A generalized scheme for the global adaptive regulation of robot manipulators with bounded inputs

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Abstract

In this work, a generalized adaptive control scheme for the global position stabilization of robot manipulators with bounded inputs is proposed. It gives rise to various families of bounded controllers with adaptive gravity compensation. Compared with the adaptive approaches previously developed in a bounded-input context, the proposed scheme guarantees the adaptive regulation objective: globally, avoiding discontinuities in the control expression as well as in the adaptation auxiliary dynamics, preventing the inputs to reach their natural saturation bounds, and imposing no *saturation-avoidance* restriction on the control gains. Experimental results corroborate the efficiency of the proposed adaptive scheme.

Keywords: Robot control, adaptive control, global regulation, bounded inputs, saturations.

1 Introduction

In an actual control system, *saturation* is an ever present nonlinear phenomenon characterizing the signal transfer from the controller outputs to the plant inputs. This is a natural consequence of the power supply limitations of real-life actuators. Disregarding such a physical constraint may lead to unexpected or undesirable consequences, as pointed out for instance in [1], [2], [3], and [4]. Control synthesis under the consideration of such inevitable

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nonlinearity has consequently become important and considerably attracted the attention of the feedback control community [5].

In an unbounded input setting, one of the simplest control techniques for the global regulation of robot manipulators is the so-called *PD with gravity compensation* [6]. In view of the above-mentioned potential undesirable effects of saturations in actual applications, extensions of such a well-known control law to the bounded input case, under various analytical frameworks, have been developed [7, 8, 9, 10, 11, 12]. For instance, assuming that the exact value of the system parameters and accurate measurements of all the system states (positions and velocities) are available, a basic approach was proposed in [7] and [8]. In these works, the P and D parts (at every joint) are, each of them, explicitly bounded through specific *saturation functions*; a continuously differentiable one —more precisely, the hyperbolic tangent function— is used in [7] and the conventional non-smooth one in [8]. Because of their structure, this type of algorithms have been denoted *SP-SD* controllers in [13]. Further, two alternative schemes, that prove to be simpler and/or give rise to improved closed-loop performances, were recently proposed in [9]. The first approach includes both the P and D parts (at every joint) within a single saturation function, while in the second one all the terms of the controller (P, D, and gravity compensation) are covered by one of such functions, with the P terms internally embedded within an additional saturation; the exclusive use of a single saturation (at every joint) including all the terms of the controller was further achieved through desired gravity compensation in [14]. Moreover, free-of-velocity-measurement versions of the SP-SD controllers in [7] and [8] —still depending on the exact values of the system parameters— are obtained through the design methodologies developed in [10] and [11]. In [10], global regulation is proved to be achieved when each velocity measurement is replaced by the *dirty derivative* [15] of the respective position in the SP-SD controller of [8]. A similar replacement in a more general form of the SP-SD controller is proved to achieve global regulation through the design procedure proposed in [11] (where an alternative type of dirty derivative, that involves a saturation function in the auxiliary dynamics giving rise to the estimated velocity, results from the application of the proposed methodology). Furthermore, an output feedback dynamic controller with a structure similar to that resulting from the methodology in [11], but which considers a single saturation function (at every joint) where both the position errors and velocity estimation states are involved, was proposed in [12] (where a *dissipative* linear term on the auxiliary state is added to the saturating velocity error dynamics involved for the dirty derivative calculation); extensions of this approach to the elastic-joint case were further developed in [16].

In view of the considered gravity compensation terms, the implementation of the above mentioned saturating schemes becomes specially problematic when the system parameters are uncertain. In view of such an additional constraint, adaptive SP-SD-type algorithms, giving rise to bounded controllers, have been developed in [17, 18, 19].

In [17], global regulation is aimed through a discontinuous scheme that switches among two different control laws under the consideration of state and output feedback. Both considered control laws keep an SP-SD structure similar to that of [8]; the first one avoids gravity compensation taking high-valued control gains (by means of which the closed-loop trajectories are lead close to the desired configuration), and the second one considers adaptive gravity compensation terms that are kept bounded by means of a discontinuous auxiliary dynamics. Each velocity measurement is replaced by the dirty derivative of the corresponding position in the output feedback version of the proposed algorithm. Unfortunately, a precise criterion to determine the switching moment (from the first control law to the second one) is not furnished for either of the proposed schemes.

In [18], semiglobal regulation is proved to be achieved through a state feedback scheme that keeps the same structure of the SP-SD controller of [7] but additionally considers adaptive gravity compensation. The adaptation algorithm is defined in terms of a discontinuous auxiliary dynamics by means of which the parameter estimators are prevented to take values beyond some pre-specified limits, which consequently keeps the adaptive gravity compensation terms bounded. This approach was further extended in [20] to the case when the control objective is defined in *task coordinates* and the kinematic parameters, additionally to those involved in the system dynamics, are considered to be uncertain too.

In [19], a controller that keeps the SP-SD structure of [7] is proposed, where each velocity measurement is replaced by the dirty derivative of the corresponding position, and an adaptive gravity compensation term, with initial-condition-dependent bounds, is considered. Based on the proof developed for the main result, semiglobal regulation is claimed to be achieved.

Bounded adaptive schemes have also been proposed in the context of tracking control, for instance in [21] and [22]. In [21], a controller with SP and SD correction terms analog to those involved in [18] and adaptive *desired* compensation terms of the system dynamics was proposed. The adaptation auxiliary dynamics is of the discontinuous type of that involved in [18]. Semiglobal stabilization was proved to be achieved for suitable trajectories. More recently, in [22], an algorithm similar to that of [21] was presented involving identical SP and SD correction terms but only adaptive *on-line* gravity compensation (no other term of the system dynamics is compensated). A discontinuous adaptation algorithm analog to that involved in [21] is considered. Unfortunately, it is not clear through such an approach how the desired trajectory can be guaranteed to be a solution of the closed-loop system.

Let us note that by the way the SP and SD terms are defined in the above mentioned adaptive schemes, the bound of the control signal at every link turns out to be defined in terms of the sum of the P and D control gains (and of an additional term involving the bounds of the parameter estimators). This limits the choice of such gains if the natural actuator bounds (or arbitrary input bounds) are aimed to be avoided. This, in turn, restricts the closed-loop region of attraction in the semiglobal stabilization cases. Let us further note that the discontinuous character of the auxiliary (adaptation) dynamics considered for instance in [17], [18], [21], and [22] is not necessarily a drawback. As a matter of fact, previous works involving adaptive control schemes where the parameter estimates are aimed to remain bounded within pre-specified values generally appeal to the same kind of discontinuous adaptation dynamics. This is seen even in recent works addressed either to manipulators in an unbounded input context [23, 24] or to systems of different nature [25, 26]. Nevertheless, a bounded adaptive scheme that solves the regulation problem globally and avoiding discontinuities has not yet been proposed for robot manipulators with saturating inputs, and would constitute a convenient alternative developed within a simpler analytical framework and through simpler and/or more natural ways to cope with the need to bound the auxiliary state variables. On the other hand, adaptive versions of PD-type saturating schemes other than the SP-SD algorithm, like those developed in [9], have not yet been proposed. These arguments have motivated the present work which aims at filling in the mentioned gaps.

It is worth adding that recent works have devoted efforts to solve the global regulation problem in the bounded-input context through nonlinear PID-type controllers. This is the case for instance of [27]; [28]; [29], where state-feedback and output-feedback schemes were presented; and [30], where a controller having the same structure of the state-feedback algorithm presented in [29] was previously proposed. Such PID-type algorithms do not only avoid the exact knowledge of the system parameters, but also disregard the structure of the robot dynamics (or of any of its components). However, in a bounded-input context, the design of an adaptive scheme that solves the regulation problem globally, avoiding input saturation, and free of discontinuities, remains an open analytical challenge. Such a challenge becomes more defiant, interesting, and innovative if it aims at giving rise to adaptive versions of a general class of bounded PD-type algorithms that includes the SP-SD type as a special case as well as others with analog energy properties but alternative saturating structures. Moreover, as will be corroborated in subsequent sections of this work, regulation towards a suitable configuration permits the adaptive scheme to provide an estimation (happening to be exact under ideal conditions) of the robot dynamic parameters, which is not the case of other types of controllers.

In this work, we propose a generalized scheme for the global adaptive regulation of robot manipulators with saturating inputs. It gives rise to various families of bounded controllers with adaptive gravity compensation, including the adaptive versions of the SP-SD algorithms in [7] and [8] as well as the schemes in [9] as particular cases. With respect to the adaptive approaches previously developed in a bounded-input context, the proposed scheme guarantees the adaptive regulation objective: globally, avoiding discontinuities in the control expression as well as in the adaptation auxiliary dynamics, preventing the inputs to attain their natural saturation bounds, and imposing no *saturation-avoidance* restriction on the choice of the P and D (positive) control gains. In addition, the approach proposed in this work is not restricted to the use of a specific saturation function to achieve the required boundedness, but can rather involve any one within a set of smooth and non-smooth (Lipschitz-continuous) bounded passive functions that include the hyperbolic tangent and the conventional saturation as particular cases. Experimental results corroborate the efficiency of the proposed adaptive scheme.

The work is organized as follows: Section 2 states the general n-degree-of-freedom (n-DOF) serial rigid robot manipulator open-loop dynamics and some of its main properties, as well as considerations, assumptions, notations, and definitions that are involved throughout the study. In Section 3, a generalized approach for the design of global regulators with exact gravity compensation is shown. Such a generalized approach proves to furnish a useful structure for the design of the proposed adaptive scheme, which is presented in Section 4. The closed-loop analysis is developed in Section 5, where global adaptive regulation is proved to be achieved avoiding input saturation. Experimental results are presented in Section 6. Finally, conclusions are given in Section 7.

2 Preliminaries

Let $X \in \mathbb{R}^{m \times n}$ and $y \in \mathbb{R}^n$. Throughout this work, X_{ij} denotes the element of X at its i^{th} row and j^{th} column, X_i represents the i^{th} row of X, and y_i stands for the i^{th} element of y. 0_n represents the origin of $\mathbb{R}^n$ and I_n the $n \times n$ identity matrix. $\|\cdot\|$ denotes the standard Euclidean norm for vectors, *i.e.* $\|y\| = \sqrt{\sum_{i=0}^{n} y_i^2}$, and induced norm for matrices, *i.e.* $\|X\| = \sqrt{\lambda_{\max}(X^T X)}$, where $\lambda_{\max}(X^T X)$ represents the maximum eigenvalue of $X^T X$. The kernel of X is denoted $\ker(X)$. Consider a continuously differentiable scalar function $\zeta : \mathbb{R} \to \mathbb{R}$ and a locally Lipschitz-continuous scalar function $\phi : \mathbb{R} \to \mathbb{R}$, both vanishing at zero, *i.e.* $\zeta(0) = \phi(0) = 0$. Let ζ' denote the derivative of ζ with respect to its argument, and $D^+\phi$ stand for the upper-right (Dini) derivative of ϕ, *i.e.* $D^+\phi(\varsigma) = \limsup_{h\to 0^+} \frac{\phi(\varsigma+h)-\phi(\varsigma)}{h}$ [31, App. C.2] [32, App. I]. Thus, $\phi(\varsigma) = \int_0^\varsigma D^+\phi(r)dr$; moreover, $(\zeta \circ \phi)(\varsigma) = \zeta(\phi(\varsigma)) = \int_0^\varsigma \zeta'(\phi(r))D^+\phi(r)dr$.

Let us consider the general n-DOF serial rigid robot manipulator dynamics with viscous friction [33, §2.1], [34, §6.2], [35, §7.2]:

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = \tau \tag{1}$$

where q, $\dot{q}$, $\ddot{q} \in \mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity, and acceleration vectors, $H(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, and $C(q, \dot{q})\dot{q}$, $F\dot{q}$, $g(q)$, $\tau \in \mathbb{R}^n$ are, respectively, the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with $F \in \mathbb{R}^{n\times n}$ being a positive definite constant diagonal matrix whose entries $f_i > 0$, $i = 1, \ldots, n$, are the viscous friction coefficients. Some well-known properties characterizing the terms of such a dynamical model are recalled here (see for instance [6, Chap. 4 & 14]). Subsequently, we denote $\dot{H}$ the change rate of H, *i.e.* $\dot{H} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n} : (q, \dot{q}) \mapsto \left(\frac{\partial H_{ij}}{\partial q}(q)\dot{q}\right)$.

