Chua's circuit and its characterization as a filter

I. Campos-Cantón[1]‡, O.A. Segura-Cisneros[1], R.E. Balderas-Navarro[2], E. Campos-Cantón[3]

[1] Facultad de Ciencias,
[2] Instituto de Investigación en Comunicación Óptica,
Universidad Autónoma de San Luis Potosí (UASLP), Alvaro Obregón 64, 78000 San Luis Potosí, SLP, México,
[3] División de Matemáticas Aplicadas,
Instituto Potosino de Investigación Científica y Tecnológica (IPICYT), Camino a la Presa San José 2055, 78216 San Luis Potosí, SLP, México

E-mail: `icampos@fciencias.uaslp.mx`

Abstract.
This article deals with the Chua's circuit characterization from the point of view of a filter based on the concept of piecewise linear functions. Furthermore, experiments are developed for teaching electronic systems that can be used for filtering. The frequency range, in which ~~is~~ tested, is from $20Hz$ to $20kHz$, due to audio spectrum is comprised in this frequency range. The associated node to the capacitor and Chua's diode is used as input, and the node comprised for the another capacitor and the coil is used as output, thereby establishing one input-output relationship for each system case given by the piecewise linear functions. The experimental result shows that Chua's circuit behaves as a bandpass filter-amplifier, with a maximum frequency around $3kHz$ and bandwidth between $1.5kHz$ and $5.5kHz$.

Submitted to: *Eur. J. Phys.*

---

‡ Corresponding author

1. Introduction

Filters are key elements in a wide variety of electronic applications. Among them we can mention the wireless transceivers [1] and information retrieval in direct transmission [2]; for which a band pass filter is used, to extract the embedded desired signal in a sum of different signals with different frequency domain of each one. The low pass filters are used to improve the signal-noise ratio of an information signal contaminated with noise, where usually it has a bigger bandwidth than the information signal. Filters are also used in applications such as ~~biological systems study~~ [3], for example, consider an instrumentation amplifier with high gain to measure electroencephalogram (EEG) signals; EEG signals are low level and are obtained by electrodes that are attached to the scalp of the individual. These electrodes are very high impedance devices, therefore, the EEG signal can be easily corrupted by additive noise 60/120 Hz, which is generated from electronic equipment connected to the electrical network. An instrumentation differential amplifier can significantly suppress the additive noise, but not ~~completely removed it~~. EEG signals have most of their energy below 60 Hz, and therefore a low pass filter is ideal to suppress these components. The use of filters in many applications is due to the ability to discriminate unwanted information signals, also, allow that two systems are coupled together due to their properties of low-high input-output impedance's.

Active filters of first and second order with low-high input impedance are of great interest because of several filters of the same kind can be directly connected in cascade to implement higher order filters [4]. Therefore, by adjusting the high-low impedance at the output of active filters, it makes easy to connect to the next state without any buffer [5]. On the other hand, the use of switched capacitors and resistors is ideal for integrated circuit implementations of this filter.

Nowadays, a topic of great importance is to give alternative filter designs, and one of the ~~bets~~ is to develop filters based on nonlinear systems [6]. The chaotic circuits are candidates to perform these tasks and we hope that they can generate different filters and also have the possibility to act as active filters due to operational amplifiers embedded in its circuitry. The first step to develop filters is to characterize these circuits in frequency and then use them to perform signal filtering as with linear filters, where the configuration is done by selecting a specific bandwidth. One of the electrical circuits which exhibit chaotic behavior is the Chua's circuit [7]. Various applications have been made with this system including secure communication systems using chaotic signals [8] [9], noise generators [10], among others.

