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Output-Feedback Adaptive Control for the Global Regulation of Robot Manipulators with Bounded Inputs

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Abstract: In this paper, an output-feedback adaptive scheme for the global position stabilization of robot manipulators with bounded inputs is proposed. Compared to the previous output-feedback adaptive approaches developed in a bounded-input context, the proposed free-of-velocity feedback controller guarantees the adaptive regulation objective: globally, avoiding discontinuities throughout the scheme, preventing the inputs to reach their natural saturation bounds, and imposing no *saturation-avoidance* restriction on the control gains. Moreover, the developed scheme is not restricted to the use of a specific saturation function to achieve the required boundedness, but may involve any one within a set of smooth and non-smooth (Lipschitz-continuous) bounded passive functions that include the hyperbolic tangent and the conventional saturation as particular cases. Experimental results corroborate the efficiency of the proposed scheme.

Keywords: Adaptive control, output feedback, global regulation, bounded inputs, robot manipulators.

1. INTRODUCTION

Since the publication of [1], the *Proportional-Derivative with gravity compensation* (PDgc) controller has proved to be a useful technique for the regulation of robot manipulators. In its original form, it achieves the global stabilization objective under ideal conditions, for instance: unconstrained input, availability of all the link positions and velocities, and exact knowledge of the system parameters. Unfortunately, in actual applications, such underlying assumptions are not generally satisfied, giving rise to unexpected or undesirable effects like input saturation and those related to such a nonlinear phenomenon [2], noisy responses and/or deteriorated performance [3], or steady-state errors [4]. However, such inconveniences have not necessarily rendered useless the PDgc technique. Inspired by this control method, researchers have developed alternative (nonlinear or dynamic) PDgc-based approaches that deal with the limitations on the actuator capabilities and/or on the available system data, while keeping the natural energy properties of the original PDgc controller: definition of a unique arbitrarily-located closed-loop equilibrium configuration and motion dissipation. For instance, extensions of the PDgc controller that cope with the input saturation phenomenon have been developed under various analytical frameworks in [5, 6, 7, 8, 9, 10]. Indeed, assuming the availability of the exact value of all the system parameters and accurate measurements of all the link positions and velocities, a bounded PDgc-based approach was proposed in [5] and [6]. In these works, the P and D terms (at every joint) are, each of them, explicitly bounded through specific *saturation functions*; a continuously differentiable one —more precisely, the hyperbolic tangent function— is used in [5] and the conventional non-smooth one in [6]. In view of their structure, this type of algorithms have been denoted *SP-SD* controllers in [11]. Further, two alternative schemes, that prove to be simpler and/or give rise to improved closed-loop performances, were recently proposed in [7]. The first approach includes both the P and D actions (at every joint) within a single saturation function, while in the second one all the terms of the controller (P, D, and gravity compensation) are covered by one of such functions, with the P terms internally embedded within an additional saturation; the exclusive use of a single saturation (at every joint) including all the terms of the controller was further achieved through desired gravity compensation in [12]. Moreover, free-of-velocity-measurement versions of the SP-SD controllers in [6] and [5] —still depending on the exact values of the system parameters— are obtained through the design methodologies developed in [8] and [9]. In [8], global regulation is proved to be achieved when each velocity measurement is replaced by the *dirty derivative* [13] of the respective position in the SP-SD controller of [6]. A similar replacement in a more general form of the SP-SD controller is proved to achieve global regulation through

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the design procedure proposed in [9] (where an alternative type of dirty derivative, that involves a saturation function in the auxiliary dynamics that gives rise to the estimated velocity, results from the application of the proposed methodology). Furthermore, an output-feedback dynamic controller with a structure similar to that resulting from the methodology in [9], but which considers a single saturation function (at every joint) where both the position errors and velocity estimation states are involved, was proposed in [10] (where a *dissipative* linear term on the auxiliary state is added to the saturating velocity error dynamics involved for the dirty derivative calculation).

Further, SP-SD-type adaptive algorithms that give rise to bounded controllers while alleviating the system parameter dependence of the gravity compensation term have been developed in [14, 15, 16]. In [14], global regulation is aimed through a discontinuous scheme that switches among two different control laws, under the consideration of state and output feedback. Both considered control laws keep an SP-SD structure similar to that of [6]; the first one avoids gravity compensation taking high-valued control gains (by means of which the closed-loop trajectories are lead close to the desired configuration), and the second one considers adaptive gravity compensation terms that are kept bounded by means of a discontinuous auxiliary dynamics. Each velocity measurement is replaced by the dirty derivative of the corresponding position in the output-feedback version of the proposed algorithm. Unfortunately, a precise criterion to determine the switching moment (from the first control law to the second one) is not furnished for either of the developed schemes.

In [15], semiglobal regulation is achieved through a state feedback scheme that keeps the SP-SD structure of [5] but additionally considers adaptive gravity compensation. The adaptation algorithm is defined in terms of a discontinuous auxiliary dynamics by means of which the parameter estimators are prevented to take values beyond some pre-specified limits, which consequently keeps the adaptive gravity compensation terms bounded.

In [16], a controller that keeps the SP-SD structure of [5] is proposed, where each velocity measurement is replaced by the dirty derivative of the corresponding position, and an adaptive gravity compensation term, with initial-condition-dependent bounds, is considered. Based on the proof developed for the main result, semiglobal regulation is claimed to be achieved.

Let us note that by the way the SP and SD terms are defined in the above mentioned adaptive schemes, the bound of the control signal at every link turns out to be defined in terms of the sum of the P and D control gains. This limits the choice of such gains if the natural actuator bounds (or arbitrary input bounds) are aimed to be avoided. This, in turn, restricts the closed-loop region of attraction in the semiglobal stabilization cases. On the other hand, as far as the authors are aware, the semiglobal and/or discontinuous approaches developed in [16] and [14] are the only output-feedback bounded adaptive algorithms proposed in the literature. Moreover, a continuous adaptive scheme, with continuous auxiliary dynamics, that achieves the regulation objective globally, avoiding input saturation, and disregarding velocity measurements in the feedback, is still missing in the literature, and consequently remains an open problem. These arguments have motivated the present work which aims at filling in the mentioned gap.

Let us further note that previous works involving adaptive control schemes where the parameter estimates are aimed to remain bounded within pre-specified values generally appeal to a discontinuous adaptation dynamics of the kind of those used in [15] and [14]. This is seen even in recent works [17, 18, 19]. The discontinuous character of such type of adaptation auxiliary dynamics is not necessarily a drawback, but a bounded adaptive scheme that avoids discontinuities constitutes a convenient alternative developed within a simpler analytical context and through simpler and/or more natural ways to cope with the need to bound the parameter estimates. This is achieved through the approach proposed in this work by considering the parameter estimators to be the output variables of the adaptation subsystem instead of assigning them the role of auxiliary states. The adaptation subsystem states are in turn liberated from having to be initiated and evolve within a constrained subset rendering the proposed approach globally stabilizing in an authentic sense. Indeed, all the closed-loop system states, including those involved in the auxiliary adaptation dynamics, can be initiated anywhere. Thus, through its authentic globally stabilizing and continuous characters, the proposed approach overcomes the limitations of the previous output-feedback bounded adaptive regulation approaches. Let us further note that the scheme developed in this work achieves the necessary exact integration to adopt a structure that does not involve velocity measurements. The work in [16] avoids this problem by involving a constant gravity-related regression matrix in the adaptation auxiliary state equation. However, the resulting parameter-estimate bounds of the controller in [16] are not arbitrarily fixed, giving a partial solution to the constrained-input problem. This is suitably solved through the structure adopted by the adaptive subsystem designed in this work. Furthermore, as far as the authors are aware, the approach developed in this paper is the first output-feedback adaptive scheme that achieves the regulation task globally through a dynamic control algorithm with non-varying structure, overcoming the semiglobal character of the result in [16] as well as the varying structure nature and analytical complications of the approach presented in [14].