Property 1 *The inertia matrix $H(q)$ is a positive definite symmetric bounded matrix, i.e. $\mu_m I_n \leq H(q) \leq \mu_M I_n$, $\forall q \in \mathbb{R}^n$, for some positive constants $\mu_m \leq \mu_M$.*

Property 2 *The Coriolis matrix $C(q, \dot{q})$ satisfies:*

2a. $\|C(q, \dot{q})\| \leq k_c \|\dot{q}\|$, $\forall (q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, *for some constant* $k_c \geq 0$;

2b. $\dot{q}^T \left[\frac{1}{2}\dot{H}(q, \dot{q}) - C(q, \dot{q})\right] \dot{q} = 0$, $\forall (q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, *and actually* $\dot{H}(q, \dot{q}) = C(q, \dot{q}) + C^T(q, \dot{q})$, $\forall (q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$.

Property 3 *The viscous friction coefficient matrix satisfies $f_m\|x\|^2 \leq x^T F x \leq f_M\|x\|^2$, $\forall x \in \mathbb{R}^n$, where $0 < f_m \triangleq \min_i\{f_i\} \leq \max_i\{f_i\} \triangleq f_M$.*

Property 4 *The gravity vector $g(q)$ is bounded, or equivalently, every element of the gravity vector, $g_i(q)$, $i = 1, \ldots, n$, satisfies $|g_i(q)| \leq B_{gi}$, $\forall q \in \mathbb{R}^n$, for some positive constants B_{gi}, $i = 1, \ldots, n$.*

Property 4 is not satisfied by all types of robot manipulators but it is for instance by those having only revolute joints [6, §4.3]. This work is addressed to manipulators satisfying Property 4.

Property 5 *The gravity vector can be rewritten as $g(q, \theta) = G(q)\theta$, where $\theta \in \mathbb{R}^p$ is a constant vector whose elements depend exclusively on the system parameters, and $G(q) \in \mathbb{R}^{n\times p}$ —the regression matrix— is a continuous matrix function whose elements depend exclusively on the configuration variables and do not involve any of the system parameters.*

Property 6 *Consider the gravity vector $g(q,\theta)$. Let θ_{Mj} represent an upper bound of $|\theta_j|$, such that $|\theta_j| \leq \theta_{Mj}$, $\forall j \in \{1,\ldots,p\}$, and let $\theta_M \triangleq \left(\theta_{M1},\ldots,\theta_{Mp}\right)^T$ and $\Theta \triangleq [-\theta_{M1},\theta_{M1}] \times \cdots \times [-\theta_{Mp},\theta_{Mp}]$. By Properties 4 and 5, there exist positive constants $B_{gi}^{\theta_M}$, $i=1,\ldots,n$, such that $|g_i(x,y)| = |G_i(x)y| \leq B_{gi}^{\theta_M}$, $i=1,\ldots,n$, $\forall x \in \mathbb{R}^n$, $\forall y \in \Theta$. Furthermore, there exist positive constans $B_{G_{ij}}$, B_{G_i}, and B_G such that $|G_{ij}(x)| \leq B_{G_{ij}}$, $\|G_i(x)\| \leq B_{G_i}$, and $\|G(x)\| \leq B_G$, $\forall x \in \mathbb{R}^n$, $i=1,\ldots,n$, $j=1,\ldots,p$.*

Let us suppose that the absolute value of each input τ_i (i^{th} element of the input vector τ) is constrained to be smaller than a given saturation bound $T_i > 0$, *i.e.* $|\tau_i| \leq T_i$, $i=1,\ldots,n$. In other words, letting u_i represent the control variable (controller output) relative to the i^{th} degree of freedom, we have that

$$\tau_i = T_i \mathrm{sat}\left(\frac{u_i}{T_i}\right) \tag{2}$$

$i=1,\ldots,n$, where $\mathrm{sat}(\cdot)$ is the standard saturation function, *i.e.* $\mathrm{sat}(\varsigma) = \mathrm{sign}(\varsigma)\min\{|\varsigma|,1\}$.

Let us note from (1)-(2) that $T_i \geq B_{gi}$ (see Property 4), $\forall i \in \{1,\ldots,n\}$, is a necessary condition for the manipulator to be stabilizable at any desired equilibrium configuration $q_d \in \mathbb{R}^n$. Thus, the following assumption turns out to be crucial within the analytical setting considered in this work:

Assumption 1 $T_i > B_{gi}$, $\forall i \in \{1,\ldots,n\}$.

The control schemes proposed in this work involve special (saturation) functions fitting the following definition.

Definition 1 *Given a positive constant M, a nondecreasing Lipschitz-continuous function $\sigma : \mathbb{R} \to \mathbb{R}$ is said to be a* ***generalized saturation*** *with bound M if:*

(a) $\varsigma\sigma(\varsigma) > 0$ *for all* $\varsigma \neq 0$*;*

(b) $|\sigma(\varsigma)| \leq M$ *for all* $\varsigma \in \mathbb{R}$*.*

If in addition

(c) $\sigma(\varsigma) = \varsigma$ *when* $|\varsigma| \leq L$*,*

for some positive constant $L \leq M$, σ is said to be a ***linear saturation*** *for (L,M) [36].*

Any function satisfying Definition 1 has the following properties.

Lemma 1 *Let $\sigma : \mathbb{R} \to \mathbb{R}$ be a generalized saturation function with bound M, and let k be a positive constant. Then*

1. $\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) = 0$*;*

2. $\exists \sigma'_M \in (0,\infty)$ *such that* $0 \leq D^+\sigma(\varsigma) \leq \sigma'_M$, $\forall \varsigma \in \mathbb{R}$*;*

3. $\frac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma'_M\varsigma^2}{2}$, $\forall \varsigma \in \mathbb{R}$*;*

4. $\int_0^\varsigma \sigma(kr)dr > 0$, $\forall\varsigma \neq 0$*;*

5. $\int_0^\varsigma \sigma(kr)dr \to \infty$ *as* $|\varsigma| \to \infty$*;*

6. if σ *is strictly increasing, then*

   (a) $\varsigma[\sigma(\varsigma+\eta) - \sigma(\eta)] > 0$, $\forall\varsigma \neq 0$, $\forall\eta \in \mathbb{R}$*;*

   (b) for any constant $a \in \mathbb{R}$, $\bar{\sigma}(\varsigma) = \sigma(\varsigma+a) - \sigma(a)$ *is a strictly increasing generalized saturation function with bound* $\bar{M} = M + |\sigma(a)|$*;*

7. if σ *is a linear saturation for* (L, M) *then, for any continuous function* $\nu : \mathbb{R} \to \mathbb{R}$ *such that* $|\nu(\eta)| < L$, $\forall\eta \in \mathbb{R}$*, we have that* $\varsigma\big[\sigma\big(\varsigma + \nu(\eta)\big) - \sigma\big(\nu(\eta)\big)\big] > 0$, $\forall\varsigma \neq 0$, $\forall\eta \in \mathbb{R}$*.*

Proof. See the Appendix. $\triangleleft$

3 Global regulation involving exact gravity compensation: A generalized approach

Let us consider the following *generalized* expression defining saturating controllers for the global regulation of system (1)-(2):

$$u(q, \dot{q}, \theta) = -s_d(\bar{q}, \dot{q}, \theta) - s_P(K_P\bar{q}) + G(q)\theta \tag{3}$$

where $\bar{q} = q - q_d$, for any constant (desired equilibrium position) vector $q_d \in \mathbb{R}^n$; $G(q)$ is the regression matrix related to the gravity vector, according to Property 5, *i.e.* such that $g(q, \theta) = G(q)\theta$; $K_P \in \mathbb{R}^{n\times n}$ is a positive definite diagonal matrix, *i.e.* $K_P = \text{diag}[k_{P1}, \ldots, k_{Pn}]$ with $k_{Pi} > 0$ for all $i = 1, \ldots, n$;

$$\begin{aligned} s_P &: \mathbb{R}^n \to \mathbb{R}^n \\ x &\mapsto \Big(\sigma_{P1}(x_1)\ ,\ \ldots\ ,\ \sigma_{Pn}(x_n)\Big)^T \end{aligned}$$

with $\sigma_{Pi}(\cdot)$, $i = 1, \ldots, n$, being (suitable) **generalized saturation functions** with bounds M_{Pi}; and $s_d : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$ is a bounded continuous vector function satisfying

$$s_d(x, 0_n, z) = 0_n \tag{4}$$

$\forall x \in \mathbb{R}^n$, $\forall z \in \mathbb{R}^p$,

$$\|s_d(x, y, z)\| \leq \kappa\|y\| \tag{5}$$

$\forall(x, y, z) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, for some positive constant κ, and, given $z \in \mathbb{R}^p$ such that $|G_i(q)z| < T_i$, $i = 1, \ldots, n$, $\forall q \in \mathbb{R}^n$:

$$y^T s_d(x, y, z) > 0 \tag{6}$$

$\forall y \neq 0_n$, $\forall x \in \mathbb{R}^n$, and

$$|u_i(x, y, z)| < T_i \tag{7}$$

$i = 1, \ldots, n$, $\forall x \in \mathbb{R}^n$, $\forall y \in \mathbb{R}^n$, for suitable bounds M_{Pi} of $\sigma_{Pi}(\cdot)$.

Proposition 1 *Consider system* (1)-(2) *taking* $u = u(q, \dot{q}, \theta)$ *as defined in Eq.* (3), *under the satisfaction of Assumption 1 and the conditions on the vector function* s_d *stated through the expressions in* (4)–(7). *Thus, for any positive definite diagonal matrix* K_P, *global asymptotic stability of the closed loop trivial solution* $\bar{q}(t) \equiv 0_n$ *is guaranteed with* $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$.

Proof. Observe that the satisfaction of (7), under the consideration of (2), shows that $T_i > |u_i(q, \dot{q}, \theta)| = |u_i| = |\tau_i|$, $i = 1, \ldots, n$, $\forall (q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$. From this expression we see that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$. This proves that under the proposed scheme, the input saturation values, T_i, are never reached. Thus, under the consideration of Property 5, the closed-loop dynamics takes the form

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} = -s_d(\bar{q}, \dot{q}, \theta) - s_P(K_P\bar{q}) \tag{8}$$

(Observe that in the error variable space $q = \bar{q} + q_d$, and consequently $H(q) = H(\bar{q} + q_d)$, $C(q, \dot{q}) = C(\bar{q} + q_d, \dot{q})$, and $G(q) = G(\bar{q} + q_d)$; however, for the sake of simplicity, $H(q)$, $C(q, \dot{q})$, and $G(q)$ are used throughout the paper.) Let us define the scalar function

$$V_0(\bar{q}, \dot{q}) = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})H(q)\dot{q} + \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr \tag{9}$$

with $\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr = \sum_{i=1}^{n} \int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i$ and ε being a positive constant satisfying

$$\varepsilon < \varepsilon_M \triangleq \min\{\varepsilon_1, \varepsilon_2\} \tag{10}$$

where

$$\varepsilon_1 \triangleq \sqrt{\frac{\mu_m}{\mu_M^2 \beta_P}} \qquad \text{and} \qquad \varepsilon_2 \triangleq \frac{f_m}{\beta_M + \frac{(f_M+\kappa)^2}{4}}$$

with

$$\beta_P \triangleq \max_i\{\sigma'_{PiM}k_{Pi}\} \quad , \quad \beta_M \triangleq k_C B_P + \mu_M \beta_P \quad , \quad B_P \triangleq \sqrt{\sum_{i=0}^{n} M_{Pi}^2}$$