In this framework, the paper's objective is to ~~characterized~~ the Chua's circuit as a ~~filter, through performing experimental measurements and present it in a similar way based on linear filter theory~~, i.e., we use the Chua's circuit as a filter and show that it exhibits similar properties to those of an active filter. In Section 2 we established the piecewise input-output model in three conditions: when Chua's diode is active (two cases) and when Chua's diode is inactive (one case). In Section 3 the instrumentation of the Chua's circuit as filter is described. The experimental results are given in Section

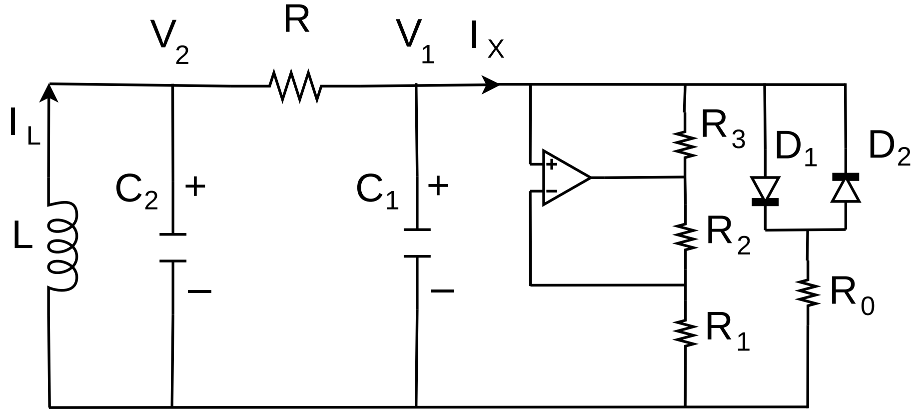

Fig. 1. Chua's circuit.

4, and finally the conclusions are in Section 5.

2. Input - Output model

In circuit theory, a filter is defined as a two-port electrical network which alters the amplitude or phase characteristics of a signal with respect to frequency. Ideally, a filter does not add new frequencies to the input signal, an also should not change the frequencies that make up the signal; however, change the relative amplitude of the various frequency components besides its phase relationship. Filters are usually used to emphasize signals that present certain range of frequencies and to reject signals that present another frequency range [11]. In practice, filters are basic elements used in communication systems such as radio, television, telephone, radar and computers. The truth is that filters have permeated modern technology, so it is not hard to find electronic devices that use them in one way or another. The goal of this section is to model the Chua's circuit and its resonator circuit.

Figure 1 shows the Chua's circuit, where the component values employed in its construction is set to have a chaotic behavior. For more information about Chua's circuit see [12]. The mathematic model is described by the following set of equations:

$$\begin{array}{l}\dot{V_1} = \frac{1}{RC_1}V_2 - \frac{1}{RC_1}V_1 - \frac{1}{C_1}I_X,\\ \dot{V_2} = \frac{1}{RC_2}V_1 - \frac{1}{RC_2}V_2 + \frac{1}{C_2}I_L,\\ \dot{I_L} = -\frac{1}{L}V_2,\\ I_X = m_1V_1 - \frac{1}{2}(m_0 - m_1)[|V_1 + V_D| - |V_1 - V_D|],\end{array} \tag{1}$$

where $m_0 = -\frac{R_2}{R_1R_3}$, $m_1 = -\frac{R_2}{R_1R_3} + \frac{1}{R_0}$, $C_1 = 10\ nF$, $C_2 = 100\ nF$, $L = 18\ mH$, $R = 1.8\ k\Omega$, $R_1 = 750\ \Omega$, $R_2 = 220\ \Omega$, $R_3 = 220\ \Omega$, and $R_0 = 1.2\ k\Omega$ are parameter. For these parameter values we have a double scroll, which is shown in Fig. 2, projected onto the planes a) (V_1,V_2), b) (V_1,i_L), and c) (V_2,i_L).