Let us further add that, within the bounded-input context, recent works have focused on global tracking through state feedback [20] and output feedback [21]. Such approaches were designed assuming exact knowledge of the system parameters. The consequences of such an assumption have actually been shown and commented for instance in [20]. In view of its effectiveness, the dirty-derivative-based velocity estimation algorithm of [21] has been included as part of the scheme developed in this work (just as the previously cited works on output-feedback control include the standard dirty derivative or some nonlinear version of this technique). Notwithstanding, this study solves the global regulation problem considering that the exact values of the system parameters are unavailable. The results obtained in this work show that the main drawback of the conventional (*exact*) gravity-compensation-based algorithms is eliminated through the proposed output-feedback adaptive approach.

In this paper, we propose an output-feedback adaptive scheme for the global regulation of robot manipulators with saturating inputs. It gives rise to a family of bounded continuous output-feedback adaptive SP-SD-type controllers that include continuous auxiliary adaptation and velocity-estimation dynamics. Moreover, the structure of the proposed scheme permits the P and D control gains to take any positive value while guaranteeing input saturation avoidance and global position stabilization. With respect to the previous output-feedback adaptive approaches developed in a bounded-input context, the proposed free-of-velocity feedback controller guarantees the adaptive regulation objective: globally, avoiding discontinuities throughout the scheme, preventing the inputs to attain their natural saturation bounds, and imposing no *saturation-avoidance* restriction on the choice of the P and D control gains. Furthermore, contrarily to the adaptive schemes of the previously cited studies, the approach proposed in this work is not restricted to the use of a specific saturation function to achieve the required boundedness, but may rather involve any one within a set of smooth and non-smooth (Lipschitz-continuous) bounded passive functions that include the hyperbolic tangent and the conventional saturation as particular cases. Experimental results corroborate the proposed contribution.

2. PRELIMINARIES

Let $X \in \mathbb{R}^{m\times n}$ and $y \in \mathbb{R}^n$. X_{ij} denotes the element of X at its i^{th} row and j^{th} column, X_i represents the i^{th} row of X, and y_i stands for the i^{th} element of y. 0_n represents the origin of $\mathbb{R}^n$ and I_n the $n\times n$ identity matrix. $\|\cdot\|$ denotes the standard Euclidean norm for vectors, *i.e.* $\|y\| = \sqrt{\sum_{i=0}^{n} y_i^2}$, and induced norm for matrices, *i.e.* $\|X\| = \sqrt{\lambda_{\max}(X^TX)}$, where $\lambda_{\max}(X^TX)$ represents the maximum eigenvalue of X^TX. The kernel of X is denoted $\ker(X)$ while, for $m = n$, $\det(X)$ denotes the determinant of X. Consider a continuously differentiable scalar function $\zeta : \mathbb{R} \to \mathbb{R}$ and a locally Lipschitz-continuous scalar function $\phi : \mathbb{R} \to \mathbb{R}$, both vanishing at zero, *i.e.* $\zeta(0) = \phi(0) = 0$. Let ζ' denote the derivative of ζ with respect to its argument, and $D^+\phi$ stand for the upper-right (Dini) derivative of ϕ, *i.e.* $D^+\phi(\varsigma) = \limsup_{h\to 0^+} \frac{\phi(\varsigma+h)-\phi(\varsigma)}{h}$ [22, App. A.1] [23, App. I]. Thus $\phi(\varsigma) = \int_0^{\varsigma} D^+\phi(r)dr$; moreover, $(\zeta\circ\phi)(\varsigma) = \zeta(\phi(\varsigma)) = \int_0^{\varsigma} \zeta'(\phi(r))D^+\phi(r)dr$.

Let us consider the general n-degree-of-freedom (n-DOF) serial rigid manipulator dynamics with viscous friction [24]:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau \tag{1}$$

where q, $\dot{q}$, $\ddot{q} \in \mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity, and acceleration vectors, $H(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, and $C(q,\dot{q})\dot{q}$, $F\dot{q}$, $g(q)$, $\tau \in \mathbb{R}^n$ are, respectively, the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with $F \in \mathbb{R}^{n\times n}$ being a positive definite constant diagonal matrix whose entries $f_i > 0$, $i = 1,\dots,n$, are the viscous friction coefficients. Some well-known properties characterizing the terms of such a dynamical model are recalled here (see for instance [25, Chap. 4]; see further [25, Chap. 14] and [26] concerning Property 3 below); the most commonly well-known are synthesized and gathered together in Property 1.

Property 1: $H(q)$, $C(q,\dot{q})$, and F satisfy:

1a. $\mu_m I_n \leq H(q) \leq \mu_M I_n$, $\forall q \in \mathbb{R}^n$, for some positive constants $\mu_m \leq \mu_M$.

1b. $\|C(q,\dot{q})\| \leq k_c\|\dot{q}\|$, $\forall (q,\dot{q}) \in \mathbb{R}^n\times\mathbb{R}^n$, for some constant $k_c \geq 0$.

1c. With $\dot{H} \triangleq \frac{d}{dt}H$: $\dot{q}^T\left[\frac{1}{2}\dot{H}(q,\dot{q}) - C(q,\dot{q})\right]\dot{q} = 0$, $\forall (q,\dot{q}) \in \mathbb{R}^n\times\mathbb{R}^n$, and actually $\dot{H}(q,\dot{q}) = C(q,\dot{q}) + C^T(q,\dot{q})$.

1d. $f_m\|\dot{q}\|^2 \leq \dot{q}^TF\dot{q} \leq f_M\|\dot{q}\|^2$, $\forall \dot{q} \in \mathbb{R}^n$, where $0 < f_m \triangleq \min_i\{f_i\} \leq \max_i\{f_i\} \triangleq f_M$.

Property 2: The gravity vector is bounded on $\mathbb{R}^n$, or equivalently, every element of the gravity vector, $g_i(q)$, $i = 1,\dots,n$, satisfies $|g_i(q)| \leq B_{gi}$, $\forall q \in \mathbb{R}^n$, for some positive constants B_{gi}, $i = 1,\dots,n$.[1]

Property 3: The gravity vector can be rewritten as $g(q,\theta) = G(q)\theta$, where $\theta \in \mathbb{R}^p$ is a constant vector whose elements depend exclusively on the system parameters, and $G(q) \in \mathbb{R}^{n\times p}$ —the regression matrix— is a continuous matrix function whose elements depend exclusively on the configuration variables and do not involve any of the system parameters. Equivalently, the potential energy function of the robot can be rewritten as $U(q,\theta) = \Upsilon(q)\theta$, where $\Upsilon(q) \in \mathbb{R}^{1\times p}$ —the regression vector— is a continuous row vector function whose elements depend exclusively on the configuration variables and do not involve any of the system parameters. Actually, $G^T(q) = \frac{\partial}{\partial q}\Upsilon^T(q)$, or equivalently, $\Upsilon_j(q) = \sum_{i=1}^{n}\int_{q_i^*}^{q_i} G_{ij}(q_1,\dots,q_{i-1},r_i,q_{i+1}^*,$

[1] Property 2 is not satisfied by all types of robot manipulators but it is for instance by those having only revolute joints [25, §4.3]. This work is addressed to robots satisfying Property 2.