σ'_{PiM} being the positive bound of $D^+\sigma_{Pi}(\cdot)$ in accordance to point 2 of Lemma 1, κ as defined through (5), and μ_m, μ_M, k_C, f_m, and f_M as defined in Properties 1, 2a, and 3. Observe that

$$\begin{aligned}V_0(\bar{q}, \dot{q}) &= \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})H(q)\dot{q} + (1 - \alpha + \alpha)\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr \\ &\geq \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|s_P(K_P\bar{q})\|\|\dot{q}\| + \alpha\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + (1 - \alpha)\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr\end{aligned}$$

for any positive less-than-unity constant α, where Property 1 has been taken into account. Moreover, from point 3 of Lemma 1, we have that $\int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i \geq \frac{\sigma_{Pi}^2(k_{Pi}\bar{q}_i)}{2k_{Pi}\sigma'_{PiM}}$, $\forall \bar{q}_i \in \mathbb{R}$, whence we get that $\alpha\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr = \alpha\sum_{i=1}^{n}\int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i \geq \alpha\sum_{i=1}^{n}\frac{\sigma_{Pi}^2(k_{Pi}\bar{q}_i)}{2k_{Pi}\sigma'_{PiM}} \geq \frac{\alpha}{2\max_i\{k_{Pi}\sigma'_{PiM}\}}\sum_{i=1}^{n}\sigma_{Pi}^2(k_{Pi}\bar{q}_i) = \frac{\alpha}{2\beta_P}\|s_P(K_P\bar{q})\|^2$, and consequently

$$V_0(\bar{q}, \dot{q}) \geq W_0(\bar{q}, \dot{q}) + (1 - \alpha)\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr \tag{11}$$

where

$$W_0(\bar{q},\dot{q}) = \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|s_P(K_P\bar{q})\|\|\dot{q}\| + \frac{\alpha}{2\beta_P}\|s_P(K_P\bar{q})\|^2$$
$$= \frac{1}{2}\begin{pmatrix}\|s_P(K_P\bar{q})\|\\ \|\dot{q}\|\end{pmatrix}^T\begin{pmatrix}\frac{\alpha}{\beta_P} & -\varepsilon\mu_M\\ -\varepsilon\mu_M & \mu_m\end{pmatrix}\begin{pmatrix}\|s_P(K_P\bar{q})\|\\ \|\dot{q}\|\end{pmatrix} \tag{12}$$

and α is chosen to be a positive constant satisfying

$$\frac{\varepsilon^2}{\varepsilon_1^2} < \alpha < 1 \tag{13}$$

(see (10)). Note further that, by (13), $W_0(\bar{q},\dot{q})$ is positive definite (since with $\varepsilon < \varepsilon_M \leq \varepsilon_1$, in accordance to (10), any α satisfying (13) renders positive definite the matrix at the right-hand side of (12)), and observe that $W_0(0_n,\dot{q}) \to \infty$ as $\|\dot{q}\| \to \infty$. From this, inequality (13), and points 4 and 5 of Lemma 1 (through which one sees that the integral term in the right-hand side of inequality (11) is a radially unbounded positive definite function of $\bar{q}$), $V_0(\bar{q},\dot{q})$ is concluded to be positive definite and radially unbounded. Its upper-right derivative along the system trajectories, $\dot{V}_0 = D^+V_0$ [32, App. I] [37, §6.1A], is given by

$$\begin{aligned}\dot{V}_0(\bar{q},\dot{q}) =\ & \dot{q}^T H(q)\ddot{q} + \frac{1}{2}\dot{q}^T\dot{H}(q,\dot{q})\dot{q} + \varepsilon s_P^T(K_P\bar{q})H(q)\ddot{q} + \varepsilon\dot{q}^T\dot{H}(q,\dot{q})s_P(K_P\bar{q})\\ & + \varepsilon\dot{q}^T H(q)s_P'(K_P\bar{q})K_P\dot{q} + s_P^T(K_P\bar{q})\dot{q}\\ =\ & \dot{q}^T\big[-C(q,\dot{q})\dot{q} - F\dot{q} - s_d(\bar{q},\dot{q},\theta) - s_P(K_P\bar{q})\big] + \frac{1}{2}\dot{q}^T\dot{H}(q,\dot{q})\dot{q}\\ & + \varepsilon s_P^T(K_P\bar{q})\big[-C(q,\dot{q})\dot{q} - F\dot{q} - s_d(\bar{q},\dot{q},\theta) - s_P(K_P\bar{q})\big] + \varepsilon\dot{q}^T\dot{H}(q,\dot{q})s_P(K_P\bar{q})\\ & + \varepsilon\dot{q}^T H(q)s_P'(K_P\bar{q})K_P\dot{q} + s_P^T(K_P\bar{q})\dot{q}\\ =\ & -\dot{q}^T F\dot{q} - \dot{q}^T s_d(\bar{q},\dot{q},\theta) - \varepsilon s_P^T(K_P\bar{q})F\dot{q} - \varepsilon s_P^T(K_P\bar{q})s_d(\bar{q},\dot{q},\theta) - \varepsilon s_P^T(K_P\bar{q})s_P(K_P\bar{q})\\ & + \varepsilon\dot{q}^T C(q,\dot{q})s_P(K_P\bar{q}) + \varepsilon\dot{q}^T H(q)s_P'(K_P\bar{q})K_P\dot{q}\end{aligned}$$

where $H(q)\ddot{q}$ has been replaced by its equivalent expression from the closed-loop dynamics in (8), Property 2b has been used, and

$$s_P'(K_P\bar{q}) \triangleq \operatorname{diag}[D^+\sigma_{P1}(k_{P1}\bar{q}_1),\ldots,D^+\sigma_{Pn}(k_{Pn}\bar{q}_n)] \tag{14}$$

Observe that from Properties 1, 2a, and 3, the satisfaction of (5), points (b) of Definition 1 and 2 of Lemma 1, and the positive definite character of K_P, we have that

$$\dot{V}_0(\bar{q},\dot{q}) \leq -\dot{q}^T s_d(\bar{q},\dot{q},\theta) - W_1(\bar{q},\dot{q})$$

with

$$\begin{aligned}W_1(\bar{q},\dot{q}) =\ & f_m\|\dot{q}\|^2 - \varepsilon f_M\|s_P(K_P\bar{q})\|\|\dot{q}\| - \varepsilon\kappa\|s_P(K_P\bar{q})\|\|\dot{q}\| + \varepsilon\|s_P(K_P\bar{q})\|^2 - \varepsilon k_C B_P\|\dot{q}\|^2\\ & - \varepsilon\mu_M\beta_P\|\dot{q}\|^2\\ =\ & \begin{pmatrix}\|s_P(K_P\bar{q})\|\\ \|\dot{q}\|\end{pmatrix}^T\begin{pmatrix}\varepsilon & -\frac{\varepsilon}{2}(f_M+\kappa)\\ -\frac{\varepsilon}{2}(f_M+\kappa) & f_m - \varepsilon\beta_M\end{pmatrix}\begin{pmatrix}\|s_P(K_P\bar{q})\|\\ \|\dot{q}\|\end{pmatrix}\end{aligned} \tag{15}$$

Note further that, from the satisfaction of (10), $W_1(\bar{q},\dot{q})$ is positive definite (since any $\varepsilon < \varepsilon_M \leq \varepsilon_2$ renders positive definite the matrix at the right-hand side of (15)). From this and (6), by Lyapunov's second method —see for instance [32, Chap. II, §6], where (generalized) statements of Lyapunov's second method are presented under the consideration of locally Lipshcitz-continuous Lyapunov functions and their upper-right derivative along the system trajectories—, the trivial solution $\bar{q}(t) \equiv 0$ is concluded to be globally asymptotically stable, which completes the proof. ◁

Remark 1 Let $K_D \in \mathbb{R}^{n\times n}$ be a positive definite diagonal matrix, *i.e.* $K_D = \mathrm{diag}[k_{D1},\dots,k_{Dn}]$ with $k_{Di} > 0$ for all $i = 1,\dots,n$. A generalized version of the SP-SD controller is retrieved from (3) by defining

$$s_d(\bar{q},\dot{q},\theta) = s_D(K_D\dot{q}) \tag{16}$$

where $s_D : \mathbb{R}^n \to \mathbb{R}^n : x \mapsto \Big(\sigma_{D1}(x_1)\ ,\ \dots\ ,\ \sigma_{Dn}(x_n)\Big)^T$, with $\sigma_{Di}(\cdot)$, $i = 1,\dots,n$, being **generalized saturation functions** with bounds M_{Di}, and the involved bound values, M_{Pi} and M_{Di}, satisfying

$$M_{Pi} + M_{Di} < T_i - B_{gi} \tag{17}$$

$i = 1,\dots,n$. Special cases of the generalized SP-SD controller in Eqs. (3) and (16) were defined in [7] and [8], taking $\sigma_{Pi}(x_i) = k_{Pi}\tanh\left(\frac{\lambda_{Pi}x_i}{k_{Pi}}\right)$ and $\sigma_{Di}(x_i) = k_{Di}\tanh\left(\frac{\lambda_{Di}x_i}{k_{Di}}\right)$, with $\lambda_{Pi} > 0$, $\lambda_{Di} > 0$, and $k_{Pi} + k_{Di} < T_i - B_{gi}$, in [7], and $\sigma_{Pi}(x_i) = \delta_{Pi}\mathrm{sat}\left(\frac{x_i}{\delta_{Pi}}\right)$ and $\sigma_{Di}(x_i) = \delta_{Di}\mathrm{sat}\left(\frac{x_i}{\delta_{Di}}\right)$, with $\delta_{Pi} > 0$, $\delta_{Di} > 0$, and $\delta_{Pi} + \delta_{Di} < T_i - B_{gi}$, in [8]. Further, generalized versions of the SPD and SPDgc-like schemes proposed in [9] are retrieved from (3) as well, by respectively defining

$$s_d(\bar{q},\dot{q},\theta) = s_P(K_P\bar{q} + K_D\dot{q}) - s_P(K_P\bar{q}) \tag{18}$$

with the generalized saturations $\sigma_{Pi}(\cdot)$, $i = 1,\dots,n$, being **strictly increasing**, and bound values satisfying

$$M_{Pi} \leq T_i - B_{gi} \tag{19}$$

$i = 1,\dots,n$, for the SPD case (note that the generalized saturations, $\sigma_{Pi}(\cdot)$, in (18) are not restricted to be continuously differentiable as originally formulated in [9]), and

$$s_d(\bar{q},\dot{q},\theta) = s_0\big(G(q)\theta - s_P(K_P\bar{q})\big) - s_0\big(G(q)\theta - s_P(K_P\bar{q}) - K_D\dot{q}\big) \tag{20}$$

where $s_0 : \mathbb{R}^n \to \mathbb{R}^n : x \mapsto \Big(\sigma_{01}(x_1)\ ,\ \dots\ ,\ \sigma_{0n}(x_n)\Big)^T$, with $\sigma_{0i}(\cdot)$, $i = 1,\dots,n$, being **linear saturation functions** for (L_{0i}, M_{0i}), and the involved linear/generalized saturation function parameters satisfying

$$B_{gi} + M_{Pi} < L_{0i} \leq M_{0i} < T_i \tag{21}$$

$i = 1,\dots,n$, for the SPDgc-like case (notice that the *internal* saturations, $\sigma_{Pi}(\cdot)$, in (20) are permitted to be any function satisfying Definition 1 and are consequently not tied to be linear saturations as originally formulated in [9]). Observe from (21) that, by virtue of point (c) of Definition 1 (under the consideration of Properties 4 and 5), we have that $s_0\big(G(q)\theta - s_P(K_P\bar{q})\big) \equiv G(q)\theta - s_P(K_P\bar{q})$ (see (20) and (3)). Furthermore, note that, from points (a) of Definition 1 and 6a and 7 of Lemma 1 (under the satisfaction of inequalities (21) in the SPDgc-like case), $s_d(\bar{q}, \dot{q}, \theta)$ in every one of the above cases in (16), (18), and (20) satisfies the expressions in (4) and (6). Further, notice that, through the satisfaction of (17), (19) (under the consideration of the strictly increasing character of the generalized saturation functions σ_{Pi} involved in the SPD case), and (21), every $s_d(\bar{q}, \dot{q}, \theta)$ in expressions (16), (18), and (20) satisfies inequalities (7) too. Moreover, from the Lipschitz-continuous character of generalized saturation functions, one sees that $s_d(\bar{q}, \dot{q}, \theta)$ in every one of the above cases in (16), (18), and (20) satisfies inequality (5) with

$$\kappa = \max_i \{\sigma'_{iM} k_{Di}\} \tag{22a}$$

where

$$\sigma'_{iM} = \begin{cases} \sigma'_{DiM} & \text{in the SP-SD case} \\ \sigma'_{PiM} & \text{in the SPD case} \\ \sigma'_{0iM} & \text{in the SPDgc-like case} \end{cases} \tag{22b}$$

σ'_{DiM}, σ'_{PiM}, and σ'_{0iM} respectively being the positive bounds of $D^+\sigma_{Di}(\cdot)$, $D^+\sigma_{Pi}(\cdot)$, and $D^+\sigma_{0i}(\cdot)$, in accordance to point 2 of Lemma 1. $\triangleleft$

4 The proposed adaptive scheme

If the accurate values of the elements of θ in $g(q, \theta)$ are unknown, exact gravity compensation is no longer possible. However, in such a situation, global position stabilization avoiding input saturation can still be accomplished through adaptive gravity compensation. This is achieved by means of suitable strict bounds on the elements of θ, as described next.

