = -\frac{c\tau^2}{2n}\omega \quad (16)$$

for $\omega \in \left(\frac{\pi}{2\tau}, \frac{\pi}{\tau} \right)$.

Proof: Firstly, we observe since $n, \tau, c > 0$ then $s = 0$ is a zero of $f(s)$ if and only if $K_p = 0$. Now, let us assume that $f(s)$ has a pure imaginary zero $s = i\omega \neq 0$. Then, a direct calculation leads to (14) and (15).

The parametrization (14)-(15) defines a continuous curve in the controller's gains space (K_p, K_d) when ω varies from 0 to ∞. The curve and coordinate axis K_d divide the plane (K_p, K_d) into an infinite (countable) set of connected open regions $\Omega_j, j = 1, 2, \ldots$, see Fig. 2.

In order to explicitly determine the regions $\Omega_j, j = 1, 2, \ldots$, one needs to compute the intersections of the curve with the axis K_d. These intersections can be found by solving for $\omega \neq 0$ the equation $K_p(\omega) = 0$, where $K_p(\omega)$ is given by (14). The equation $K_p(\omega) = 0$ has as solutions those of the equation (16). Since the equation (16) is transcendental we then directly search for a numerical solution. The solutions can be found by plotting the functions $\tan(\tau\omega), -\frac{c\tau^2}{2n}\omega$. It follows that there is an infinite number of solutions $\bar{\omega}_k, k = 0, 1, \ldots$, of the equation (16) which satisfy

$$\bar{\omega}_k \in \left((2k+1)\frac{\pi}{2\tau}, (2k+2)\frac{\pi}{2\tau} \right).$$

With these $\bar{\omega}_k, k = 0, 1, \ldots$, the boundary of each region $\Omega_j, j = 1, 2, \ldots$, is explicitly determined.
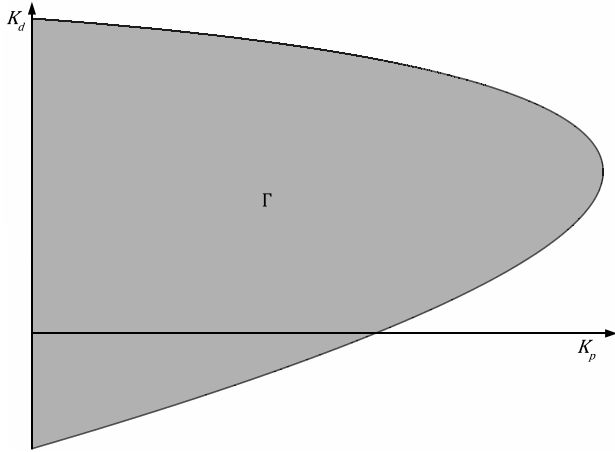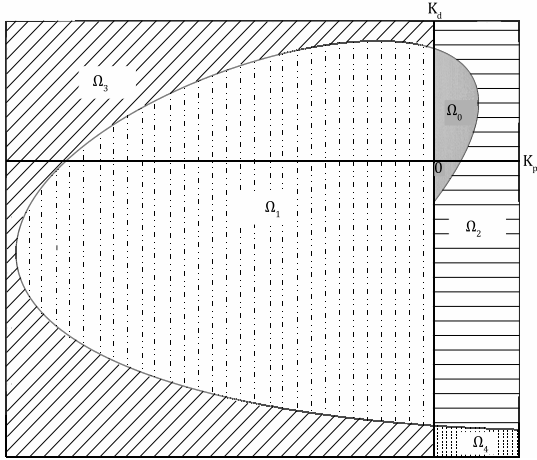
By using the Mikhailov's stability criterion we show that for all (K_p, K_d) inside of the open region $\Gamma = \Omega_0$, whose boundary is given by the curve obtained from the parametrization (14)-(15) for varying $\omega \in (0, \bar{\omega}_0)$ and the segment $[K_d(0), K_d(\bar{\omega}_0)]$ of the coordinate axis K_d as defined by (13), the function $f(s)$ has not zeros with non-negative real parts, which ends the proof. ∎

A. Numerical example

Let us consider network parameters in the same setup as in [1], where n=100 TCP flows, τ=0.3250 s, and c=800 packets/s. For these network parameters the pair of gains