$\dots, q_n^*)dr_i$, $\forall j \in \{1,\dots,p\}$, with $q^* = (q_1^*,\dots,q_n^*)^T$ being the reference configuration where $U(q^*,\theta) = 0$.[2]

Property 4: Consider the gravity vector $g(q,\theta)$. Let θ_{Mj} represent an upper bound of θ_j, such that $\theta_j \leq \theta_{Mj}$, $\forall j \in \{1,\dots,p\}$, and let $\theta_M \triangleq \left(\theta_{M1},\dots,\theta_{Mp}\right)^T$ and $\Theta \triangleq [-\theta_{M1},\theta_{M1}] \times \cdots \times [-\theta_{Mp},\theta_{Mp}]$. By Properties 2 and 3, there exist positive constants $B_{gi}^{\theta_M} \geq B_{gi}$, $i = 1,\dots,n$, such that $|g_i(x,y)| = |G_i(x)y| \leq B_{gi}^{\theta_M}$, $i = 1,\dots,n$, $\forall x \in \mathbb{R}^n$, $\forall y \in \Theta$. Furthermore, there exist positive constants $B_{G_{ij}}$, B_{G_i}, and B_G such that $|G_{ij}(x)| \leq B_{G_{ij}}$, $\|G_i(x)\| \leq B_{G_i}$, and $\|G(x)\| \leq B_G$, $\forall x \in \mathbb{R}^n$, $i = 1,\dots,n$, $j = 1,\dots,p$.

Let us suppose that the absolute value of each input τ_i (i^{th} element of the input vector τ) is constrained to be smaller than a given saturation bound $T_i > 0$, *i.e.* $|\tau_i| \leq T_i$, $i = 1,\dots,n$. In other words, letting u_i represent the control signal (controller output) relative to the i^{th} degree of freedom, we have that

$$\tau_i = T_i \text{sat}\left(\frac{u_i}{T_i}\right) \tag{2}$$

$i = 1,\dots,n$, where sat$(\cdot)$ is the standard saturation function, *i.e.* $\text{sat}(\varsigma) = \text{sign}(\varsigma)\min\{|\varsigma|,1\}$.

Let us note from (1)-(2) that $T_i \geq B_{gi}$ (see Property 2), $\forall i \in \{1,\dots,n\}$, is a necessary condition for the manipulator to be stabilizable at any desired equilibrium configuration $q_d \in \mathbb{R}^n$. Thus, the following assumption turns out to be crucial within the analytical setting considered here:

Assumption 1: $T_i > B_{gi}$, $\forall i \in \{1,\dots,n\}$.

The control scheme proposed in this work involves special functions fitting the following definition.

Definition 1: Given a positive constant M, a nondecreasing Lipschitz-continuous function $\sigma : \mathbb{R} \to \mathbb{R}$ is said to be a ***generalized saturation*** with bound M if

(a) $\varsigma\sigma(\varsigma) > 0$ for all $\varsigma \neq 0$;
(b) $|\sigma(\varsigma)| \leq M$ for all $\varsigma \in \mathbb{R}$.

Functions meeting Definition 1 own the next properties.

Lemma 1: Let $\sigma : \mathbb{R} \to \mathbb{R}$ be a generalized saturation function with bound M, and k be a positive constant. Then

1. $\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) = 0$;
2. $\exists \sigma_M' \in (0,\infty)$ such that $0 \leq D^+\sigma(\varsigma) \leq \sigma_M'$, $\forall \varsigma \in \mathbb{R}$;
3. $\frac{\sigma^2(k\varsigma)}{2k\sigma_M'} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma_M'\varsigma^2}{2}$, $\forall \varsigma \in \mathbb{R}$;
4. $\int_0^\varsigma \sigma(kr)dr > 0$, $\forall \varsigma \neq 0$;
5. $\int_0^\varsigma \sigma(kr)dr \to \infty$ as $|\varsigma| \to \infty$;
6. if σ is strictly increasing, then, for any constant a, $\bar{\sigma}(\varsigma) = \sigma(\varsigma + a) - \sigma(a)$ is a strictly increasing generalized saturation function with bound $\bar{M} = M + |\sigma(a)|$.

Proof: See [27]. □

[2] The reference configuration q^* is the generalized position with respect to which $U(q,\theta)$ is quantified. In other words, $U(q,\theta)$ represents the amount of work needed to relocate the system configuration at q departing from q^*.

3. THE PROPOSED CONTROLLER

Let $M_a \triangleq \left(M_{a1},\dots,M_{ap}\right)^T$ and $\Theta_a \triangleq [-M_{a1},M_{a1}] \times \cdots \times [-M_{ap},M_{ap}]$, with M_{aj}, $j = 1,\dots,p$, being positive constants such that

$$\theta_j < M_{aj} \tag{3a}$$

$\forall j \in \{1,\dots,p\}$, and

$$B_{gi}^{M_a} < T_i \tag{3b}$$

$\forall i \in \{1,\dots,n\}$, where, in accordance to Property 4, $B_{gi}^{M_a}$ are positive constants such that $|g_i(x,y)| = |G_i(x)y| \leq B_{gi}^{M_a}$, $i = 1,\dots,n$, $\forall x \in \mathbb{R}^n$, $\forall y \in \Theta_a$. Let us note that Assumption 1 ensures the existence of such positive values M_{aj}, $j = 1,\dots,p$, satisfying inequalities (3). Notice further that inequalities (3b) are satisfied if $\sum_{j=1}^p B_{G_{ij}} M_{aj} < T_i$, $B_{G_i}\|M_a\| < T_i$, or $B_G\|M_a\| < T_i$, $i = 1,\dots,n$; as a matter of fact, $\sum_{j=1}^p B_{G_{ij}} M_{aj}$, $B_{G_i}\|M_a\|$, or $B_G\|M_a\|$, may be taken as the value of $B_{gi}^{M_a}$ as long as inequality (3b) is satisfied.

The proposed output-feedback adaptive control scheme is defined as

$$u(q,\vartheta,\hat{\theta}) = -s_P(K_P\bar{q}) - s_D(K_D\vartheta) + G(q)\hat{\theta} \tag{4}$$

where $\bar{q} = q - q_d$, for any constant (desired equilibrium position) vector $q_d \in \mathbb{R}^n$; $G(q)$ is the regression matrix related to the gravity vector, according to Property 3, *i.e.* such that $g(q,\theta) = G(q)\theta$; $K_P \in \mathbb{R}^{n\times n}$ and $K_D \in \mathbb{R}^{n\times n}$ are positive definite diagonal matrices, *i.e.* $K_P = \text{diag}[k_{P1},\dots,k_{Pn}]$ and $K_D = \text{diag}[k_{D1},\dots,k_{Dn}]$ with $k_{Pi} > 0$ and $k_{Di} > 0$ $\forall i = 1,\dots,n$; for all $x \in \mathbb{R}^n$: $s_P(x) = \left(\sigma_{P1}(x_1),\dots,\sigma_{Pn}(x_n)\right)^T$ and $s_D(x) = \left(\sigma_{D1}(x_1),\dots,\sigma_{Dn}(x_n)\right)^T$, with $\sigma_{Pi}(\cdot)$ and $\sigma_{Di}(\cdot)$, $i = 1,\dots,n$, being **generalized saturation functions** with bounds M_{Pi} and M_{Di} such that

$$M_{Pi} + M_{Di} < T_i - B_{gi}^{M_a} \tag{5}$$

$i = 1,\dots,n$;[3] $\vartheta \in \mathbb{R}^n$ (the velocity estimator) and $\hat{\theta} \in \Theta_a \subset \mathbb{R}^p$ (the parameter estimator) are the output vector variables of auxiliary dynamic subsystems defined as

$$\dot{q}_c = -AK_D^{-1}s_D\left(K_D(q_c + B\bar{q})\right) \tag{6a}$$

$$\vartheta = q_c + B\bar{q} \tag{6b}$$

and

$$\dot{\phi}_c = -\varepsilon\Gamma G^T(q)s_P(K_P\bar{q}) \tag{7a}$$

$$\hat{\theta} = s_a\left(\phi_c - \Gamma\Upsilon^T(q)\right) \tag{7b}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$, and $\Gamma \in \mathbb{R}^{p\times p}$ are positive definite diagonal matrices, *i.e.* $A = \text{diag}[a_1,\dots,a_n]$ and $B =$