Let $M_a \triangleq \big(M_{a1}, \ldots, M_{ap}\big)^T$ and $\Theta_a \triangleq [-M_{a1}, M_{a1}] \times \cdots \times [-M_{ap}, M_{ap}]$, with M_{aj}, $j = 1, \ldots, p$, being positive constants such that

$$|\theta_j| < M_{aj} \tag{23a}$$

$\forall j \in \{1, \ldots, p\}$, and

$$B_{gi}^{M_a} < T_i \tag{23b}$$

$\forall i \in \{1, \ldots, n\}$, where, in accordance to Property 6, $B_{gi}^{M_a}$ are positive constants such that $|g_i(x, y)| = |G_i(x)y| \leq B_{gi}^{M_a}$, $i = 1, \ldots, n$, $\forall x \in \mathbb{R}^n$, $\forall y \in \Theta_a$. Let us note that Assumption 1 ensures the existence of such positive values M_{aj}, $j = 1, \ldots, p$, satisfying inequalities (23). Notice further that inequalities (23b) are satisfied if $\sum_{j=1}^p B_{G_{ij}} M_{aj} < T_i$, $B_{G_i}\|M_a\| < T_i$, or $B_G\|M_a\| < T_i$, $i = 1, \ldots, n$; actually, $\sum_{j=1}^p B_{G_{ij}} M_{aj}$, $B_{G_i}\|M_a\|$, or $B_G\|M_a\|$, may be taken as the value of $B_{gi}^{M_a}$ as long as inequality (23b) is satisfied.

Based on the generalized algorithm in Eq. (3), the proposed adaptive control scheme is defined as

$$u(q, \dot{q}, \hat{\theta}) = -s_d(\bar{q}, \dot{q}, \hat{\theta}) - s_P(K_P\bar{q}) + G(q)\hat{\theta} \tag{24}$$

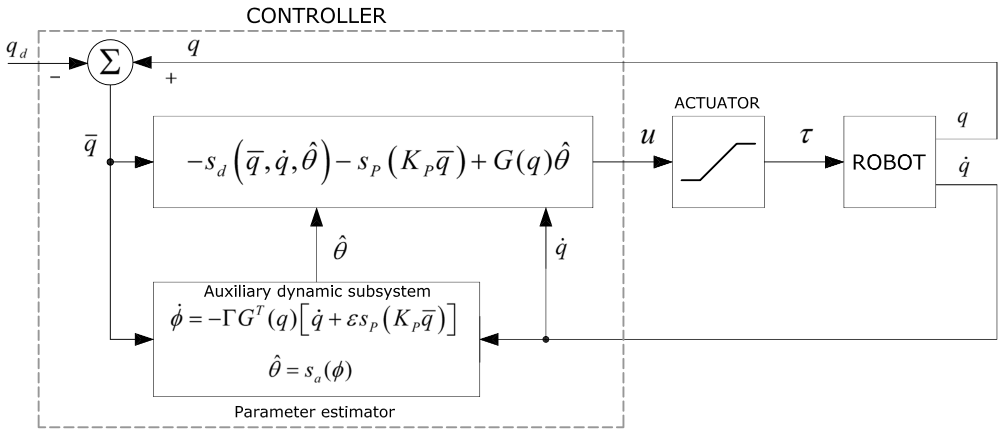

Figure 1: Block diagram of the proposed adaptive control scheme

with $s_P(\cdot)$, K_p, and $s_d(\cdot,\cdot,\cdot)$ being as defined in Section 3, and $\hat{\theta}$ (the parameter estimator) being the output variable of an auxiliary (adaptation) dynamic subsystem defined as

$$\dot{\phi} = -\Gamma G^T(q)\big[\dot{q} + \varepsilon s_P(K_P\bar{q})\big] \tag{25a}$$
$$\hat{\theta} = s_a(\phi) \tag{25b}$$

where ϕ is the (internal) state of the auxiliary dynamics in Eq. (25a);

$$\begin{aligned} s_a : \mathbb{R}^p &\to \mathbb{R}^p \\ x &\mapsto \Big(\sigma_{a1}(x_1)\ ,\ \ldots\ ,\ \sigma_{ap}(x_p)\Big)^T \end{aligned}$$

$\sigma_{aj}(\cdot)$, $j = 1,\ldots,p$, being **strictly increasing generalized saturation functions** with bounds M_{aj} as defined above, *i.e.* satisfying inequalities (23); $\Gamma \in \mathbb{R}^{p\times p}$ is a positive definite diagonal constant matrix, *i.e.* $\Gamma = \mathrm{diag}[\gamma_1 \ldots, \gamma_p]$ with $\gamma_j > 0$ for all $j = 1,\ldots,p$; and ε is a positive constant satisfying inequality (10), *i.e.* (for ease on the reading, inequality (10) is restated here)

$$\varepsilon < \varepsilon_M \triangleq \min\{\varepsilon_1, \varepsilon_2\} \tag{26}$$

where

$$\varepsilon_1 \triangleq \sqrt{\frac{\mu_m}{\mu_M^2\beta_P}} \qquad \text{and} \qquad \varepsilon_2 \triangleq \frac{f_m}{\beta_M + \frac{(f_M+\kappa)^2}{4}}$$

with

$$\beta_P \triangleq \max_i\{\sigma'_{PiM}k_{Pi}\} \quad , \quad \beta_M \triangleq k_C B_P + \mu_M\beta_P \quad , \quad B_P \triangleq \sqrt{\sum_{i=0}^{n} M_{Pi}^2}$$

σ'_{PiM} being the positive bound of $D^+\sigma_{Pi}(\cdot)$ in accordance to point 2 of Lemma 1, κ as defined through (5), and μ_m, μ_M, k_C, f_m, and f_M as defined in Properties 1, 2a, and 3. A block diagram of the proposed adaptive control scheme is shown in Fig. 1.

Remark 2 Observe that the control scheme in (24)-(25) does not involve the exact values of the elements of θ. It only requires the satisfaction of inequalities (23). In other words, only strict bounds M_{aj} of $|\theta_j|$, $j = 1, \ldots, p$, —*i.e.* any set of them satisfying inequalities (23b)— are involved. Notice further that a suitable choice of ε does not require the exact knowledge of the system parameters either. Indeed, observe, on the one hand, that an estimation of the right-hand side of inequality (26) may be obtained by means of upper and lower bounds of the system parameters and viscous friction coefficients (more precisely, nonzero lower bounds of μ_m and f_m, and upper bounds of μ_M, k_C, and f_M; see Properties 1, 2a, and 3). On the other hand, the satisfaction of inequality (26) is not necessary but only sufficient for the closed-loop analysis to hold, as shown in the following section, which permits the consideration of values of ε higher than ε_M (up to certain limit) without destabilizing the closed loop. $\triangleleft$

Remark 3 Let us note that the existence of suitable values of ε satisfying inequality (26) is ensured through the consideration of the viscous friction terms in the manipulator open-loop dynamics (see specifically the definition of ε_2). An approach independent of the consideration of friction may be considered a stronger result. However, viscous friction is an ever-present phenomenon in mechanical systems [38]. Moreover, such a dissipative phenomenon has also been considered in previous works like that in [18]. In spite of the dependence on the friction terms, the result developed in this work states an important contribution since it is the first one to guarantee global stabilization —preventing input saturation— while stating a design formulation that considers a generalized controller structure. Let us further highlight the continuous nature of the adaptation algorithm in Eqs. (25). This constitutes another main difference of this work with respect to previous studies, like those in [17] and [18], where discontinuous adaptation algorithms were involved. Through the (Lipschitz-)continuous nature of the proposed control scheme, theoretical issues naturally raised in the context of discontinuous dynamic systems, such as existence and uniqueness of the closed-loop solutions [31, 39], are avoided. The consideration of more complex stability analytical frameworks addressed to systems with discontinuous right-hand sides [39, 40] is avoided too. Moreover, the implementation of the continuous adaptation dynamic subsystem in Eqs. (25) is released from the logical operations inherent to the switching performed by the discontinuous adaptation algorithms of the previous works. Thus, the continuous nature of the adaptation algorithm developed in this work constitutes a convenient alternative developed within a simpler analytical context and through simpler and/or more natural ways to cope with the need to bound the auxiliary state variables. $\triangleleft$

5 Closed-loop analysis

Consider system (1)-(2) taking $u = u(q, \dot{q}, \hat{\theta})$ as defined through Eqs. (24)-(25). Observe that —under Assumption 1, the satisfaction of inequalities (23), and the consideration of (2)— the fulfilment of (7) shows that

$$T_i > \big|u_i\big(q, \dot{q}, s_a(\phi)\big)\big| = |u_i| = |\tau_i| \quad i = 1, \ldots, n \quad \forall (q, \dot{q}, \phi) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p \tag{27}$$

Thus, under the consideration of Property 5, the closed-loop system takes the form

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} = -s_d\big(\bar{q}, \dot{q}, s_a(\phi)\big) - s_P(K_P\bar{q}) + G(q)\bar{s}_a(\bar{\phi}) \tag{28a}$$

$$\dot{\bar{\phi}} = -\Gamma G^T(q)\big[\dot{q} + \varepsilon s_P(K_P\bar{q})\big] \tag{28b}$$

where $\bar{\phi} = \phi - \phi^*$ and

$$\bar{s}_a(\bar{\phi}) = s_a(\bar{\phi} + \phi^*) - s_a(\phi^*) \tag{29}$$

with $\phi^* = \left(\phi_1^*, \ldots, \phi_p^*\right)^T$ such that $s_a(\phi^*) = \theta$, or equivalently, $\phi_j^* = \sigma_{aj}^{-1}(\theta_j)$, $j = 1, \ldots, p$ —notice that their strictly increasing character renders the generalized saturation functions σ_{aj}, $j = 1, \ldots, p$, (involved in the definition of s_a) invertible—. Observe that, by point 6b of Lemma 1, the elements of $\bar{s}_a(\bar{\phi})$ in (29), *i.e.*

$$\bar{\sigma}_{aj}(\bar{\phi}_j) = \sigma_{aj}(\bar{\phi}_j + \phi_j^*) - \sigma_{aj}(\phi_j^*)$$

$j = 1, \ldots, p$, turn out to be strictly increasing generalized saturation functions.

Remark 4 Let us note that, from Eqs. (28) under stationary conditions, *i.e.* by considering $\ddot{q} = \dot{q} = 0_n$ and $\dot{\bar{\phi}} = 0_p$, q_d proves to be the unique equilibrium position of the closed-loop system —or equivalently, 0_n is the unique equilibrium position error of the closed loop—, while the parameter estimation error equilibrium vector $\bar{\phi}_e$ turns out to be defined by the solutions of the equation $G(q_d)\bar{s}_a(\bar{\phi}_e) = 0_n$, and consequently $\bar{s}_a(\bar{\phi}_e) \in \ker(G(q_d))$. ◁

Proposition 2 *Consider the closed-loop system in Eqs.* (28)*, under the satisfaction of Assumption 1 and the conditions on the vector function s_d stated through the expressions in* (4)–(7)*. Thus, for any positive definite diagonal matrices K_P and Γ, and any ε satisfying inequality* (26)*, the trivial solution $(\bar{q}, \bar{\phi})(t) \equiv (0_n, 0_p)$ is stable and, for any initial condition $(\bar{q}, \dot{q}, \bar{\phi})(0) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, $\bar{q}(t) \to 0_n$ as $t \to \infty$, and $\bar{s}_a(\bar{\phi}(t)) \to \ker(G(q_d))$ as $t \to \infty$, with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$.*

Proof. By (27), we see that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $\forall t \geq 0$. This proves that, under the proposed adaptive scheme, input saturation is avoided. Now, in order to develop the stability/convergence analysis, let us define the scalar function

$$V_1(\bar{q}, \dot{q}, \bar{\phi}) = V_0(\bar{q}, \dot{q}) + \int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr \tag{30}$$

where $\int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr = \sum_{j=1}^{p} \int_0^{\bar{\phi}_j} \bar{\sigma}_{aj}(r_j)\gamma_j^{-1}dr_j$, and $V_0(\bar{q}, \dot{q})$ is as defined in Eq. (9); the complete expression is given as

$$V_1(\bar{q}, \dot{q}, \bar{\phi}) = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})H(q)\dot{q} + \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + \int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr$$