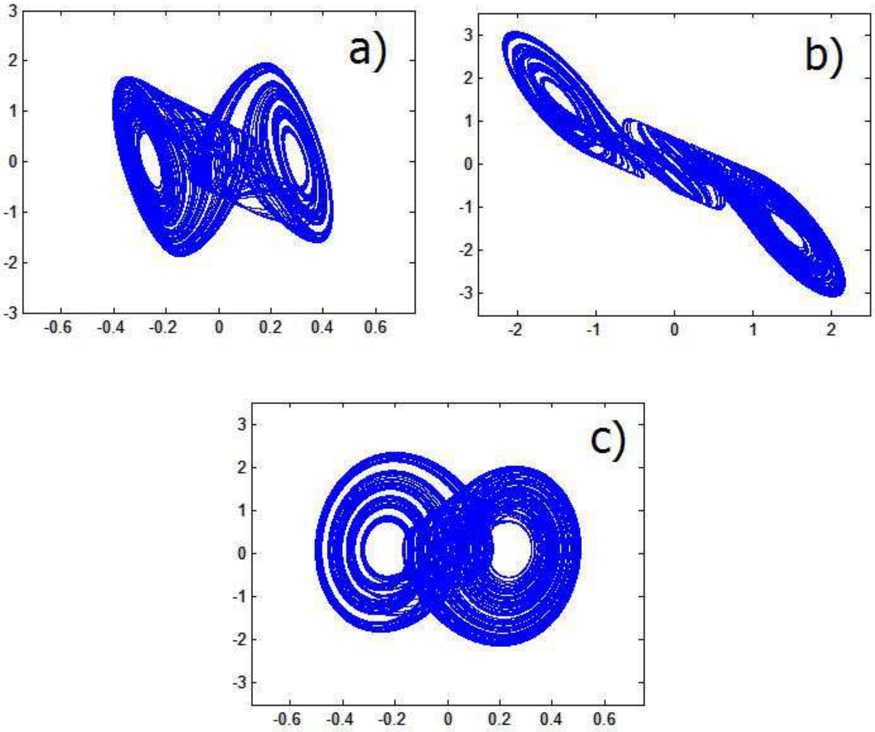

Fig. 2. Chua's circuit phase projections. a) Projection on the plane $V_1 - V_2$. b) Projection on the plane $V_1 - I_L$. c) Projection on the plane $V_2 - I_L$.

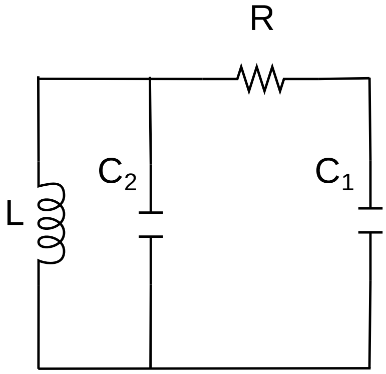

Fig. 3. Chua's circuit RLC grid.

2.1. Chua's circuit resonance frequency

Resonance is the tendency of a system to oscillate with greater amplitude in a specific frequency than others. The way to find analytically the resonance frequency is that the system reactance goes to zero, i.e., the impedance behaves as purely resistive.

In Fig. 3 the RLC network of the Chua's circuit is shown, and its impedance is given by:

$$Z_T = \frac{RLC_2s^2 + Ls + R}{RLC_1C_2s^3 + L(C_1 + C_2)s^2 + RC_1s + 1}, \tag{2}$$

and the reactance of Eq. 2 is given as follows:

$$X(\omega) = \frac{R(1-\omega^2 LC_2)(\omega^3 RLC_1C_2-\omega RC_1)+\omega L(1-\omega^2 L(C_1+C_2))}{(1-\omega^2 L(C_1+C_2))^2+(\omega^3 RLC_1C_2-\omega RC_1)^2}, \tag{3}$$

thus its resonance frequency is given by:

$$\begin{aligned}\omega_1 &= \pm 16\ 666\ rad/s,\\ \omega_2 &= \pm 22\ 222\ rad/s,\\ \omega_3 &= 0\ rad/s.\end{aligned} \tag{4}$$

When the Chua's diode is added to RLC network, a natural question that arises is the following: what frequency given by (4) prevail?. We analyze this situation in the next section.