[3] Note that the satisfaction of inequalities (3) guarantees positivity of the right-hand side of inequalities (5). As will be seen, inequalities (5) constitute the tuning criterion on M_{Pi} and M_{Di} through which the input variables u_i are prevented to reach their natural saturation bound T_i along the closed loop trajectories.

diag$[b_1,\ldots,b_n]$ with $a_i>0$ and $b_i>0$ for all $i=1,\ldots,n$, and $\Gamma=\text{diag}[\gamma_1,\ldots,\gamma_p]$ with $\gamma_j>0$ for all $j=1,\ldots,p$; q_c and ϕ_c are the state vectors of the auxiliary dynamics in Eqs. (6a) and (7a) respectively; $\Upsilon(q)$ is the regression vector related to the potential energy function, according to Property 3, *i.e.* such that $U(q,\theta)=\Upsilon(q)\theta$; for all $x\in\mathbb{R}^n$: $s_a(x)=\left(\sigma_{a1}(x_1),\ldots,\sigma_{ap}(x_p)\right)^T$, with $\sigma_{aj}(\cdot)$, $j=1,\ldots,p$, being **strictly increasing generalized saturation functions** with bounds M_{aj} satisfying inequalities (3); and ε is a positive constant satisfying

$$\varepsilon<\varepsilon_M\triangleq\min\{\varepsilon_0,\varepsilon_1,\varepsilon_2\} \tag{8}$$

where $\varepsilon_0\triangleq\sqrt{\frac{\mu_m}{\mu_M^2\beta_P}}$, $\varepsilon_1\triangleq\frac{f_m}{\beta_M+f_M^2/2}$, $\varepsilon_2\triangleq 2\beta_m$, with $\beta_P\triangleq\max_i\{\sigma'_{PiM}k_{Pi}\}$, $\beta_m\triangleq\min_i\left\{\frac{a_i}{b_ik_{Di}}\right\}$, $\beta_M\triangleq k_cB_P+\mu_M\beta_P$, $B_P\triangleq\sqrt{\sum_{i=0}^n M_{Pi}^2}$, σ'_{PiM} being the positive bound of $D^+\sigma_{Pi}(\cdot)$ in accordance to point 2 of Lemma 1, and μ_m, μ_M, k_c, f_m, and f_M as defined in Property 1.

Remark 1: Observe that the control scheme in (4),(6)-(7) does not involve the exact values of the elements of θ. It only requires the satisfaction of inequalities (3). In other words, only strict bounds M_{aj} of θ_j, $j=1,\ldots,p$, —*i.e.* any set of them satisfying inequalities (3b)— are involved. Notice further that a suitable choice of ε does not require the exact knowledge of the system parameters either. Indeed, observe, on the one hand, that an estimation of the right-hand side of inequality (8) may be obtained by means of upper and lower bounds of the system parameters and viscous friction coefficients (more precisely, nonzero lower bounds of μ_m and f_m, and upper bounds of μ_M, k_c, and f_M; see Property 1). On the other hand, the satisfaction of inequality (8) is not necessary but only sufficient for the closed-loop analysis to hold, as shown in the following section, which permits the consideration of values of ε higher than ε_M (up to certain limit) without destabilizing the closed loop. Note further that the velocity vector $\dot q$ is not involved in any of the expressions in Eqs. (4),(6)-(7) either. ◁

Remark 2: The auxiliary subsystem in Eqs. (6) is an alternative version of the dirty derivative (applied to $\bar q$) involving the saturation vector function $s_D(\cdot)$ in its dynamics. In its conventional form, where the function $s_D(\cdot)$ is not included (or equivalently, which is obtained by replacing $s_D(\cdot)$ in (6a) by the identity function), it leads (through its output variable ϑ) to the derivative of $\bar q$ (or equivalently, to the velocity vector $\dot q$) with every of its components going through a first-order low-pass filter. This is commonly done in practice to bound the high frequency gains, giving rise to a causal (approximated) derivative operator. The consideration of $s_D(\cdot)$ in (6a) proves to be helpful to show the expected stability/convergence closed-loop properties, as will be seen in Section 4. The auxiliary subsystem in Eqs. (7), for its part, is the adaptation algorithm. Its particular form gives rise to parameter estimates evolving within pre-specified limits, avoiding discontinuities throughout its dynamical structure. Let us note that the ε-term in the adaptation subsystem forces q_d to be the unique equilibrium configuration of the closed-loop system. This eliminates the steady-state position error generated by conventional approaches that include *exact* gravity compensation through generally-inexact (or biassed) parameter estimates. Further, inequality (8) states a (sufficient) condition that guarantees the required stability/convergence properties. It is obtained from the closed-loop analysis, by looking for the conditions through which the involved Lyapunov function adopts the required analytical properties. This will be corroborated later on in the following section. ◁

4. CLOSED-LOOP ANALYSIS

Consider system (1)-(2) taking $u=u(q,\vartheta,\hat\theta)$ as defined through (4),(6)-(7). Define the variable transformation

$$\begin{pmatrix}\bar q\\ \vartheta\\ \bar\phi\end{pmatrix}=\begin{pmatrix}q-q_d\\ q_c+B(q-q_d)\\ \phi_c-\Gamma\Upsilon^T(q)-\phi^*\end{pmatrix} \tag{9}$$

with $\phi^*=\left(\phi_1^*,\ldots,\phi_p^*\right)^T$ such that $s_a(\phi^*)=\theta$, or equivalently, $\phi_j^*=\sigma_{aj}^{-1}(\theta_j)$, $j=1,\ldots,p$.[4] Observe that, from the satisfaction of inequalities (3) and (5), we have that $\left|u_i\left(\bar q+q_d,\vartheta,s_a(\bar\phi+\phi^*)\right)\right|\le M_{Pi}+M_{Di}+B_{gi}^{M_a}<T_i$, $i=1,\ldots,n$, $\forall(\bar q,\vartheta,\bar\phi)\in\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n$, whence, in view of (2), one sees that

$$\begin{aligned}T_i>\left|u_i\left(\bar q+q_d,\vartheta,s_a(\bar\phi+\phi^*)\right)\right|=|u_i|=|\tau_i|\ ,\\ i=1,\ldots,n\ ,\ \forall(\bar q,\vartheta,\bar\phi)\in\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\end{aligned} \tag{10}$$

Thus, under the consideration of Property 3 and the variable transformation (9), the closed-loop dynamics adopts the (equivalent) form

$$\begin{aligned}H(q)\ddot q+C(q,\dot q)\dot q+F\dot q\\ =-s_P(K_P\bar q)-s_D\left(K_D\vartheta\right)+G(q)\bar s_a(\bar\phi)\end{aligned} \tag{11a}$$

$$\dot\vartheta=-AK_D^{-1}s_D(K_D\vartheta)+B\dot q \tag{11b}$$

$$\dot{\bar\phi}=-\Gamma G^T(q)\left[\varepsilon s_P(K_P\bar q)+\dot q\right] \tag{11c}$$

where $\bar s_a(\bar\phi)=s_a(\bar\phi+\phi^*)-s_a(\phi^*)$. Observe that, by point 6 of Lemma 1, the elements of $\bar s_a(\bar\phi)$, *i.e.* $\bar\sigma_{aj}(\bar\phi_j)=\sigma_{aj}(\bar\phi_j+\phi_j^*)-\sigma_{aj}(\phi_j^*)$, $j=1,\ldots,p$, turn out to be strictly increasing generalized saturation functions.