Note that, from the positive definite and radially unbounded characters of $V_0(\bar{q}, \dot{q})$ (shown in the proof of Proposition 1) and points 6b, 4, and 5 of Lemma 1 (through which the integral term in the right-hand side of Eq. (30) is concluded to be a radially unbounded positive definite function of $\bar{\phi}$), $V_1(\bar{q}, \dot{q}, \bar{\phi})$ proves to be positive definite and radially unbounded. Its upper-right derivative along the system trajectories, $\dot{V}_1 = D^+V_1$, is given by

$$\begin{aligned}
\dot{V}_1(\bar{q},\dot{q},\bar{\phi}) =\ & \dot{q}^T H(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{H}(q,\dot{q})\dot{q} + \varepsilon s_P^T(K_P\bar{q})H(q)\ddot{q} + \varepsilon\dot{q}^T\dot{H}(q,\dot{q})s_P(K_P\bar{q}) \\
& + \varepsilon\dot{q}^T H(q)s_P'(K_P\bar{q})K_P\dot{q} + s_P^T(K_P\bar{q})\dot{q} + \bar{s}_a^T(\bar{\phi})\Gamma^{-1}\dot{\bar{\phi}} \\
=\ & \dot{q}^T\big[-C(q,\dot{q})\dot{q} - F\dot{q} - s_d\big(\bar{q},\dot{q},s_a(\phi)\big) - s_P(K_P\bar{q}) + G(q)\bar{s}_a(\bar{\phi})\big] + \frac{1}{2}\dot{q}^T\dot{H}(q,\dot{q})\dot{q} \\
& + \varepsilon s_P^T(K_P\bar{q})\big[-C(q,\dot{q})\dot{q} - F\dot{q} - s_d\big(\bar{q},\dot{q},s_a(\phi)\big) - s_P(K_P\bar{q}) + G(q)\bar{s}_a(\bar{\phi})\big] \\
& + \varepsilon\dot{q}^T\dot{H}(q,\dot{q})s_P(K_P\bar{q}) + \varepsilon\dot{q}^T H(q)s_P'(K_P\bar{q})K_P\dot{q} + s_P^T(K_P\bar{q})\dot{q} \\
& - \bar{s}_a^T(\bar{\phi})G^T(q)\big[\dot{q} + \varepsilon s_P(K_P\bar{q})\big] \\
=\ & -\dot{q}^T F\dot{q} - \dot{q}^T s_d\big(\bar{q},\dot{q},s_a(\phi)\big) - \varepsilon s_P^T(K_P\bar{q})F\dot{q} - \varepsilon s_P^T(K_P\bar{q})s_d\big(\bar{q},\dot{q},s_a(\phi)\big) \\
& - \varepsilon s_P^T(K_P\bar{q})s_P(K_P\bar{q}) + \varepsilon\dot{q}^T C(q,\dot{q})s_P(K_P\bar{q}) + \varepsilon\dot{q}^T H(q)s_P'(K_P\bar{q})K_P\dot{q}
\end{aligned}$$

where $H(q)\ddot{q}$ and $\dot{\bar{\phi}}$ have been replaced by their equivalent expressions from the closed-loop manipulator dynamics in Eqs. (28), Property 2b has been used, and $s_P'(K_P\bar{q})$ was defined in (14). Observe that from Properties 1, 2a, and 3, the satisfaction of (5), points (b) of Definition 1 and 2 of Lemma 1, and the positive definite character of K_P, we have that

$$\dot{V}_1(\bar{q},\dot{q},\bar{\phi}) \leq -\dot{q}^T s_d\big(\bar{q},\dot{q},s_a(\phi)\big) - W_1(\bar{q},\dot{q})$$

where $W_1(\bar{q},\dot{q})$ was defined in (15) as

$$W_1(\bar{q},\dot{q}) = \begin{pmatrix} \|s_P(K_P\bar{q})\| \\ \|\dot{q}\| \end{pmatrix}^T \begin{pmatrix} \varepsilon & -\frac{\varepsilon}{2}(f_M+\kappa) \\ -\frac{\varepsilon}{2}(f_M+\kappa) & f_m - \varepsilon\beta_M \end{pmatrix} \begin{pmatrix} \|s_P(K_P\bar{q})\| \\ \|\dot{q}\| \end{pmatrix}$$

and (based on the satisfaction if inequality (26)) shown to be a positive definite function in the proof of Proposition 1. From this and (6), we have that $\dot{V}_1(\bar{q},\dot{q},\bar{\phi}) \leq 0$, $\forall(\bar{q},\dot{q},\bar{\phi}) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, with $\dot{V}_1(\bar{q},\dot{q},\bar{\phi}) = 0 \iff (\bar{q},\dot{q}) = (0_n, 0_n)$. Therefore, by Lyapunov's second method [32, Chap. II, §6], the trivial solution $(\bar{q},\bar{\phi})(t) \equiv (0_n, 0_p)$ is concluded to be stable. Now, in view of the radially unbounded character of $V_1(\bar{q},\dot{q},\bar{\phi})$, the set $\Omega \triangleq \{(\bar{q},\dot{q},\bar{\phi}) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p : V_1(\bar{q},\dot{q},\bar{\phi}) \leq c\}$ is compact for any positive constant c [31, p. 128]. Moreover, in view of the negative semidefinite character of $\dot{V}_1(\bar{q},\dot{q},\bar{\phi})$, Ω is positively invariant with respect to the closed-loop dynamics [31, p. 115]. Furthermore, from previous arguments, we have that $E \triangleq \{(\bar{q},\dot{q},\bar{\phi}) \in \Omega : \dot{V}_1(\bar{q},\dot{q},\bar{\phi}) = 0\} = \{(\bar{q},\dot{q},\bar{\phi}) \in \Omega : \bar{q} = \dot{q} = 0_n\}$. Further, from Remark 4, the largest invariant set in E, denoted $\mathcal{M}$, is given as $\mathcal{M} = \{(\bar{q},\dot{q},\bar{\phi}) \in E : \bar{s}_a(\bar{\phi}) \in \ker(G(q_d))\}$. Thus, by the invariance theory [37, §7.2] —more specifically, by [37, Theorem 7.2.1]—, we have that $(\bar{q},\dot{q},\bar{\phi})(0) \in \Omega \implies (\bar{q},\dot{q},\bar{\phi})(t) \to \mathcal{M}$ as $t \to \infty$. Since this holds for any $c > 0$ and $V_1(\bar{q},\dot{q},\bar{\phi})$ is radially unbounded (in view of which Ω may be rendered arbitrarily large), we conclude that, for any $(\bar{q},\dot{q},\bar{\phi})(0) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, $\bar{q}(t) \to 0_n$ as $t \to \infty$ and $\bar{s}_a(\bar{\phi}(t)) \to \ker(G(q_d))$ as $t \to \infty$, which completes the proof. ◁

Corollary 1 *If $G^T(q_d)G(q_d)$ is nonsingular, then the trivial solution $(\bar{q},\bar{\phi})(t) \equiv (0_n, 0_p)$ is globally asymptotically stable.*

Proof. This is concluded by noting on the one hand that non-singularity of $G^T(q_d)G(q_d)$ implies that $\ker(G(q_d)) = \{0_p\}$, and on the other hand that $\bar{s}_a(\bar{\phi}) = 0_p \iff \bar{\phi} = 0_p$. Then, from Proposition 2, we have that, for any $(\bar{q}, \dot{q}, \bar{\phi})(0) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, $(\bar{q}, \bar{\phi})(t) \to (0_n, 0_p)$ as $t \to \infty$, whence the stability of the trivial solution $(\bar{q}, \bar{\phi})(t) \equiv (0_n, 0_p)$ is concluded to be globally asymptotical [31, §4.1], [41, §26], [32, Chap. I, §2.10–2.11], [42, §2.3.1]. ◁

Remark 5 Adaptive versions of the SP-SD controller and of the SPD and SPDgc-like algorithms of [9] are obtained by considering in the proposed design method the expressions in (16), (18), and (20), respectively, with suitable adjustments on the saturation function parameter conditions. Thus, the SP-SD controller with adaptive gravity compensation is obtained from (24) taking

$$s_d(\bar{q}, \dot{q}, \hat{\theta}) = s_D(K_D\dot{q}) \tag{31}$$

with $s_D(\cdot)$ and K_D as defined in Remark 1, and the involved bound values, M_{Pi} and M_{Di}, satisfying

$$M_{Pi} + M_{Di} < T_i - B_{gi}^{Ma} \tag{32}$$

$i = 1, \ldots, n$, the adaptive SPD scheme is gotten taking

$$s_d(\bar{q}, \dot{q}, \hat{\theta}) = s_P(K_P\bar{q} + K_D\dot{q}) - s_P(K_P\bar{q}) \tag{33}$$

with $s_P(\cdot)$ as defined for this case in Remark 1, and bound values satisfying

$$M_{Pi} \leq T_i - B_{gi}^{Ma} \tag{34}$$

$i = 1, \ldots, n$, and the adaptive SPDgc-like algorithm is obtained taking

$$s_d(\bar{q}, \dot{q}, \hat{\theta}) = s_0\big(G(q)\hat{\theta} - s_P(K_P\bar{q})\big) - s_0\big(G(q)\hat{\theta} - s_P(K_P\bar{q}) - K_D\dot{q}\big) \tag{35}$$

with $s_0(\cdot)$ as defined in Remark 1, and the involved saturation function parameters satisfying

$$B_{gi}^{M_a} + M_{Pi} < L_{0i} \leq M_{0i} < T_i \tag{36}$$

$i = 1, \ldots, n$. For these cases, κ in (26) remains as specified in Remark 1, through Eqs. (22), *i.e.* (for ease of reading, Eqs. (22) are restated here)

$$\kappa = \max_i \{\sigma'_{iM} k_{Di}\} \tag{37a}$$

where

$$\sigma'_{iM} = \begin{cases} \sigma'_{DiM} & \text{in the SP-SD case} \\ \sigma'_{PiM} & \text{in the SPD case} \\ \sigma'_{0iM} & \text{in the SPDgc-like case} \end{cases} \tag{37b}$$

σ'_{DiM}, σ'_{PiM}, and σ'_{0iM} respectively being the positive bounds of $D^+\sigma_{Di}(\cdot)$, $D^+\sigma_{Pi}(\cdot)$, and $D^+\sigma_{0i}(\cdot)$, in accordance to point 2 of Lemma 1. ◁

Figure 2: Experimental setup

6 Experimental results

In order to experimentally corroborate the efficiency of the developed adaptive approach, real-time control implementations were carried out on a 2-DOF direct-drive manipulator. The experimental setup, shown in Fig. 2, is a prototype of the 2-revolute-joint robot arm used in [43, 44], located at the *Instituto Tecnológico de la Laguna*. The actuators are direct-drive brushless motors operated in torque mode, so they act as torque source and accept an analog voltage as a reference of torque signal. The control algorithm is executed at a 2.5 ms sampling period in a control board (based on a DSP 32-bit floating point microprocessor from Texas Instrument) mounted on a PC-host computer. The robot software is in open architecture, whose platform is based in C language to run the control algorithm in real time.

For the considered experimental manipulator, Properties 1, 2a, 3–5 are satisfied with

$$G(q) = \begin{pmatrix} \sin q_1 & \sin(q_1 + q_2) \\ 0 & \sin(q_1 + q_2) \end{pmatrix} \quad , \quad \theta = \begin{pmatrix} 38.465 \\ 1.825 \end{pmatrix} \quad [\text{Nm}] \tag{38}$$

$\mu_m = 0.088$ kg m^2, $\mu_M = 2.533$ kg m^2, $k_C = 0.1455$ kg m^2, $B_{g1} = 40.29$ Nm, $B_{g2} = 1.825$ Nm, $f_m = 0.175$ kg m^2/s, and $f_M = 2.288$ kg m^2/s. The maximum allowed torques (input saturation bounds) are $T_1 = 150$ Nm and $T_2 = 15$ Nm for the first and second links respectively. From these data, one easily corroborates that Assumption 1 is fulfilled.