3. Chua's Filter

In Section 2 the Chua's circuit mathematical model (1) that describes its behavior was given, now in this section we look the circuit (Fig. 1) as a two-port system, where V_1 and V_2 are its input and output, respectively. With this in mind, we can write the following equations for nodes 1 and 2:

$$\begin{aligned}V_1 &= \left(\frac{V_2-RI_X}{1+RC_1 s}\right),\\ V_1 &= V_2\left(\frac{Ls+RLC_2s^2-R}{Ls}\right),\end{aligned} \tag{5}$$

where from

$$V_2\left(\frac{RLC_1C_2s^3+L(C_1+C_2)s^2+RC_1s-1}{Ls}\right) = -I_X. \tag{6}$$

The transfer function can be given by Eq. 6, and taking into account that I_X is a piecewise linear function determined by (1). By considering this, we have three functions for the following cases:

case I: $V_1 < -V_D$

$$\frac{V_2}{V_1} = \frac{(R_0R_2+R_1R_3)Ls}{R\alpha LC_1s^3+\alpha L(C_1+C_2)s^2+(R_1R_3LV_D-R\alpha C_1)s-\alpha},$$

case II: $|V_1| < V_D$

$$\frac{V_2}{V_1} = \frac{R_2Ls}{RR_1R_3C_1C_2s^3+R_1R_3L(C_1+C_2)s^2-RR_1R_3C_1s-R_1R_3},$$

case III: $V_1 > V_D$

$$\frac{V_2}{V_1} = \frac{(R_0R_2+R_1R_3)Ls}{R\alpha LC_1s^3+\alpha L(C_1+C_2)s^2-(R_1R_3LV_D+R\alpha C_1)s-\alpha},$$

$\alpha = R_0R_1R_3.$

With the above expressions, especially considering circuit parameters and $V_D = 0.7$, the following transfer functions are obtained:

$$\begin{aligned}
&\textbf{case I:}\\
&H(s) = \frac{1782s}{6.415\times10^{-6}s^3+0.392s^2-1485s-1.98\times10^8},\\
&\textbf{case II:}\\
&H(s) = \frac{3.96s}{5.346\times10^{-9}s^3+3.267\times10^{-4}s^2-2.97s-165\times10^8},\\
&\textbf{case III:}\\
&H(s) = \frac{1782s}{6.415\times10^{-6}s^3+0.392s^2-5643s-1.98\times10^8}.
\end{aligned} \tag{7}$$

Fig. 4 shows Bode diagrams of Eq. 7, in this figure we can see a bandpass filter response with cutoff frequency around 3.5 kHz and corresponds to the expected response of the Chua's circuit (ω_2). A noticeable aspect of these cases is the gain difference; for each case Chua's circuit oscillates chaotic, therefore, it is expected that response magnitude of the filter does not have a fixed gain, i.e. it oscillates between a peak and a trough.