Remark 3: Let us note, from Eqs. (11) under stationary conditions: $\ddot q=\dot q=\dot\vartheta=0_n$ and $\dot{\bar\phi}=0_p$, that q_d proves to be the unique equilibrium position of the closed-loop

[4] Notice that their strictly increasing character renders invertible the generalized saturations σ_{aj}, $j=1,\ldots,p$.

system —or equivalently, 0_n is the unique equilibrium position error of the closed loop— while the parameter estimation error equilibrium vector $\bar{\phi}_e$ turns out to be defined by the solutions of the equation $G(q_d)\bar{s}_a(\bar{\phi}_e) = 0_n$, and consequently $\bar{s}_a(\bar{\phi}_e) \in \ker(G(q_d))$. $\triangleleft$

Proposition 1: Consider the closed-loop system in Eqs. (11) under the satisfaction of Assumption 1 and inequalities (3) and (5). Then, for any positive definite diagonal matrices K_P, K_D, A, B, and Γ, and any ε satisfying inequality (8), the trivial solution $(\bar{q},\vartheta,\bar{\phi})(t) \equiv (0_n,0_n,0_p)$ is stable and, for any initial condition $(\bar{q},\dot{q},\vartheta,\bar{\phi})(0) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, $(\bar{q},\vartheta)(t) \to (0_n,0_n)$ as $t \to \infty$, and $\bar{s}_a(\bar{\phi}(t)) \to \ker(G(q_d))$ as $t \to \infty$, with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1,\ldots,n$, $\forall t \geq 0$.

Proof: By (10), one sees that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $\forall t \geq 0$. This proves that under the proposed output-feedback adaptive scheme, the input saturation values, T_i, are never reached. Now, in order to develop the stability/convergence analysis, let us define the scalar function

$$V(\bar{q},\dot{q},\vartheta,\bar{\phi}) = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})H(q)\dot{q} + \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + \int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr + \int_{0_n}^{\vartheta} s_D^T(K_D r)B^{-1}dr \tag{12}$$

where $\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i$, $\int_{0_n}^{\vartheta} s_D^T(K_D r)B^{-1}dr = \sum_{i=1}^n \int_0^{\vartheta_i} \sigma_{Di}(k_{Di}r_i)b_i^{-1}dr_i$, and $\int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr = \sum_{j=1}^p \int_0^{\bar{\phi}_j} \bar{\sigma}_{aj}(r_j)\gamma_j^{-1}dr_j$. Observe that, under the consideration of Property 1a, we have that

$$V(\bar{q},\dot{q},\vartheta,\bar{\phi}) \geq \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|s_P(K_P\bar{q})\|\|\dot{q}\| + \alpha_0\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + W_{01}(\bar{q},\vartheta,\bar{\phi})$$

with

$$W_{01}(\bar{q},\vartheta,\bar{\phi}) = \int_{0_n}^{\vartheta} s_D^T(K_D r)B^{-1}dr + \int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr + (1-\alpha_0)\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr$$

for any constant $\alpha_0 \in (0,1)$. Moreover, from point 3 of Lemma 1, we have: $\int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i \geq \frac{\sigma_{Pi}^2(k_{Pi}\bar{q}_i)}{2k_{Pi}\sigma'_{PiM}}$, $\forall \bar{q}_i \in \mathbb{R}$, whence we get: $\alpha_0\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr = \alpha_0\sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i$ $\geq \alpha_0\sum_{i=1}^n \frac{\sigma_{Pi}^2(k_{Pi}\bar{q}_i)}{2k_{Pi}\sigma'_{PiM}} \geq \frac{\alpha_0}{2\max_i\{k_{Pi}\sigma'_{PiM}\}}\sum_{i=1}^n \sigma_{Pi}^2(k_{Pi}\bar{q}_i)$ $= \frac{\alpha_0}{2\beta_P}\|s_P(K_P\bar{q})\|^2$, and consequently

$$V(\bar{q},\dot{q},\vartheta,\bar{\phi}) \geq \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|s_P(K_P\bar{q})\|\|\dot{q}\| + \frac{\alpha_0}{2\beta_P}\|s_P(K_P\bar{q})\|^2 + W_{01}(\bar{q},\vartheta,\bar{\phi})$$

which may be rewritten as

$$V(\bar{q},\dot{q},\vartheta,\bar{\phi}) \geq W_{00}(\bar{q},\dot{q}) + W_{01}(\bar{q},\vartheta,\bar{\phi}) \triangleq W_0(\bar{q},\dot{q},\vartheta,\bar{\phi})$$

with

$$W_{00}(\bar{q},\dot{q}) = \frac{1}{2}\begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}^T Q_0 \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}$$

where $Q_0 = \begin{pmatrix}\frac{\alpha_0}{\beta_P} & -\varepsilon\mu_M \\ -\varepsilon\mu_M & \mu_m\end{pmatrix}$ and α_0 is chosen such that

$$\frac{\varepsilon^2}{\varepsilon_0^2} < \alpha_0 < 1 \tag{13}$$

Note that (8) guarantees the existence of positive values α_0 satisfying (13) (since $\varepsilon < \varepsilon_M \leq \varepsilon_0 \implies \frac{\varepsilon^2}{\varepsilon_0^2} < 1$). Moreover, by (13), W_{00} is a positive definite function of $(\bar{q},\dot{q})$,[5] while from point 4 of Lemma 1, one sees that $W_{01}(\bar{q},\vartheta,\bar{\phi}) \geq 0$, $\forall(\bar{q},\vartheta,\bar{\phi}) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, with $W_{01}(\bar{q},\vartheta,\bar{\phi}) = 0 \iff (\bar{q},\vartheta,\bar{\phi}) = (0_n,0_n,0_p)$. Hence, $W_0(\bar{q},\dot{q},\vartheta,\bar{\phi})$ is concluded to be positive definite. Taking this into account, by noting that $W_{00}(0_n,\dot{q}) \to \infty$ as $\|\dot{q}\| \to \infty$, and from point 5 of Lemma 1 that $W_{01}(\bar{q},0_n,0_p) \to \infty$ as $|\bar{q}_i| \to \infty$ for every $i \in \{1,\ldots,n\}$, $W_{01}(0_n,\vartheta,0_p) \to \infty$ as $|\vartheta_i| \to \infty$ for every $i \in \{1,\ldots,n\}$, and $W_{01}(0_n,0_n,\bar{\phi}) \to \infty$ as $|\bar{\phi}_j| \to \infty$ for every $j \in \{1,\ldots,p\}$, $W_0(\bar{q},\dot{q},\vartheta,\bar{\phi})$ additionally proves to be radially unbounded [22, p. 115]. Therefore, $V(\bar{q},\dot{q},\vartheta,\bar{\phi})$ is concluded to be positive definite and radially unbounded. Its upper-right derivative along the system trajectories, $\dot{V} = D^+V$ [23, App. I] [28, §6.1A], is given by