The proposed adaptive scheme in Eqs. (24)-(25) was tested in its SP-SD, SPD, and SPDgc-like forms, under the respective consideration of expressions (31)-(32), (33)-(34), and (35)-(36). The involved saturation functions were defined as

$$\sigma_{Pi}(\varsigma) = M_{Pi}\,\text{sat}(\varsigma/M_{P_i}) \quad \text{and} \quad \sigma_{Di}(\varsigma) = M_{Di}\,\text{sat}(\varsigma/M_{D_i})$$

$i = 1, 2$, in the SP-SD case;

$$\sigma_{Pi}(\varsigma) = \begin{cases} \varsigma & \forall|\varsigma| \leq L_{Pi} \\ \text{sign}(\varsigma)L_{Pi} + (M_{Pi} - L_{Pi})\tanh\left(\frac{\varsigma - \text{sign}(\varsigma)L_{Pi}}{M_{Pi} - L_{Pi}}\right) & \forall|\varsigma| > L_{Pi} \end{cases}$$

with $0 < L_{Pi} < M_{Pi}$, $i = 1, 2$, in the SPD case;

$$\sigma_{Pi}(\varsigma) = M_{Pi}\,\text{sat}(\varsigma/M_{P_i}) \quad \text{and} \quad \sigma_{0i}(\varsigma) = M_{0i}\,\text{sat}(\varsigma/M_{0_i})$$

$i = 1, 2$, in the SPDgc-like case; and

$$\sigma_{aj}(\varsigma) = \begin{cases} \varsigma & \forall |\varsigma| \leq L_{aj} \\ \mathrm{sign}(\varsigma) L_{aj} + (M_{aj} - L_{aj}) \tanh\left(\frac{\varsigma - \mathrm{sign}(\varsigma) L_{aj}}{M_{aj} - L_{aj}}\right) & \forall |\varsigma| > L_{aj} \end{cases}$$

with $0 < L_{aj} < M_{aj}$, $j = 1, 2$, in all the three cases. Let us note that with these saturation functions we have $\sigma'_{PiM} = \sigma'_{DiM} = \sigma'_{0iM} = 1$, $\forall i \in \{1, 2\}$, and that in consequence, for the three controllers, inequality (5) is satisfied with $\kappa = \max_i\{k_{Di}\}$ (see Eqs. (37)). The experimental implementations were run fixing the following saturation parameter values (all of them expressed in Nm): $M_{P1} = 58$, $M_{D1} = 38$, $M_{P2} = 7$, and $M_{D2} = 4$ in the SP-SD case; $M_{P1} = 50$, $M_{P2} = 7$, and $L_{Pi} = 0.9 M_{Pi}$, $i = 1, 2$, in the SPD case; $M_{01} = 120$, $M_{P1} = 50$, $M_{02} = 12$, and $M_{P2} = 7$ in the SPDgc-like case; and $M_{a1} = 50$, $M_{a2} = 3$, and $L_{aj} = 0.9 M_{aj}$, $j = 1, 2$, in all the three cases. These saturation function parameter values were corroborated to satisfy inequalities (23), (32), (34), and (36), taking $B_{gi}^{M_a} = \sum_{j=1}^{2} B_{G_{ij}} M_{aj}$, $i = 1, 2$, *i.e.* $B_{g1}^{M_a} = 53$ and $B_{g2}^{M_a} = 3$.

For comparison purposes, additional experiments were implemented considering the adaptive controller proposed by [18] —referred to as Z$_e$00— (choice made in terms of the analog nature of the compared algorithms: bounded adaptive; comparison of controllers of different nature looses coherence), *i.e.*

$$\begin{aligned} u &= G(q)\hat{\theta} - K_P T_h(\Lambda_P \bar{q}) - K_D T_h(\Lambda_D \dot{q}) \\ \dot{\hat{\theta}} &= P\big(Q(\bar{q}, \dot{q}), \hat{\theta}\big) \end{aligned}$$

where $T_h(x) = \big(\tanh(x_1), \ldots, \tanh(x_n)\big)^T$; $\Lambda_P = \mathrm{diag}[\lambda_{P1}, \ldots, \lambda_{Pn}]$ and $\Lambda_D = \mathrm{diag}[\lambda_{D1}, \ldots, \lambda_{Dn}]$ with $\lambda_{Pi} = 1$ [rad]$^{-1}$ and $\lambda_{Di} = 1$ s/rad, $\forall i \in \{1, \ldots, n\}$; $Q(\bar{q}, \dot{q}) = -\Gamma G^T(q)[\dot{q} + \varepsilon T_h(\bar{q})]$; the elements of P are defined as

$$P_j(Q, \hat{\theta}) = \begin{cases} Q_j & \text{if } \theta_{jm} < \hat{\theta}_j < \theta_{jM} \text{ or } \big(\hat{\theta}_j \leq \theta_{jm} \text{ and } Q_j \geq 0\big) \text{ or } \big(\hat{\theta}_j \geq \theta_{jM} \text{ and } Q_j \leq 0\big) \\ 0 & \text{if } \big(\hat{\theta}_j \leq \theta_{jm} \text{ and } Q_j < 0\big) \text{ or } \big(\hat{\theta}_j \geq \theta_{jM} \text{ and } Q_j > 0\big) \end{cases}$$

$j = 1, \ldots, p$, with θ_{jm} and θ_{jM} being known lower and upper bounds of θ_j respectively; and the initial auxiliary state values are taken such that $\hat{\theta}_j(0) \in [\theta_{jm}, \theta_{jM}]$, $j = 1, \ldots, p$. The parameter bounds were fixed at $\theta_{1m} = 10$, $\theta_{1M} = 70$, $\theta_{2m} = 0.5$, and $\theta_{2M} = 5$ [Nm].

The results of two experimental tests (for every implemented controller) are presented. The initial and desired link positions at all the executed experiments were $q_1(0) = q_2(0) = \dot{q}_1(0) = \dot{q}_2(0) = 0$ and $q_{d1} = q_{d2} = \pi/4$ [rad], while the auxiliary state variable initial values were taken as $\phi_1(0) = \phi_2(0) = 0$ for the SP-SD, SPD, and SPDgc-like algorithms, and $\phi_1(0) = 20$, $\phi_2(0) = 2$ [Nm] for the Z$_e$00 controller. Let us notice that through the selected desired configurations, the condition stated by Corollary 1 is satisfied (one can verify from $G(q)$ in (38) that, for the considered manipulator, the desired configurations that satisfy the condition stated by Corollary 1 are those such that $q_{d1} \neq m_1\pi$ and $q_{d1} + q_{d2} \neq m_2\pi$, for any $m_1, m_2 = 0, \pm 1, \pm 2, \ldots$).

With the aim at getting fast position responses, in the first implementation —referred to as Test 1—, high control gains were taken for the SP-SD, SPD, and SPDgc-like algorithms,

Table 1: Control parameter values: Test 1

prmtr.	SP-SD	SPD	SPDgc-like	Z_e00	*units*
k_{P1}	2900	3500	3700		N m/rad
				70	N m
k_{P2}	225	250	250		N m/rad
				9	N m
k_{D1}	40	80	40		N m s/rad
				6.5	N m
k_{D2}	3	6	3		N m s/rad
				2.5	N m
γ_1	2.5	2.5	9	500	N m/rad
γ_2	0.05	0.05	0.15	2	
ε	0.000021	0.000014	0.000017		rad/N m s
				0.0005	rad/s

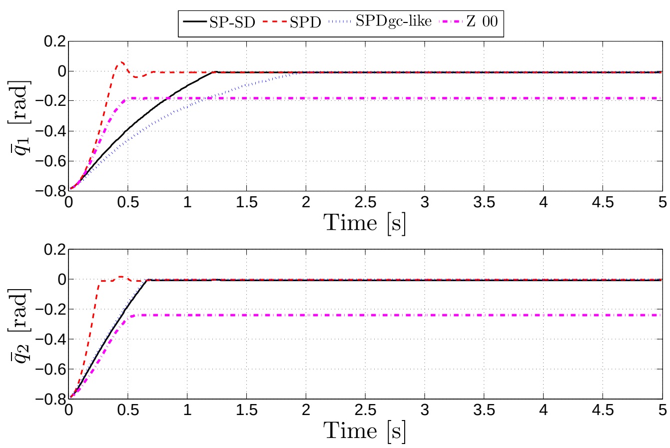

Figure 3: Test 1: position errors

and a consequent considerably small value of ε satisfying inequality (26) was fixed. As for the Z_e00 scheme, a relatively small value of ε was also taken (although several times higher than for the other algorithms) and reasonable values of the rest of the tuning parameters were fixed disregarding the tuning procedure stated in [18, Theorem 2] in order to prevent considerably slower responses, with control gains small enough to avoid input saturation (recall that they fix the bounds of the SP and SD actions). Under the stated considerations, the tuning parameter combination giving rise to the best closed-loop performance —in terms mainly of stabilization time (as short as possible) and transient response (avoiding or lowering down overshoot and oscillations as much as possible)— was determined from numerous trial-and-error experiments for every implemented controller. The resulting values are presented in Table 1.

Figures 3–5 show the results of Test 1 for every implemented controller. Observe that the SP-SD, SPD, and SPDgc-like algorithms achieve the position regulation objective —avoiding input saturation— in less than 2 seconds. On the other hand, the parameter estimators

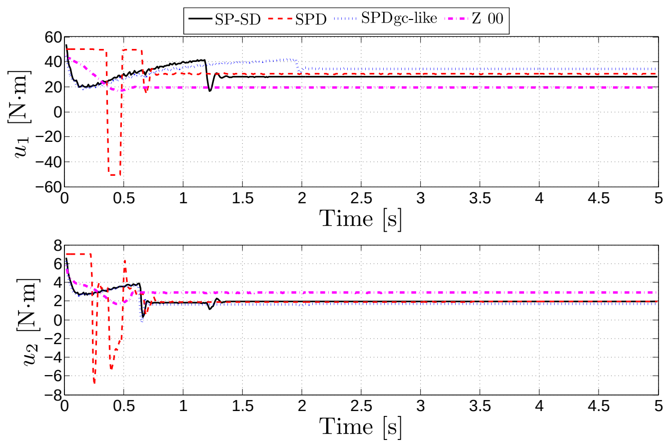

Figure 4: Test 1: control signals

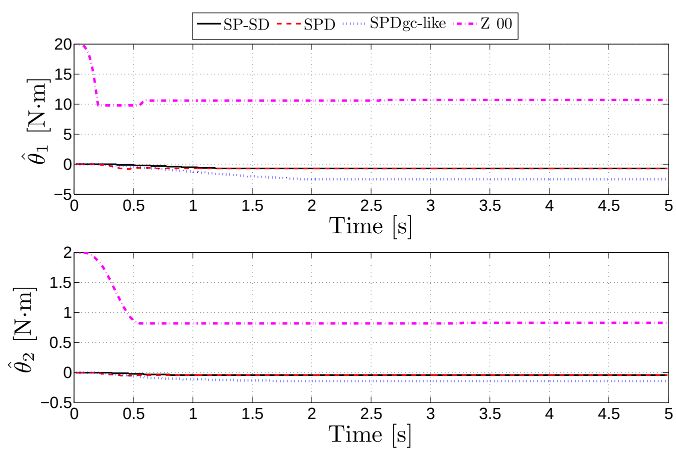

Figure 5: Test 1: parameter estimators

Table 2: Control parameter values: Test 2

prmtr.	SP-SD	SPD	SPDgc-like	Z_e00	*units*
k_{P1}	2100	1600	1850	75	N m/rad N m
k_{P2}	225	125	250	9	N m/rad N m
k_{D1}	40	80	40	6.5	N m s/rad N m
k_{D2}	3	6	3	2.5	N m s/rad N m
γ_1	0.5	0.5	9	375	N m/rad
γ_2	0.015	0.015	0.25	15	
ε	3	2.5	0.31	0.1	rad/N m s rad/s

present important steady-state errors. These parametric convergence errors are mainly due to the unmodeled phenomena such as the static friction. It is worth pointing out that the small value of ε importantly reduces the ability of the adaptation auxiliary dynamics to decrease the parameter estimation steady-state error. However, it is important to note that this does neither prevent the position regulation objective to be achieved (avoiding input saturation), nor to do it in a considerably short time. As for the additional implementation, notice that the Z_e00 controller generates lower bias in the parameter estimator steady-state values but the size of the errors is however observed to remain considerable and, more importantly, the position responses could not be stabilized throughout the duration of the test.