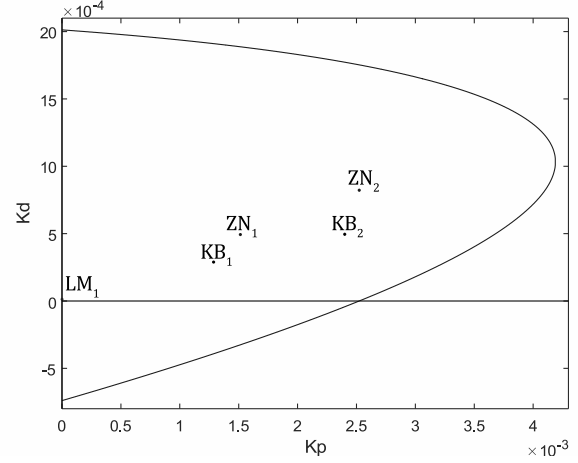$$LM = (K_p, K_d) = (3.7 \times 10^{-7}, 1.2610 \times 10^{-5})$$

is obtained via solution of linear matrix inequalities in [1].

Fig. 1. Stability region Γ in the plane (K_p, K_d).

Fig. 3. Stability region Γ for the numerical example along with PD stabilizing controllers proposed by [1], [6] and the Ziegler-Nichols method.

Fig. 2. First four regions Ω_j partitioning the plane (K_p, K_d).

On the other hand, the approach proposed in [6] for designing a PD stabilizing controller that minimize a linear quadratic cost function presents a parametrization for the controller's gains $K_p(\lambda)$ and $K_d(\lambda)$, where λ is real negative parameter, see the Propositions 1 and 3 in [6]. By using this approach with $\lambda = -8$ and $\lambda = -10$ we respectively obtain the pairs of gains

$$
\begin{aligned}
KB_1 &= (K_p, K_d) = (1.2866 \times 10^{-3}, 2.8878 \times 10^{-4}), \\
KB_2 &= (K_p, K_d) = (2.4 \times 10^{-3}, 4.9530 \times 10^{-4}).
\end{aligned}
$$

For a further comparison we design a PD controller by using the classical Ziegler-Nichols approach. By following [10] (see page 235) we have a stabilizing PD controller for $K_p \in [0.6K_u, K_u]$ and $K_d = 0.125K_pT_u$, where K_u is the proportional gain for which the output starts to oscillate under a step input and $K_d = 0$, and T_u is the corresponding

oscillation period. Note that this tunig provides a set of stabilizing PD controllers as K_p can be selected in the interval $[0.6K_u, K_u]$. For $K_p = 0.6K_u$ and $K_p = K_u$ the following pairs of gains are obtained:

$$
\begin{aligned}
ZN_1 &= (K_p, K_d) = (1.51407 \times 10^{-3}, 4.93398 \times 10^{-4}), \\
ZN_2 &= (K_p, K_d) = (2.52345 \times 10^{-3}, 8.22393 \times 10^{-4}).
\end{aligned}
$$

In Fig. 3 we plot the stability region Γ in the controller's gains space (K_p, K_d) along with the pairs LM, KB_1, KB_2, ZN_1 and ZN_2. As expected, the PD controllers proposed in [1] and [6] as well as the ones obtained by means of the classical Ziegler-Nichols method belong to the complete set of PD controllers that stabilizes (12).

V. FRAGILITY ANALYSIS

In this section we address the fragility analysis of PD stabilizing controllers, i.e. the robustness to perturbations in the controller gains K_p and K_d.

Although negative gains can stabilize the system (12) it makes sense only consider positive ones for the practical application to the networks, see [14] for discussions in the case of PI controllers. Thus, for the fragility analysis we consider controller's gains (K_p, K_d) belonging to the stability region Γ_p whose boundary in the controller's gains space is given by

$$
\begin{aligned}
\partial \Gamma_p &= C \cup \{(K_p, K_d) : K_d \in [0, K_d(\bar{\omega})] \text{ and } K_p = 0\} \\
&\quad \cup \{(K_p, K_d) : K_p \in [0, K_p(\hat{\omega})] \text{ and } K_d = 0\},
\end{aligned}
$$

where

$$
C = \{(K_p(\omega), K_d(\omega)) : \omega \in (\hat{\omega}, \bar{\omega})\},
$$

with $K_p(\omega)$ and $K_d(\omega)$ respectively given by (14) and (15), $\bar{\omega}$ the solution of (16) for $\omega \in \left(\frac{\pi}{2\tau}, \frac{\pi}{\tau}\right)$ and $\hat{\omega}$ is the solution of

$$
\tan(\tau\omega) = \frac{2n}{\omega c\tau^2}, \quad \omega \in \left(0, \frac{\pi}{2\tau}\right).
$$