$$\begin{aligned}
&\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) \\
&= \dot{q}^T H(q)\ddot{q} + \frac{1}{2}\dot{q}^T\dot{H}(q,\dot{q})\dot{q} + \varepsilon s_P^T(K_P\bar{q})H(q)\ddot{q} \\
&\quad + \varepsilon s_P^T(K_P\bar{q})\dot{H}(q,\dot{q})\dot{q} + \varepsilon\dot{q}^T H(q)s'_P(K_P\bar{q})K_P\dot{q} \\
&\quad + s_P^T(K_P\bar{q})\dot{q} + s_D^T(K_D\vartheta)B^{-1}\dot{\vartheta} + \bar{s}_a^T(\bar{\phi})\Gamma^{-1}\dot{\bar{\phi}} \\
&= -\dot{q}^T F\dot{q} - \varepsilon s_P^T(K_P\bar{q})F\dot{q} - \varepsilon s_P^T(K_P\bar{q})s_P(K_P\bar{q}) \\
&\quad - \varepsilon s_P^T(K_P\bar{q})s_D(K_D\vartheta) + \varepsilon\dot{q}^T C(q,\dot{q})s_P(K_P\bar{q}) \\
&\quad + \varepsilon\dot{q}^T H(q)s'_P(K_P\bar{q})K_P\dot{q} \\
&\quad - s_D^T(K_D\vartheta)B^{-1}AK_D^{-1}s_D(K_D\vartheta)
\end{aligned}$$

where $H(q)\ddot{q}$, $\dot{\vartheta}$, and $\dot{\bar{\phi}}$ have been replaced by their equivalent expression from the closed-loop manipulator dynamics in Eqs. (11), Property 1c has been used, and $s'_P(K_P\bar{q}) \triangleq \mathrm{diag}[D^+\sigma_{P1}(k_{P1}\bar{q}_1),\ldots,D^+\sigma_{Pn}(k_{Pn}\bar{q}_n)]$. Observe that from Properties 1a, 1b, 1d, and points (b) of Definition 1 and 2

[5] By (13), it follows that $\varepsilon^2\left(\frac{\mu_M^2\beta_P}{\mu_m}\right) = \frac{\varepsilon^2}{\varepsilon_0^2} < \alpha_0 \implies \varepsilon^2\mu_M^2 < \frac{\alpha_0\mu_m}{\beta_P} \implies 0 < \frac{\alpha_0\mu_m}{\beta_P} - \varepsilon^2\mu_M^2 = \det(Q_0)$ whence (taking into account that $\frac{\alpha_0}{\beta_P} > 0$, by the leading principal minor criterion) Q_0 is concluded to be a positive definite symmetric matrix.

of Lemma 1, we have that

$$\begin{aligned}\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) \leq & -f_m\|\dot{q}\|^2+\varepsilon f_M\|s_P(K_P\bar{q})\|\|\dot{q}\| \\ & -\varepsilon\|s_P(K_P\bar{q})\|^2+\varepsilon\|s_P(K_P\bar{q})\|\|s_D(K_D\vartheta)\| \\ & +\varepsilon\left(k_cB_P+\mu_M\beta_P\right)\|\dot{q}\|^2-\beta_m\|s_D(K_D\vartheta)\|^2\end{aligned}$$

which may be rewritten as

$$\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) \leq -W_1(\bar{q},\dot{q})-W_2(\bar{q},\vartheta)$$

where

$$W_1(\bar{q},\dot{q}) = \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}^T Q_1 \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}$$

$$W_2(\bar{q},\vartheta) = \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|s_D(K_D\vartheta)\|\end{pmatrix}^T Q_2 \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|s_D(K_D\vartheta)\|\end{pmatrix}$$

with $Q_1 = \begin{pmatrix}\frac{\varepsilon}{2} & -\frac{\varepsilon f_M}{2} \\ -\frac{\varepsilon f_M}{2} & f_m-\varepsilon\beta_M\end{pmatrix}$ and $Q_2 = \begin{pmatrix}\frac{\varepsilon}{2} & -\frac{\varepsilon}{2} \\ -\frac{\varepsilon}{2} & \beta_m\end{pmatrix}$. Let us note that the fulfillment of (8) renders W_1 and W_2 positive definite functions of $(\bar{q},\dot{q})$ and $(\bar{q},\vartheta)$ respectively.[6] Hence, $\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) \leq 0$, $\forall(\bar{q},\dot{q},\vartheta,\bar{\phi}) \in \mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^p$, with $\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) = 0 \iff (\bar{q},\dot{q},\vartheta) = (0_n,0_n,0_n)$. Thus, by Lyapunov's 2nd method,[7] the trivial solution $(\bar{q},\vartheta,\bar{\phi})(t) \equiv (0_n,0_n,0_p)$ is concluded to be stable. Now, in view of the radially unbounded character of $V(\bar{q},\dot{q},\vartheta,\bar{\phi})$, the set $\Omega \triangleq \{(\bar{q},\dot{q},\vartheta,\bar{\phi}) \in \mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^p : V(\bar{q},\dot{q},\vartheta,\bar{\phi}) \leq c\}$ is compact for any positive constant c [22, p. 115]. Moreover, in view of the negative semidefinite character of $\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi})$, Ω is positively invariant with respect to the closed-loop dynamics [22, p. 101]. Furthermore, from previous arguments, we have that $E \triangleq \{(\bar{q},\dot{q},\vartheta,\bar{\phi}) \in \Omega : \dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) = 0\} = \{(\bar{q},\dot{q},\vartheta,\bar{\phi}) \in \Omega : \bar{q} = \dot{q} = \vartheta = 0_n\}$. Further, from Remark 3, the largest invariant set in E is given as $\mathscr{M} = \{(\bar{q},\dot{q},\vartheta,\bar{\phi}) \in E : \bar{s}_a(\bar{\phi}) \in \ker(G(q_d))\}$. Thus, by the invariance theory [28, §7.2] —more specifically, by [28, Theorem 7.2.1]—, we have that $(\bar{q},\dot{q},\vartheta,\bar{\phi})(0) \in \Omega \implies (\bar{q},\dot{q},\vartheta,\bar{\phi})(t) \to \mathscr{M}$ as $t\to\infty$. Since this holds for any $c > 0$ and $V(\bar{q},\dot{q},\vartheta,\bar{\phi})$ is radially unbounded (in view of which Ω may be rendered arbitrarily large), we conclude that, for any $(\bar{q},\dot{q},\vartheta,\bar{\phi})(0) \in \mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^p$, $(\bar{q},\vartheta)(t) \to (0_n,0_n)$ as $t\to\infty$ and $\bar{s}_a(\bar{\phi}(t)) \to \ker(G(q_d))$ as $t\to\infty$, which completes the proof. □

Corollary 1: If $G^T(q_d)G(q_d)$ is nonsingular, then the trivial solution $(\bar{q},\vartheta,\bar{\phi})(t) \equiv (0_n,0_n,0_p)$ is globally asymptotically stable.

[6]By (8), it follows that $\varepsilon < \varepsilon_M \leq \varepsilon_1 = \frac{f_m}{\beta_M+f_M^2/2} \implies \frac{\varepsilon^2}{2}\left(\beta_M+\frac{f_M^2}{2}\right) < \frac{\varepsilon f_m}{2} \implies 0 < \frac{\varepsilon}{2}(f_m-\varepsilon\beta_M)-\left(\frac{\varepsilon f_M}{2}\right)^2 = \det(Q_1)$, and $\varepsilon < \varepsilon_M \leq \varepsilon_2 = 2\beta_m \implies 0 < \frac{\varepsilon\beta_m}{2}-\frac{\varepsilon^2}{4} = \det(Q_2)$ whence (taking into account that $\frac{\varepsilon}{2} > 0$, by the leading principal minor criterion) Q_1 and Q_2 are concluded to be positive definite symmetric matrices.

[7]See for instance [23, Ch. II, §6], where (generalized) statements of Lyapunov's 2nd method are presented under the consideration of locally Lipschitz-continuous Lyapunov functions and their upper-right derivative along the system trajectories.