In order to get an improved parameter estimation, in the second implementation —referred to as Test 2—, a higher value of ε was fixed for the SP-SD, SPD, and SPDgc-like algorithms (considerably higher than in the precedent test) disregarding inequality (26) (recall that the condition stated by inequality (26) is only sufficient) keeping large control gains; in this context, for every one of the mentioned controllers, the tuning parameter combination giving rise to the best closed-loop performance was determined from numerous trial-and-error experiments. As for the Z_e00 scheme, an increased value of ε was also taken and adjustments in a control and the adaptation gains were done, keeping the rest of the control parameters with the same value taken in Test 1 but gains λ_{Pi}, $i = 1, 2$, (inside the hyperbolic tangent functions involved in the SP action) greater than unity were fixed; specifically: $\lambda_{P1} = 1.75$, $\lambda_{P2} = 3.5$ [rad^{-1}]. The resulting values are presented in Table 2.

Figures 6–8 show the results of Test 2 for every implemented controller. Observe that, as in Test 1, the SP-SD, SPD, and SPDgc-like algorithms achieved the position regulation objective —avoiding input saturation— in less than 2 seconds. Moreover, an improved parameter estimation took place. In this direction, observe that, among the referred schemes, the algorithm with the greatest parameter estimation bias is the one with the lowest value assigned to ε, corresponding to the SPDgc-like controller. As for the Z_e00 scheme, an improved parameter estimation, comparable to that obtained through the algorithms that

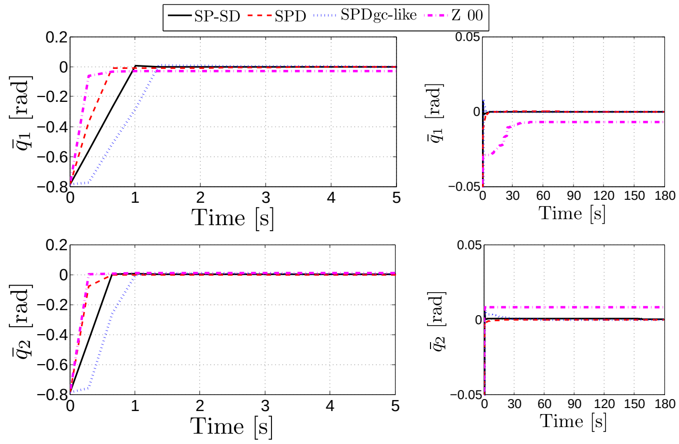

Figure 6: Test 2: position errors

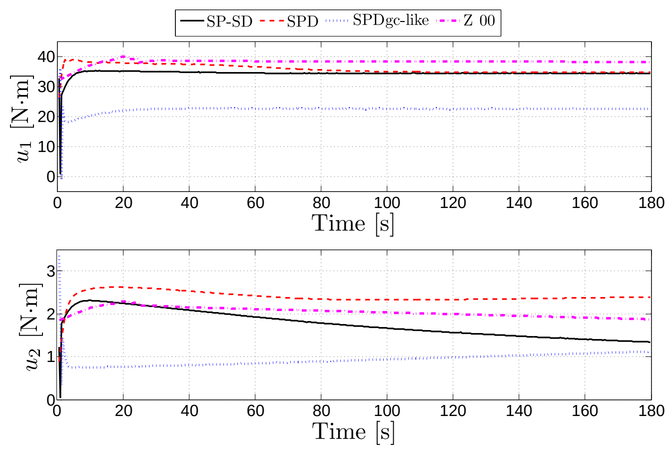

Figure 7: Test 2: control signals

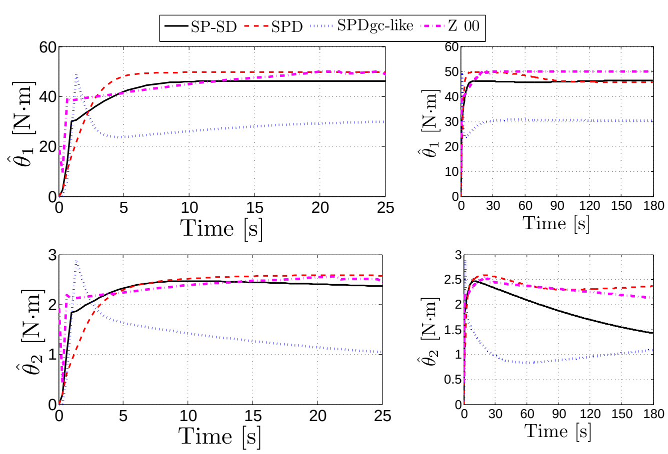

Figure 8: Test 2: parameter estimators

took the highest value of ε *i.e.* the SP-SD and SPD controllers, is observed too. Nevertheless, position stabilization was not completely achieved throughout the duration of the test.

7 Conclusions

In this work, a generalized adaptive control scheme for the global regulation of robot manipulators with bounded inputs was proposed. Compared to the adaptive approaches previously developed in a bounded-input context, the proposed scheme guarantees the adaptive regulation objective: globally, avoiding discontinuities in the control expression as well as in the adaptation auxiliary dynamics, preventing the inputs to attain their natural saturation bounds, imposing no *saturation-avoidance* restriction on the choice of the P and D (positive) control gains, and giving rise to adaptive versions of a general class of bounded PD-type controllers that include the so-called SP-SD algorithms among others. The efficiency of the proposed adaptive scheme was corroborated through real-time experimental tests on an actual 2-DOF manipulator. These results showed that fast enough position regulation is always possible —avoiding input saturation— through sufficiently high control gains. On the other hand, the unmodeled phenomena —such as the static friction— produce steady-state errors in the parameter estimations. These may be decreased —when a suitable desired configuration is defined (according to Corollary 1)— through a higher value of the adaptation subsystem gain ε.

Appendix: Proof of Lemma 1

1. Since σ is a Lipschitz-continuous function that keeps the sign of its argument (according to point (a) of Definition 1), and is nondecreasing and bounded by M, there exist positive constants $c^- \leq M$ and $c^+ \leq M$ such that

$$\lim_{|\varsigma|\to\infty} \sigma(\varsigma) = \frac{(\text{sign}(\varsigma) - 1)c^- + (\text{sign}(\varsigma) + 1)c^+}{2} \triangleq \sigma_\infty$$

Hence, we have that:

$$\begin{aligned}
\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) &= \lim_{|\varsigma|\to\infty} \limsup_{h\to 0^+} \frac{\sigma(\varsigma + h) - \varsigma(\varsigma)}{h} \\
&= \limsup_{h\to 0^+} \lim_{|\varsigma|\to\infty} \frac{\sigma(\varsigma + h) - \varsigma(\varsigma)}{h} \\
&= \limsup_{h\to 0^+} \frac{\sigma_\infty - \sigma_\infty}{h} = 0
\end{aligned}$$

2. Since σ is a Lipschitz-continuous nondecreasing function, we have that $D^+\sigma(\varsigma)$ exists and is piecewise-continuous on $\mathbb{R}$, and that $D^+\sigma(\varsigma) \geq 0$, $\forall \varsigma \in \mathbb{R}$. On the other hand, in view of its piecewise-continuity, $D^+\sigma(\varsigma)$ is bounded on any compact interval on $\mathbb{R}$. Thus, its boundedness holds on $\mathbb{R}$ if $\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) < \infty$. Since $\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) = 0$

(according to point 1 of the statement), we conclude boundedness of $D^+\sigma(\varsigma)$ (on $\mathbb{R}$), *i.e.* there exists a non-negative finite scalar σ'_M such that $D^+\sigma(\varsigma) \leq \sigma'_M$, $\forall\varsigma \in \mathbb{R}$. Finally, observe that by virtue of point (a) of Definition 1, there exists $a \in (0,\infty]$ such that $D^+\sigma(\varsigma) > 0$, $\forall\varsigma \in (-a,a)\setminus\{0\}$, whence we conclude that $\sigma'_M > 0$.

3. From Lipschitz-continuity of σ, its satisfaction of point (a) of Definition 1, and the boundedness of $D^+\sigma$ by a positive constant σ'_M (according to point 2 of the statement), it follows that

$$\frac{D^+\sigma(k\varsigma)}{\sigma'_M}|\sigma(k\varsigma)| \leq |\sigma(k\varsigma)| \leq \sigma'_M|k\varsigma|$$

$\forall\varsigma \in \mathbb{R}$, whence —considering that σ has the sign of its argument (according to point (a) of Definition 1)— we have that

$$\int_0^\varsigma \frac{\sigma(kr)}{\sigma'_M}D^+\sigma(kr)dr \leq \int_0^\varsigma \sigma(kr)dr \leq \int_0^\varsigma k\sigma'_M r dr$$

wherefrom we get

$$\frac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma'_M\varsigma^2}{2}$$

$\forall\varsigma \in \mathbb{R}$.

4. Strict positivity of $\int_0^\varsigma \sigma(kr)dr$ on $\mathbb{R}\setminus\{0\}$ follows from points 3 of the statement and (a) of Definition 1, by noting that $\sigma^2(k\varsigma) > 0$, $\forall\varsigma \neq 0$.

5. From the Lipschitz-continuous and nondecreasing characters of σ, and its satisfaction of point (a) of Definition 1, we have that there exist constants $a > 0$, $k_a > 0$, and $c \geq 1$ such that $|\sigma(\varsigma)| \geq k_a\,|a\,\mathrm{sat}(\varsigma/a)|^c$, whence we get

$$S_a(\varsigma) \triangleq \int_0^\varsigma \mathrm{sign}(r)k_a\,|a\,\mathrm{sat}(r/a)|^c\,dr \leq \int_0^\varsigma \sigma(k\varsigma)dr$$

$\forall\varsigma \in \mathbb{R}$, with

$$S_a(\varsigma) = \begin{cases} \frac{k_a}{c+1}\,|\varsigma|^{c+1} & \forall|\varsigma| \leq a \\ k_a a^c\left(|\varsigma| - \frac{ac}{c+1}\right) & \forall|\varsigma| > a \end{cases}$$

Thus, from these expressions we observe, on the one hand, that

$$\lim_{|\varsigma|\to\infty} S_a(\varsigma) \leq \lim_{|\varsigma|\to\infty}\int_0^\varsigma \sigma(kr)dr$$

and, on the other, that $S_a(\varsigma) \to \infty$ as $|\varsigma| \to \infty$, wherefrom we conclude that $\int_0^\varsigma \sigma(kr)dr \to \infty$ as $|\varsigma| \to \infty$.

6. Suppose σ is strictly increasing. Let $\psi, \eta, \varsigma \in \mathbb{R}$.

(a) Since σ is strictly increasing, we have that

$$\sigma(\psi) > \sigma(\eta) \iff \psi > \eta$$

and

$$\sigma(\psi) < \sigma(\eta) \iff \psi < \eta$$

Let $\psi = \varsigma + \eta$. Then

$$\sigma(\varsigma + \eta) - \sigma(\eta) > 0 \iff \varsigma > 0 \quad \forall \eta \in \mathbb{R}$$

and

$$\sigma(\varsigma + \eta) - \sigma(\eta) < 0 \iff \varsigma < 0 \quad \forall \eta \in \mathbb{R}$$

whence it follows that $\varsigma[\sigma(\varsigma + \eta) - \sigma(\eta)] > 0$, $\forall \varsigma \neq 0$, $\forall \eta \in \mathbb{R}$.

(b) For any constant $a \in \mathbb{R}$, let $\bar{\sigma}(\varsigma) = \sigma(\varsigma + a) - \sigma(a)$.

- *Lipschitz-continuity.* From the Lipschitz-continuity of σ and point 2 of the statement, we have that $|\sigma(\varsigma) - \sigma(\eta)| \leq \sigma'_M |\varsigma - \eta|$, $\forall \varsigma, \eta \in \mathbb{R}$. Then

$$\begin{aligned} \big|\bar{\sigma}(\varsigma) - \bar{\sigma}(\eta)\big| &= \big|\big(\sigma(\varsigma + a) - \sigma(a)\big) - \big(\sigma(\eta + a) - \sigma(a)\big)\big| \\ &= \big|\sigma(\varsigma + a) - \sigma(\eta + a)\big| \\ &\leq \sigma'_M \big|(\varsigma + a) - (\eta + a)\big| \\ &\leq \sigma'_M \big|\varsigma - \eta\big| \end{aligned}$$

  $\forall \varsigma, \eta \in \mathbb{R}$, which shows that $\bar{\sigma}$ is Lipschitz-continuous.