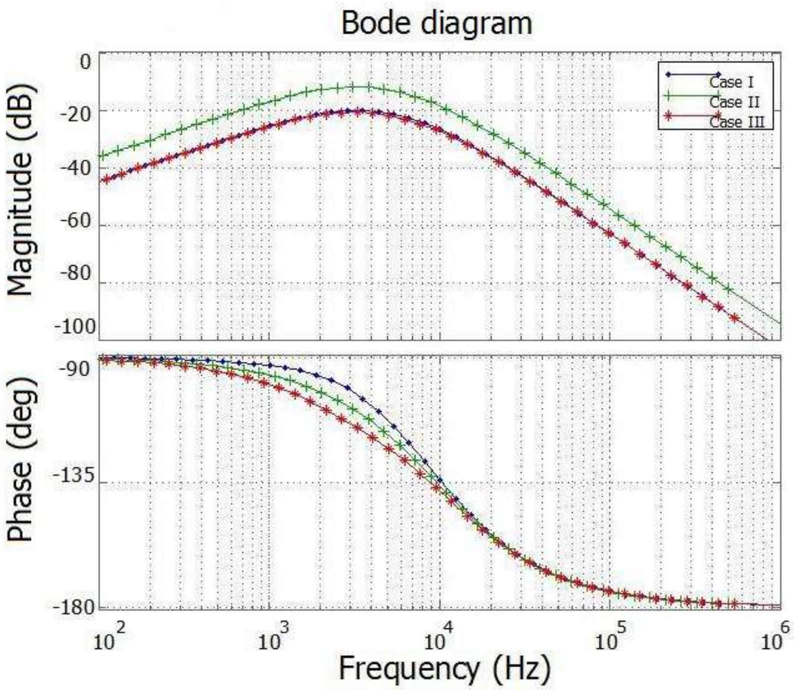

Fig. 4. Bode diagram for equation 7.

In the next section we show experimentally this $3.5kHz$ frequency. Also the Chua's circuit is analyzed as a filter based on its electronic implementation.

4. Experimental Results

The electronic circuit was implemented on phenolic board, component values are described in Section 2, we have used power supplies of ±15V, 3A model PS280 of

Tektronix Company, to capture the signal an oscilloscope TDS2004B of Tektronix Company has been employed, to generate an external signal a signal generator DG 2041A of Rigol Company. And an analog oscilloscope model 2190B of the BK Precision Company was used to observe double scroll oscillations (Fig.5).

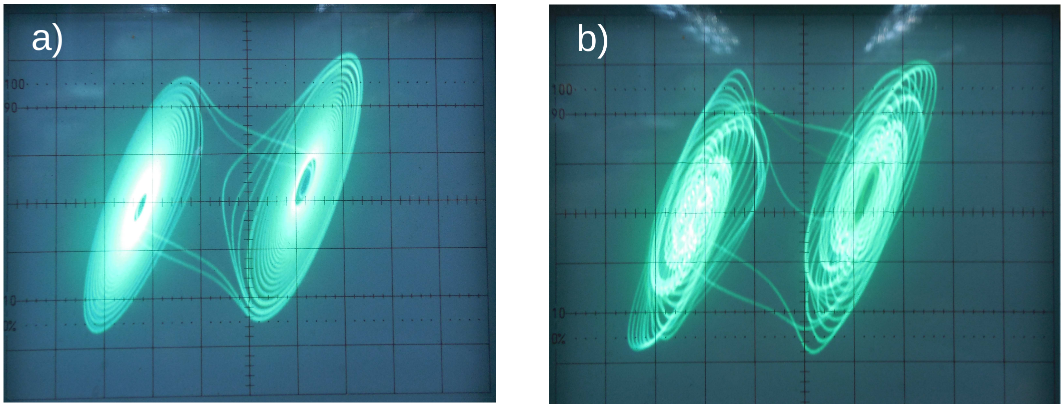

Fig. 5. Double scroll oscillation: a) Without external signal applied, b) with external sinusoidal signal applied.

In order to experimentally verify the results previously computed in Sec.2.1 about the resonance frequency of the RLC Chua's circuit, we analyzed the V_2 signal to zero input response, with the embedded FFT algorithm on TDS2004B oscilloscope. The result is shown in Fig. 6.