Fig 1: Experimental setup

Proof: Note on the one hand that non-singularity of $G^T(q_d)G(q_d)$ implies that $\ker(G(q_d)) = \{0_p\}$, and on the other hand that $\bar{s}_a(\bar{\phi}) = 0_p \iff \bar{\phi} = 0_p$. Then, from Proposition 1, we have that, for any $(\bar{q},\dot{q},\vartheta,\bar{\phi})(0) \in \mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^p$, $(\bar{q},\vartheta,\bar{\phi})(t) \to (0_n,0_n,0_p)$ as $t\to\infty$, whence the stability of the trivial solution $(\bar{q},\vartheta,\bar{\phi})(t) \equiv (0_n,0_n,0_p)$ is concluded to be globally asymptotical [22, §3.1], [23, Chap. I, §2.10–2.11]. □

5. EXPERIMENTAL RESULTS

In order to experimentally corroborate the efficiency of the proposed output-feedback adaptive scheme —referred to as the SP-SD$_c$-g$_a$ controller—, real-time control implementations were carried out on a 2-DOF direct-drive manipulator. The experimental setup, shown in Fig. 1, is a prototype of the 2-revolute-joint robot arm used in [29], located at the *Instituto Tecnológico de la Laguna*. The actuators are direct-drive brushless motors operated in torque mode, so they act as torque source and accept an analog voltage as a reference of torque signal. The control algorithm is executed at a 2.5 ms sampling period in a control board (based on a DSP 32-bit floating point microprocessor) mounted on a PC-host computer.

For the considered experimental manipulator, Properties 1–3 are satisfied with (details on the dynamic model and parameter values are given in [29]):

$$G(q) = \begin{pmatrix}\sin q_1 & \sin(q_1+q_2) \\ 0 & \sin(q_1+q_2)\end{pmatrix}, \quad \theta = \begin{pmatrix}38.465 \\ 1.825\end{pmatrix} \text{[Nm]} \tag{14}$$

$\mu_m = 0.088$ kg m^2, $\mu_M = 2.533$ kg m^2, $k_c = 0.1455$ kg m^2, $B_{g1} = 40.29$ Nm, $B_{g2} = 1.825$ Nm, $f_m = 0.175$ kg m^2/s, $f_M = 2.288$ kg m^2/s, and

$$\Upsilon(q) = \left(\cos q_1^*-\cos q_1 \;,\; \cos(q_1^*+q_2^*)-\cos(q_1+q_2)\right)$$

with $q^* = (q_1^*,q_2^*)^T$ being the reference configuration referred to in Property 3; in particular, for the experimental implementations reported in this section, $q_1^* = \pi/2$ and $q_2^* = 0$ were taken. The maximum allowed torques (input saturation bounds) are $T_1 = 150$ Nm and $T_2 = 15$ Nm for the first and second links respectively. From these data, one easily corroborates that Assumption 1 is fulfilled.

The saturation functions involved at the implementations were defined as

$$\sigma_{Pi}(\varsigma) = M_{Pi}\,\mathrm{sat}(\varsigma/M_{P_i}) \tag{15a}$$

$$\sigma_{Di}(\varsigma) = M_{Di}\,\mathrm{sat}(\varsigma/M_{D_i}) \tag{15b}$$

$i = 1,2$, and

$$\sigma_{aj}(\varsigma) = \begin{cases} \varsigma & \forall|\varsigma| \leq L_{aj} \\ \rho_{aj}(\varsigma) & \forall|\varsigma| > L_{aj} \end{cases}$$

$j = 1,2$, where

$$\rho_{aj}(\varsigma) = \mathrm{sign}(\varsigma)L_{aj} + (M_{aj} - L_{aj})\tanh\left(\frac{\varsigma - \mathrm{sign}(\varsigma)L_{aj}}{M_{aj} - L_{aj}}\right)$$

with $0 < L_{aj} < M_{aj}$. Let us note that with these saturations we have $\sigma'_{PiM} = \sigma'_{DiM} = 1$, $\forall i \in \{1,2\}$. The saturation parameter values fixed at every implementation of the SP-SD$_c$-g$_a$ were corroborated to satisfy inequalities (3) and (5), taking $B_{gi}^{M_a} = \sum_{j=1}^{2} B_{G_{ij}} M_{aj}$, $i = 1,2$.

For comparison purposes, additional experiments were run implementing the output-feedback adaptive algorithm proposed in [16] —referred to as the L00 controller— (choice made in terms of the analog nature of the compared algorithms: output-feedback adaptive developed in a bounded input context; comparison of controllers of different nature looses coherence):

$$u = -K_P T_h(\lambda\bar{q}) - K_D T_h(\delta\vartheta) + G_d\hat{\theta} \tag{16a}$$

where $G_d = G(q_d)$, $T_h(x) = \big(\tanh(x_1),\ldots,\tanh(x_n)\big)^T$, $K_P \in \mathbb{R}^{n\times n}$ and $K_D \in \mathbb{R}^{n\times n}$ are positive definite diagonal matrices, λ and δ are positive constants, and $\vartheta \in \mathbb{R}^n$ and $\hat{\theta} \in \mathbb{R}^p$ are the output variables of (interconnected) auxiliary dynamic subsystems that take the form:

$$\dot{q}_c = -\alpha K(q_c + K\bar{q}) \tag{16b}$$

$$\vartheta = q_c + K\bar{q} \tag{16c}$$

and

$$\dot{\phi}_c = \beta G_d^T\left[\eta T_h(\delta\vartheta) - \mu T_h(\lambda\bar{q})\right] \tag{16d}$$

$$\hat{\theta} = \phi_c - \beta G_d^T\bar{q} \tag{16e}$$

where $K \in \mathbb{R}^{n\times n}$ is a positive definite diagonal matrix, and α, β, η, and μ are positive constants. Arguing simplicity of development, the gain matrices involved in this control algorithm are taken in [16] as $K_P = k_P I_n$, $K_D = k_D I_n$, and $K = kI_n$, with k_P, k_D, and k being positive constants. At every implementation of the L00 algorithm, the P and D control gains, *i.e.* k_P and k_D, were fixed small enough to avoid input saturation (note that they fix the bounds of the SP and SD actions).

Furthermore, in order to perceive the advantages of the proposed approach over a conventional (non-adaptive) output-feedback saturating PD-type algorithm with standard dirty derivative, numerical results were obtained with the free-of-velocity scheme presented in [8] —referred to as the SK97 controller— *i.e.*

$$u = S_P(K_P\bar{q}) + S_D(K_D\vartheta) + G(q)\theta_e \tag{17a}$$

where $S_P(x) = \big(\sigma_{P1}(x_1),\ldots,\sigma_{Pn}(x_n)\big)^T$, $S_D(x) = \big(\sigma_{D1}(x_1),\ldots,\sigma_{Pn}(x_n)\big)^T$, with $\sigma_{Pi}(\cdot)$ and $\sigma_{Di}(\cdot)$, $i = 1,\ldots,n$, as defined in Eqs. (15), and θ_e is the *theoretically-exact-but-practically-estimated* gravity-term parameter vector, with

$$\dot{q}_c = -A(q_c + B\bar{q}) \tag{17b}$$

$$\vartheta = q_c + B\bar{q} \tag{17c}$$

The considered experimental setup (described above) was simulated (taking the model as presented for instance in [29]) including its input constraints but no additional unmodelled phenomenon on the system dynamics. Estimated parameter values were taken as $\theta_e = 1.2\theta$ in (17a).