The fragility problem of a given PD stabilizing controller can be formulated as follows: Given nominal controller's gains $(K_{p0}, K_{d0}) \in \Gamma_p$, find the maximum $\rho_0 > 0$ such that for any $K_p, K_d \geq 0$ the following condition holds:

$$B_{\rho_0}(K_{p0}, K_{d0}) = \{(K_p, K_d):$$
$$\sqrt{(K_p - K_{p0})^2 + (K_d - K_{d0})^2} < \rho_0\} \subset \Gamma_p.$$

The problem is equivalent to find the minimum distance between (K_{p0}, K_{d0}) and $\partial\Gamma_p$. The distance from (K_{p0}, K_{d0}) to the curve C is given by the minimum of the function

$$d(\omega) = \sqrt{(K_p(\omega) - K_{p0})^2 + (K_d(\omega) - K_{d0})^2}, \ \omega \in [\hat{\omega}, \bar{\omega}].$$

Since $\omega \to d(\omega)$ is a continuous function there always exists $\tilde{\omega} \in [\hat{\omega}, \bar{\omega}]$ such that $d(\tilde{\omega}) \leq d(\omega)$ for all $\omega \in [\hat{\omega}, \bar{\omega}]$. Then, it is easily seen that the minimum distance from the given point (K_{p0}, K_{d0}) to $\partial\Gamma_p$ is given by

$$\rho_0 = \min\{K_{p0}, K_{d0}, d(\tilde{\omega})\}. \tag{17}$$

The formula (17) determines a numerical procedure to determine the l_2 parametric stability margin around a nominal point (K_{p0}, K_{d0}) which allows us to examine the fragility of a given stabilizing controller, whereas a large ρ_0 leads to a less fragile controller while a small ρ_0 yields a more fragile one.

A. An algorithm for computing a non-fragile controller

Once a procedure to compute the l_2 parametric stability margin around a nominal point (K_{p0}, K_{d0}) is given, we can now address the problem of finding the controller's gains $(K_p^*, K_d^*) \in \Gamma_p$ at the center of the circle $B_{\rho_0}(K_{p0}, K_{d0})$ of maximum $\rho_0 > 0$ such that $B_{\rho_0}(K_{p0}, K_{d0}) \subset \Gamma_p$. This $\rho_0 > 0$ represents the maximum l_2 parametric stability margin in the controller's gains space (K_p, K_d), see [5].

To this aim we propose the following algorithm:

- Choose $K_p^* = \frac{1}{2} K_p(\hat{\omega})$,
- Sweep K_d over the interval $[0, K_d(\omega^*)]$, where $\omega^* \in [\hat{\omega}, \bar{\omega}]$ such that $K_p(\omega^*) = K_p^*$ and determine $\rho_0 > 0$ by using the formula (17).

This procedure determines a family of circles having different radii and centres from which we select the one with the maximum radius. Finally, we choose K_d^* to be at the center of this circle.

B. Numerical example

Consider the same numerical example as in section III. By applying the proposed algorithm in subsection A we obtain the pair

$$PD = (K_p, K_d) = (1.26172 \times 10^{-3}, 9.5 \times 10^{-4}).$$

The Table I presents the computed $\rho_0 > 0$ for the pairs LM, KB_1, ZN_1 and PD while the Fig. 4 illustrates the fragility of such controllers in the complete region Γ_p of stabilizing PD controllers.

TABLE I
FRAGILITY OF PD CONTROLLERS

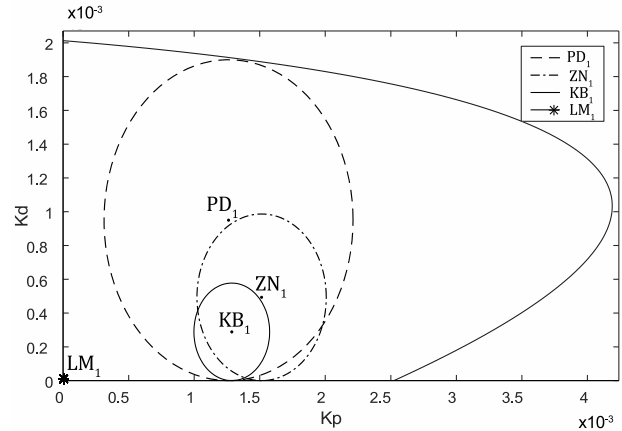|	ρ_0
LM	3.70000×10^{-7}
KB_1	2.88778×10^{-4}
ZN_1	4.93398×10^{-4}
PD	9.50000×10^{-4}

Fig. 4. Fragility comparison of the stabilizing PD controllers in Table I

As seen in Table I and also shown in Fig. 4 the controller LM designed by [1] is more fragile than the other controllers, the controllers KB_1 and ZN_1 respectively designed by [6] and the Ziegler-Nichols approach may not suffer fragility as they have an acceptable fragility margin while the controller PD designed by our proposed algorithm has the maximum fragility margin.