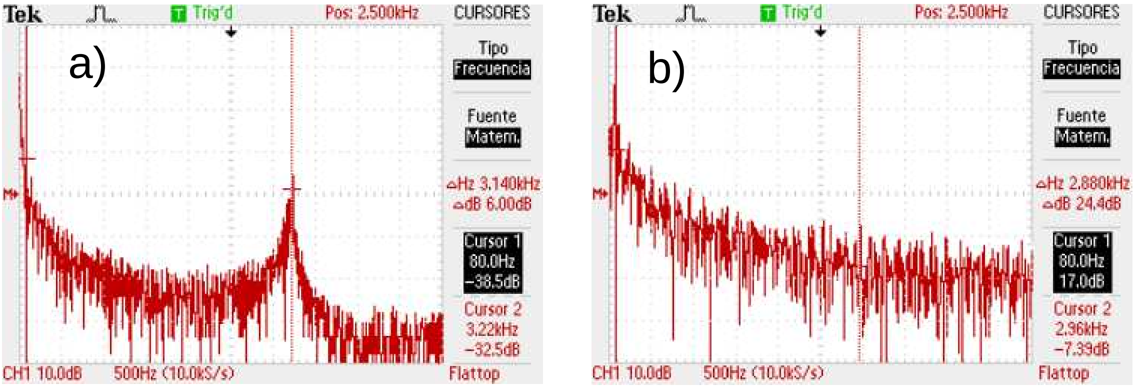

Fig. 6. Resonance frequency: a) Frequency component at 3.22 kHz and energy concentration in bands below 200 Hz, b) energy concentration below 200 Hz.

In this Fig. 6 there is a frequency component at 3.22 kHz and energy concentration in bands below 200 Hz, one of them is due to the resonant frequency of RLC circuit, the other is due to switching between circuit diodes in Fig. 1. To determine which case corresponds to each observed frequency band, we passed the V_2 state signal through a zero crossing detector, shown in Fig. 7a). Its response is shown in Fig. 7b) in yellow, and its FFT response is shown in Fig. 6b). From this, we can deduce that low-frequency components correspond to a frequency which the signal switches between its two equilibrium points given by the diode's potential barrier, while component of 3.2 kHz corresponds to RLC network resonance frequency.

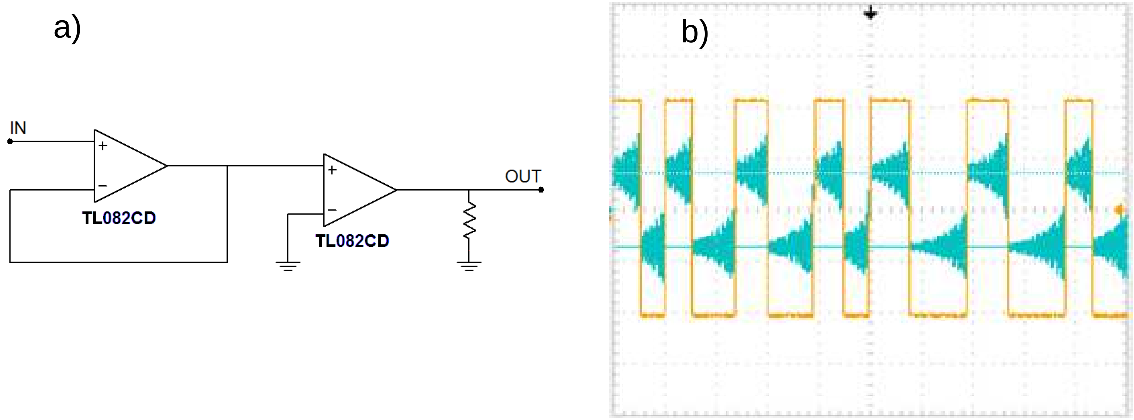

Fig. 7. Zero crossing detector: a)circuit b) the yellow signal is the circuit output

The methodology for Chua's circuit non-linear filter characterization, consists on introduce a signal at node V_1 via a buffer to ensure a good coupling to the system. A sinusoidal signal of 2 Vpp was injected into the frequency range from 10 Hz to 20 kHz with logarithmic increments. Fig. 5b) shows the Chua's attractor disturbed by a sinusoidal signal of 6 kHz. Once the signal is applied, this is monitored by the spectrum analyzer as shown in Fig. 8a), where its spectrum embedded in V_1 is shown and its output response is observed in Fig. 8b) at node V_2.