The initial conditions and desired link positions at all the implementations, for every tested controller, were: $q_i(0) = \dot{q}_i(0) = q_{ci}(0) = \phi_{ci}(0) = 0$,[8] $i = 1,2$, and $q_{d1} = q_{d2} = \pi/4$ [rad]. Let us note that, through these desired configurations, the condition stated by Corollary 1 is satisfied.[9]

With the aim at getting fast position responses, high control gains were taken for the SP-SD$_c$-g$_a$ scheme. The fast position response goal proved to be achieved for sufficiently small as well as for high values of ε (recall Remark 1). The difference among these two cases was observed to be on the parameter estimation convergence, with considerably smaller steady-state errors (due to unmodelled phenomena such as the static friction) in the latter case. Because of space limitations, only the results obtained for the latter case will be shown below. As for the L00 algorithm, reasonable values of the tuning parameters were fixed disregarding the tuning procedure stated in [16, Expressions (19)] in order to prevent extremely slow responses. For each of the three implemented controllers, the selected parameter combination was found through simulation tests, so as to have as good closed-loop responses as possible —in terms mainly of stabilization time (as short as possible) and transient response (avoiding or lowering down overshoot and oscillations as much as possible)—, and further refining the tuning experimentally in the SP-SD$_c$-g$_a$ and L00 cases. For the SP-SD$_c$-g$_a$ scheme, the resulting values were: $K_P = \mathrm{diag}[2100, 225]$ Nm/rad, $K_D =$

[8] All the experiments, for every tested controller, were initiated with the robot links oriented vertically downwards and at rest. Notice, by comparing (7b) with (16e) and (6b) with (16c), that the through the initial values assigned to the auxiliary states, $\vartheta(0)$ and $\hat{\theta}(0)$ do not necessarily take the same vector values for all the tested control algorithms. Nevertheless, the initial-value fairness criterion adopted in regard to the involved auxiliary subsystems was to initiate all the auxiliary states at zero for every implemented controller.

[9] One can verify from $G(q)$ in (14) that, for the considered manipulator, the desired configurations that satisfy the condition stated by Corollary 1 are those such that $q_{d1} \neq m_1\pi$ and $q_{d1} + q_{d2} \neq m_2\pi$, for any $m_1, m_2 = 0, \pm1, \pm2, \ldots$

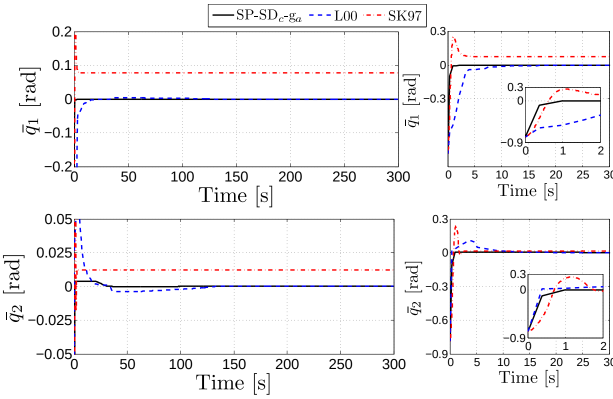

Fig 2: Position errors

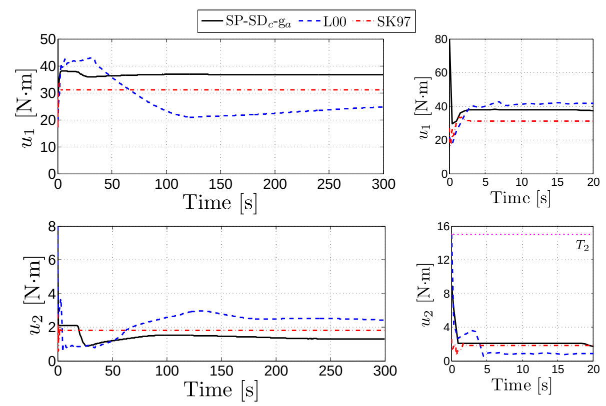

Fig 3: Control signals

diag[50,8] Nms/rad, $A = \text{diag}[80,60]$ s^{-1}, $B = \text{diag}[20,20]$ s^{-1}, $\Gamma = \text{diag}[1,0.1]$ Nm, $\varepsilon = 3$ [Nms]$^{-1}$; and the saturation function bounds (all of them in Nm) were: $M_{P1} = 30$, $M_{P2} = 3$, $M_{D1} = 50$, $M_{D1} = 6$, $M_{a1} = 50$, and $M_{a2} = 3$, with $L_{aj} = 0.9M_{aj}$, $j = 1,2$. For the L00 controller: $k_P = 9.2$ Nm, $k_D = 2.7$ Nm, $\lambda = 20$ [rad]$^{-1}$, $\delta = 10$ s/rad, $k = 20$ s^{-1}, $\alpha = 5$, $\beta = 5$ Nm/rad, $\eta = 1$ rad/s, and $\mu = 2$ rad/s. For the SK97 algorithm: $K_P = \text{diag}[80,30]$ Nm/rad, $K_D = \text{diag}[3,1]$ Nms/rad, $A = \text{diag}[1,3]$ s^{-1}, $B = \text{diag}[6,4]$ s^{-1}; and the same saturation function bound values (M_{Pi}, M_{Di}, $i = 1,2$) used for the SP-SD$_c$-g$_a$ scheme were kept for this controller.

Figures 2–4 show the results for both experimentally implemented controllers and the simulated algorithm: in Figs. 2 and 3, where the position errors and control signals are shown respectively, the left-large graphs correspond to the whole test while the right-small ones show a zoom on the transient of the same signals; in Fig. 4 the left-large graphs correspond the variation of the parameter estimations, $(\hat{\theta}_1, \hat{\theta}_2)$, while the right-small ones show the evolution of the adaptation subsystem states, (ϕ_{c1}, ϕ_{c2}). Observe that the proposed SP-SD$_c$-g$_a$ scheme achieved the position regulation objective —avoiding input saturation— in around 1 second. On the contrary, in the case of the L00 controller, longer position stabilization and parameter estimator convergence times are observed. Notice further that the SK97 controller could not avoid steady-state position errors due to the use of estimated parameters θ_e.

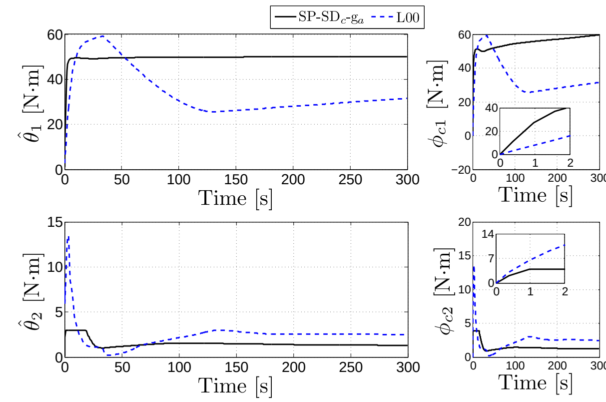

Fig 4: Parameter estimates

6. CONCLUSIONS

In this work, an output-feedback adaptive control scheme for the global regulation of robot manipulators with bounded inputs was proposed. With respect to the previous output-feedback adaptive approaches developed in a bounded-input context, the proposed free-of-velocity feedback controller guarantees the adaptive regulation objective: globally, avoiding discontinuities throughout the scheme, preventing the inputs to reach their natural saturation limits, and imposing no saturation-avoidance restriction on the control gains. Moreover, the developed scheme is not restricted to the use of a specific saturation function to achieve the required boundedness, but may rather involve any one within a set of smooth and non-smooth (Lipschitz-continuous) bounded passive functions that include the hyperbolic tangent and the conventional saturation as particular cases. The efficiency of the proposed scheme was corroborated through experimental tests. Good results were obtained, which were observed to improve those gotten through an algorithm that was previously developed in an analog analytical context.

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