VI. CONCLUDING REMARKS

In this paper, we studied the stability of TCP/AQM delay models by using a PD controller as AQM strategy. Firstly, we showed that the corresponding closed-loop delay system is of neutral type and a formal justification for transforming the neutral delay system to a retarded delay one is given. Then, the complete stability region of PD controllers is provided. This now allows the designers to select the controller gains for achieving some performance specifications based on the exact stability region and not in an estimate of it as occurring in the existing works.

The knowledge of the boundary of the stability region in the controller's parameters space allowed us to give a precise solution to the fragility problem of a given stabilizing PD controller and also to propose a simple methodology for determining the controller gains providing a non-fragile PD controller which admits controller coefficients perturbations.

REFERENCES

[1] M. Azadegan, M.T.H. Beheshti, and B. Tavassoli, "Design of Proportional-Derivative-Type State Feedback Controllers for Congestion Control of Transmission Control Protocol Networks," *Int. J. Syst. Sci.*, vol. 46, no. 10, pp. 1774–1779, Sep. 2013.

[2] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*. Boston: Birkhäuser, 2003.

[3] C.V. Hollot and Y. Chait, "Nonlinear Stability Analysis for a Class of TCP/AQM Networks," in *Proc. of the 40th IEEE Conference on Decision and Control,*" Orlando, USA, 2001.

[4] C. V. Hollot, V. Misra, D. Towsley, and W. B. Gong, "Analysis and Design of Controllers for AQM Routers Supporting TCP Flows," *IEEE Trans. Autom. Control*, vol. 47, no. 6, pp. 945-959, Jun. 2002.

[5] L. H. Keel and S.P. Bhattacharyya, "Robust, Fragile or Optimal?," *IEEE Trans. Autom. Control*, vol. 42, no. 8, pp. 1098-1105, Aug. 1997.

[6] K. B. Kim, "Design of Feedback Controls Supporting TCP Based on the State-space Approach," *IEEE Trans. Autom. Control*, vol. 51, no. 7, pp. 1086-1099, Jul. 2006.

[7] D. Melchor-Aguilar and S.-I. Niculescu, "Computing Non-fragile PI Controllers for Delay Models of TCP/AQM Networks," *Int. J. Control*, vol. 82, no. 12, pp. 2249-2259, Dec. 2009.

[8] W. Michiels, D. Melchor-Aguilar, and S. -I. Niculescu, "Stability Analysis Some Classes of TCP/AQM Networks," *Int. J. Control*, vol. 79, no. 9, pp. 1136-1144, Sep. 2006.

[9] J. Neimark, "D-Subdivisions and Spaces of Quasipolynomials," *Prikladnaya Matematika i Mekhanika*, vol.13, pp. 349-380, 1949.

[10] A. O'Dwyer, *Handbook of PI and PID Controller Tuning Rules*. London: Imperial College Press, 2006.

[11] P. F. Quet and H. Özbay, "On the Design of AQM Supporting TCP Flows Using Robust Control Theory," *IEEE Trans. Autom. Control*, vol. 49, no. 6, pp. 1031-1036, Jun. 2004.

[12] G. J. Silva, A. Datta, and S. P. Bhattacharyya, *PID Controllers for Time-Delay Systems*. Boston: Birkhäuser, 2005.

[13] J. Sun, K. King-Tim, G. Chen, S. Chan, and M. Zukerman, "PD-RED: To Improve the Performance of RED," *IEEE Commun. Lett.*, vol. 7, no. 8, pp. 406-408, Aug. 2003.

[14] H. U. Unal, D. Melchor-Aguilar, D. Üstebay, S. -I. Niculescu, and H. Özbay, "Comparison of PI Controllers Designed for the Delay Model of TCP/AQM networks," *Comput. Commun.*, vol. 36, pp. 1225-1234, Mar. 2013.

[15] D. Üstebay and H. Özbay, "Switching Resilient PI Controllers for Active Queue Management of TCP Flows," in *Proc. of the IEEE International Conference on Networking, Sensing and Control*, London, 2007, pp. 574–578.

[16] T. Vyhlídal, W. Michiels, and P. MacGahan, "Stability Impact of Small Delays in Proportional–Derivative State Feedback," *Control Eng. Practice,* vol. 17, no. 3, pp. 382-393, Mar. 2009.