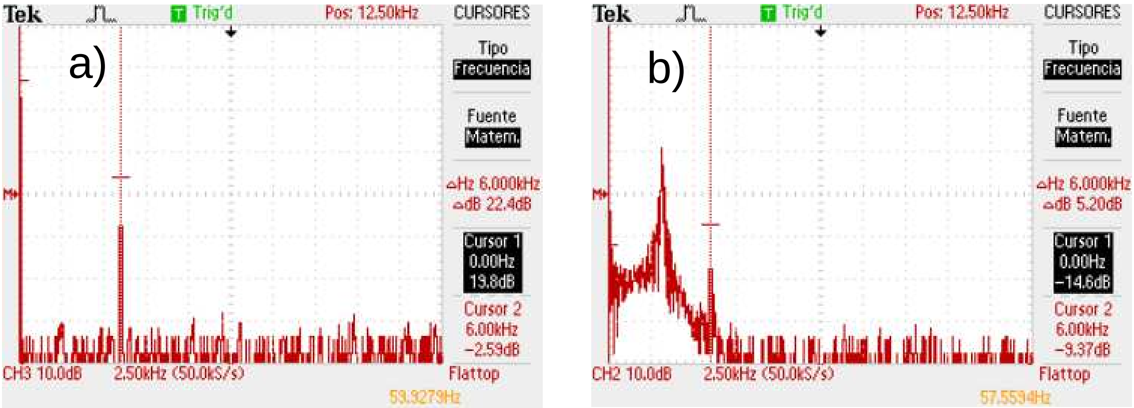

Fig. 8. Signal spectrum: a) in V1 (input), b) in V2 (output).

This procedure was repeated for each frequency within a logarithmic scale of 20 Hz to 20 kHz. In Table 1 data obtained are shown.

During response measurement to each observed frequency the system has values ranging between a maximum and a minimum. This maximum and minimum obtained are shown in Table 1, plus an average between these two. To observe the filter response a semi-logarithmic scale were plotted data shown in Fig. 9. From measurements taken, we can conclude that Chua's circuit has a bandpass filter response when the output

Table 1. Results obtained on nonlinear filter characterization at V_2 node.

Frequency Hz	Max. dB	Min. dB	Prom. dB
10	-11.8	-15	-13.4
20	-8.57	-13	-10.785
30	-4.57	-7.77	-6.17
40	-4.87	-5.37	-5.12
50	-4.97	-5.77	-5.37
60	-4.17	-6.17	-5.17
70	-4.57	-6.17	-5.37
80	-4.57	-6.17	-5.37
90	-4.97	-6.97	-5.97
100	-4.57	-8.17	-6.37
200	-7.37	-13	-10.185
300	-7.77	-15	-11.385
400	-10.6	-11.4	-11
500	-9.37	-11	-10.185
600	-8.57	-13.4	-10.985
700	-9.37	-13	-11.185
800	-8.97	-13	-10.985
900	-6.17	-9.77	-7.97
1000	-8.57	-10.6	-9.585
2000	0.631	-2.17	-0.7695
3000	12.2	10.6	11.4
4000	1.43	0.6	1.015
5000	-4.97	-5.37	-5.17
6000	-8.97	-9.97	-9.47
7000	-11.8	-14.2	-13
8000	-14.2	-15	-14.6
9000	-15.4	-17	-16.2
10000	-17	-19	-18
20000	-29	-31	-30

signal is observed at node V_2. With a bandwidth of 4 kHz between frequencies ranging from 1500 Hz to 5500 Hz, besides having the ability to amplify.

5. Conclusions

The possibility of using Chua's circuit as a filter was analyzed in this paper; it was implemented physically, and the RLC circuit response is displayed, in the same way the complete circuit response. Its theoretic behavior was experimentally found. Finally,