- *Strictly increasing monotonicity.* From the strictly increasing monotonicity of σ, we have that

$$\begin{aligned} \bar{\sigma}(\varsigma) > \bar{\sigma}(\eta) &\iff \sigma(\varsigma + a) - \sigma(a) > \sigma(\eta + a) - \sigma(a) \\ &\iff \sigma(\varsigma + a) > \sigma(\eta + a) \\ &\iff \varsigma + a > \eta + a \\ &\iff \varsigma > \eta \end{aligned}$$

  which shows that $\bar{\sigma}$ is strictly increasing.

- $\varsigma\bar{\sigma}(\varsigma) > 0$, $\forall \varsigma \neq 0$. From point 6a of the Lemma, we have that $\varsigma\bar{\sigma}(\varsigma) = \varsigma[\sigma(\varsigma + a) - \sigma(a)] > 0$, for all $\varsigma \neq 0$ and any $a \in \mathbb{R}$.
- $|\bar{\sigma}(\varsigma)| \leq \bar{M} = M + |\sigma(a)|$, $\forall \varsigma \in \mathbb{R}$. Since $|\sigma(\varsigma)| \leq M$, $\forall \varsigma \in \mathbb{R}$, we have that

$$|\bar{\sigma}(\varsigma)| = |\sigma(\varsigma + a) - \sigma(a)| \leq |\sigma(\varsigma + a)| + |\sigma(a)| \leq M + |\sigma(a)| = \bar{M}$$

Thus, according to Definition 1, $\bar{\sigma}$ is concluded to be a strictly increasing generalized saturation with bound $\bar{M} = M + |\sigma(a)|$.

7. Let us begin by noting, from point (c) of Definition 1, that $|\nu(\eta)| < L \implies \sigma\big(\nu(\eta)\big) \equiv \nu(\eta)$, $\forall \eta \in \mathbb{R}$. Furthermore, $|\varsigma + \nu(\eta)| < L \implies \sigma\big(\varsigma + \nu(\eta)\big) = \varsigma + \nu(\eta)$, $\forall \eta \in \mathbb{R}$. Hence,

$$\varsigma\big[\sigma\big(\varsigma + \nu(\eta)\big) - \sigma\big(\nu(\eta)\big)\big] = \varsigma^2 > 0 \;\text{ for all } \varsigma \neq 0 \text{ such that } |\varsigma + \nu(\eta)| < L\text{, and all } \eta \in \mathbb{R}. \tag{39}$$

On the other hand, if $\varsigma + \nu(\eta) \geq L$, which implies that

$$\varsigma \geq L - \nu(\eta) \geq L - |\nu(\eta)| > 0$$

$\forall \eta \in \mathbb{R}$, then (from point (c) of Definition 1 and the nondecreasing character of σ)

$$\sigma\big(\varsigma + \nu(\eta)\big) - \sigma\big(\nu(\eta)\big) \geq L - \nu(\eta) \geq L - |\nu(\eta)| > 0$$

$\forall \eta \in \mathbb{R}$, while if $\varsigma + \nu(\eta) \leq -L$, which implies that

$$\varsigma \leq -L - \nu(\eta) \leq -L + |\nu(\eta)| < 0$$

$\forall \eta \in \mathbb{R}$, then

$$\sigma\big(\varsigma + \nu(\eta)\big) - \sigma\big(\nu(\eta)\big) \leq -L - \nu(\eta) \leq -L + |\nu(\eta)| < 0$$

$\forall \eta \in \mathbb{R}$, and consequently

$$\varsigma\big[\sigma\big(\varsigma + \nu(\eta)\big) - \sigma\big(\nu(\eta)\big)\big] > 0 \quad \text{for all } \varsigma \in \mathbb{R} \text{ such that } |\varsigma + \nu(\eta)| \geq L, \text{ and all } \eta \in \mathbb{R}. \tag{40}$$

Thus, from (39) and (40), it follows that $\varsigma\big[\sigma\big(\varsigma + \nu(\eta)\big) - \sigma\big(\nu(\eta)\big)\big] > 0$, $\forall \varsigma \neq 0$, $\forall \eta \in \mathbb{R}$.

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References

[1] N.J. Krikelis and S.K. Barkas, "Design of tracking systems subject to actuator saturation and integrator wind-up," *International Journal of Control*, **39**(4), 667–682 (1984).

[2] B.S. Chen and S.S. Wang, "The stability of feedback control with nonlinear saturating actuator: time domain approach," *IEEE Transactions on Automatic Control*, **33**(5), 483–487 (1988).

[3] P. Kapasouris and M. Athans, "Control systems with rate and magnitude saturation for neutrally stable open loop systems," *Proc. 29th IEEE Conference on Decision and Control*, Honolulu, HI (1990), pp. 3404–3409.

[4] R.L. Kosut, "Design of linear systems with saturating linear control and bounded states," *IEEE Transactions on Automatic Control*, **28**(1), 121–124 (1983).

[5] D.S. Bernstein and A.N. Michel, "A choronological bibliography on saturating actuators," *International Journal of Robust and Nonlinear Control*, **5**, 375–380 (1995).

[6] R. Kelly, V. Santibáñez, and A. Loría, *Control of Robot Manipulators in Joint Space* (Springer, London, 2005).

[7] R. Kelly, V. Santibáñez, and H. Berghuis, "Point-to-point robot control under actuator constraints," *Control Engineering Practice*, **5**(11), 1555–1562 (1997).

[8] V. Santibáñez, R. Kelly, and F. Reyes, "A new set-point controller with bounded torques for robot manipulators," *IEEE Transactions on Industrial Electronics*, **45**(1), 126–133 (1998).

[9] A. Zavala-Río and V. Santibáñez, "Simple extensions of the PD-With-Gravity-Compensation control law for robot manipulators with bounded inputs," *IEEE Trans. on Control Systems Technology*, **14**(5), 958–965 (2006).

[10] V. Santibáñez and R. Kelly, "On global regulation of robot manipulators: Saturated linear state feedback and saturated linear output feedback," *European Journal of Control*, **3**(2), 104–113 (1997).

[11] A. Loría, R. Kelly, R. Ortega, and V. Santibáñez, "On global output feedback regulation of Euler-Lagrange systems with bounded inputs," *IEEE Transactions on Automatic Control*, **42**(8), 1138–1143 (1997).

[12] I.V. Burkov, "Stabilization of mechanical systems *via* bounded control and without velocity measurements," *Proc. 2nd Russian-Swedish Control Conference*, St. Petersburg, Russia (1995), pp. 37–41.

[13] V. Santibáñez and R. Kelly, "Global regulation for robot manipulators under SP-SD feedback," *Proc. 1996 IEEE Int. Conference on Robotics and Automation*, Minneapolis, MN (1996), pp. 927–932.

[14] A. Zavala-Río and V. Santibáñez, "A natural saturating extension of the PD-with-desired-gravity-compensation control law for robot manipulators with bounded inputs," *IEEE Transactions on Robotics*, **23**(2), 386–391 (2007).

[15] R. Ortega, A. Loría, R. Kelly, and L. Praly, "On passivity-based output feedback global stabilization of Euler-Lagrange systems," *Proc. 33th IEEE Conference on Decision and Control*, Lake Buena Vista, FL (1994), pp. 381–386.

[16] I.V. Burkov and L.B. Freidovich, "Stabilization of the position of a Lagrangian system with elastic elements and bounded control with and without measurement of velocities," *J. Appl. Maths. Mechs.*, **61**(3), 433–441 (1997).

[17] R. Colbaugh, E. Barany, and K. Glass, "Global regulation of uncertain manipulators using bounded controls," *Proc. 1997 IEEE International Conference on Robotics and Automation*, Albuquerque, NM (1997), pp. 1148–1155.

[18] E. Zergeroglu, W. Dixon, A. Behal, and D. Dawson, "Adaptive set-point control of robotic manipulators with amplitud-limited control inputs," *Robotica*, **18**(2), 171–181 (2000).

[19] A. Laib, "Adaptive output regulation of robot manipulators under actuator constraints," *IEEE Transactions on Robotics and Automation*, **16**(1), 29–35 (2000).

[20] W.E. Dixon, "Adaptive regulation of amplitud limited robot manipulators with uncertain kinematics and dynamics," *IEEE Transactions on Automatic Control*, **52**(3), 448–493 (2007).

[21] W.E. Dixon, M.S. de Queiroz, F. Zhang, and D.M. Dawson, "Tracking control of robot manipulators with bounded torque inputs," *Robotica*, **17**(2), 121–129 (1999).

[22] J.Z. Xiao, H.R. Wang, W. Zhang, and H.R. Wei, "Adaptive robotic control based on a filter function under the saturation of actuators," *Proc. Fifth International Conference on Machine Learning and Cybernetics*, Dalian, China (2006), pp. 283–287.

[23] S. Liuzzo and P. Tomei, "A global adaptive learning control for robotic manipulators," *Automatica*, **44**(5), 1379–1384 (2008).

[24] S. Liuzzo and P. Tomei, "Global adaptive learning control of robotic manipulators by output error feedback," *International Journal of Adaptive Control and Signal Processing*, **23**(1), 97–109 (2009).

[25] Y. Hong and B. Yao, "A globally stable high-performance adaptive robust control algorithm with input saturation for precision motion control of linear motor drive systems," *IEEE/ASME Transactions on Mechatronics*, **12**(2), 198–207 (2007).

[26] C. Hu, B. Yao, Z. Chen, and Q. Wang, "Adaptive robust repetitive control of an industrial biaxial precision gantry for contouring tasks," *IEEE Transactions on Control Systems Technology*, **19**(6), 1559–1568 (2011).

[27] A. Yarza, V. Santibáñez, and J. Moreno-Valenzuela, "Global asymptotic stability of the classical PID controller by considering saturation effects in industrial robots," *International Journal of Adavanced Robotic Systems*, **8**(4), 34–42 (2011).

[28] V. Santibáñez, R. Kelly, A. Zavala-Río, and P. Parada, "A new saturated nonlinear PID global regulator for robot manipulators," *Proc. 17th IFAC World Congress*, Seoul, Korea (2008), pp. 11690–11695.

[29] Y. Su and P.C. Müller, and C. Zheng, "Global asymptotic saturated PID control for robot manipulators," *IEEE Transactions on Control Systems Technology*, **18**(6), 1280–1288 (2010).

[30] J.L. Meza, V. Santibáñez, and V.M. Hernández, "Saturated nonlinear PID global regulator for robot manipulators: passivity-based analysis," *Proc. 16th IFAC World Congress*, Prague, Czech Republic (2005).

[31] H.K. Khalil, *Nonlinear Systems* (Prentice-Hall, Upper Saddle River, 2002).

[32] N. Rouche, P. Habets, and M. Laloy, *Stability Theory by Lyapunov's Direct Method* (Springer-Verlag, New York, 1977).

[33] S. Arimoto, *Control Theory of Non-Linear Mechanical Systems: A Passivity-Based and Circuit-Theoretic Approach* (Oxford University Press, Oxford, 1996).

[34] L. Sciavicco and B. Siciliano, *Modelling and Control of Robot Manipulators* (Springer, London, 2000).

[35] F.L. Lewis, D.M. Dawson, and C.T. Abdallah, *Robot Manipulator Control: Theory and Practice* (Marcel Dekker, Inc., New York, 2004).

[36] A.R. Teel, "Global stabilization and restricted tracking for multiple integrators with bounded controls," *Systems and Control Letters*, **18**(1), 165–171 (1992).

[37] A.N. Michel, L. Hou, and D. Liu, *Stability of Dynamical Systems* (Birkhäuser, Boston, 2008).

[38] B. Armstrong-Hélouvry, *Control of Machines with Friction* (Kluwer Academic Publishers, Boston, 1991).

[39] A.F. Filippov, *Differential Equations with Discontinuous Righthand Sides* (Kluwer Academic Publishers, Dordrecht, 1988).

[40] D. Shevitz and B. Paden, "Lyapunov stability theory of nonsmooth systems," *IEEE Transactions on Automatic Control*, **39**(9), 1910–1914 (1994).

[41] W. Hahn, *Stability of Motion* (Springer-Verlag, Berlin, 1967).

[42] R. Sepulchre, M. Janković, and P. Kokotovic, *Constructive Nonlinear Control* (Springer-Verlag, London, 1997).

[43] F. Reyes and R. Kelly, "Experimental evaluation of idetification schemes on a direct-drive robot," *Robotica*, **15**(5), 563–571 (1997).

[44] F. Reyes and R. Kelly, "Experimental evaluation of model-based controllers on a direct-drive robot arm," *Mechatronics*, **11**(3), 267–282 (2001).