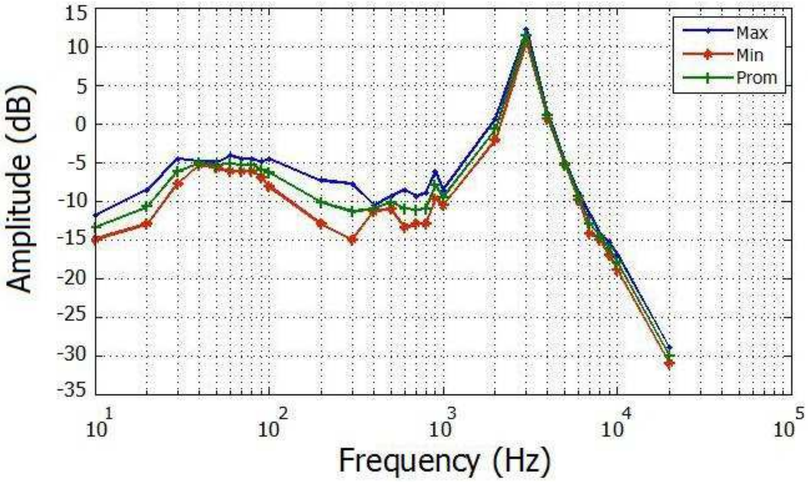

Fig. 9. Frequency response of bandpass nonlinear Chua filter.

we show the methodology under which frequency response are obtained of a non-linear filter. Based on these results, it was shown that Chua's circuit has a band pass filter behavior if we observe the V_2 state as an output. We also observed that around a $3kHz$ frequency we have 12 dB gain; this fact indicates the possibility to amplify frequency in a band between 2 kHz to 4 kHz. Therefore, we could filtering and amplifying in the same circuit, at the same time, and not needed two separate circuits.

This work contributes to help students to understand the filtered process in electronic systems. As future work, we propose to modify Chua's circuit to increase its resonant frequency. This is due to the importance of bandwidth in communication systems, which determines the type and amount of information that can be transmitted. It is also of interest characterized other chaotic oscillators as filters, specifically Lorenz's family oscillators.

acknowledgements

O. A. Segura-Cisneros is a master fellows of CONACYT (Mexico) in the Graduate Program on Applied Science at IICO-UASLP.

E. Campos-Cantón has been supported by the CONACYT through the research grant no. 181002..

References

[1] Gierkink S L J, Levantino S, Frye R C, Samori C, Boccuzzi V 2003 A low-phase-noise 5-GHz CMOS quadrature VCO using superharmonic coupling, *IEEE J. Solid-State Circuits* **38**, 1148-1154.

[2] Razavi B 1999 RF transmitter architectures and circuits, *Proceedings of the Custom Integrated Circuits Conference*, 197-204.

[3] Kuperman M N, Zanette D H 2009 Synchronization of multi-phase oscillators: an Axelrod-inspired model, *Eur. Phys. J.* **70**, 243-248.

[4] Chui W Y, Horng J W 2007 High-Input and Low-Output Impedance Voltage-Mode Universal Biquadratic Filter Using DDCCs, *IEEE Trans. Circ. Syst. II: Exp. Briefs* **54**, 649-652.

[5] Minaei S, Ibrahim M A 2009 A mixed-mode KHN-biquad using DVCC and grounded passive elements suitable for direct cascading, *Int. J. Circuits Theor. Appl.* **37**, 793-810.

[6] González-Salas J S, Campos-Cantón E, Ordaz-Salazar F C, Jiménez-López E 2013 Nonlinear filtering preserves chaotic synchronization via master-slave system, *Abstract and Applied Analysis*, 398293.

[7] Chua Leon O 1992 The Genesis of Chua's Circuit, *AEÜ* **46**, 250-257.

[8] Campos Cantón I 2009 Sistema de comunicación seguro usando señales caóticas, *RIIT* **X**, 21-27.

[9] Ren H, Baptista M, Grebogi C 2013 Wireless Communication with Chaos, *Phys. Rev. Lett.* **110**, 184101.

[10] Grigoras V 2011 Chaotic noise generators with periodic nonlinearities, *Signals, Circuits and Systems (ISSCS)* 10th International Symposium.

[11] Application Note 779 2011 *A Basic Introduction to Filters Active, Passive, and Switched Capacitor* Texas Instruments Incorporated.

[12] Chua L O, Kocarev L, Eckert K, Itoh M 1992 Experimental Chaos Synchronization in Chua's Circuit, *Int. J. Bifur. & chaos* **12**, 705-708.