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Stabilization of bed inclination angle in rotary drums by using computer vision

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Abstract

The computer vision systems have had great importance on research of different industrial processes. Videos and photographs have been used to characterize the bed behavior in a rotary drum, in this way, the bed motion, repose angles, trajectories and particles velocity can be determined. This work shows the development of a computer-vision system that measures the bed inclination angle in industrial rotary drums. The vision system purpose is to capture the images when the angular velocity on the drum changes. Eventually, with the data measured during the two experiments in industrial rotary drums, we have identified two input-output models representing the slumping motion. Based on control objective, we considered two approaches: regulation and tracking. PI and PII^2 controllers have been designed to show that a bed inclination angle can be stabilized for rolling motion considering plants that represent slumping motion and the inclination angle as the measured variable.

Introduction

The mixing of granular materials in a rotary drum has a noticeable industrial importance, especially in the coating process (pharmaceutical and food industry) and grinding of minerals. The scientific field has been interested in studying the motions that occur in the rotary drum. In this sense, the particles bed has various transverse motions; these motions depend on the drum angular velocity, diameter, filling percentage of the drum, the friction between the drum's wall and particles, moreover, the friction between each particle and other physical properties of particles as size, form, etc.[?] The motions are slipping, slumping, rolling, cascading, catarating and centrifugation.[?] The rolling motion is the most effective in mixing and coating processes due to the heat and mass transport phenomena are favored.[?] Moreover, for an optimum mixing process in a rotary drum it is necessary to consider, among others, the following factors: (i) the number of revolutions per minute of the drum, (ii) the segregation, where the particles tend to form clusters due to their density or size and (iii) filling percentage of the drum where a slight increase in the bed mass can make the rolling motion disappear.[?] If we are considering the continuous coating process in a rotary drum, then we should consider the sprayed surface by solution, the amount of sprayed mass and the rate of particles on the bed, which reach their maximum value in the rolling motion.[?]

Equally important, the transport phenomena in the rolling motion are caused by the avalanches that appear in the bed surface. Therefore, diverse studies have been focused in understanding the physical mechanics of these avalanches. These researchers consider the repose angle as the start of an avalanche,[?] which has allowed them to obtain 2D and 3D mathematical models to describe its dynamics.^{???} Poschel T. and Volkhard B.^{??} showed through molecular simulation that the granular flow is different for regular and irregular particles under the same conditions. Rajchenbach,[?] showed experimentally, that the bed inclination angle depends nonlinearly on the angular velocity of the rotary drum.

Years later, Rajchenbach[?] used a fast camera (250 frames/second) and determined the

nucleation and propagation of avalanches; Nitin Jain et al[?] used a laser flash which could determinate the displacements of particles in the avalanches. Currently, the modern cameras have allowed to take faster images and with the best quality that has been obtained by means of processing image software, this has helped to predict flow profiles on bed,[?] [?] the transition zone between slumping and rolling motions[?] and the thickness of cascading layer where the avalanches appear.[?] [?] Also, the use of video allows to obtain information of bed motions by analyzing the recording with a software tool as Canny Algorithm[?] to identify the bed surface, the inclination angle or repose angle dynamics; as well as to determinate the velocity and particle trajectories, analyzing the particles dynamics frame by frame[?] or with image techniques as PEPT.[?]

In this paper the development of a computer vision system is exploited as an instrumentation of measurement of the bed inclination angle. The main purpose is to stabilize the bed inclination angle by using images when the angular velocity of a rotary drum has changed; two industrial rotary drums were used in a couple of experiments, the difference is that one of them has baffles and the other does not have any ones. The data are used to identify two input-output models for slumping motion. Feedback controllers, PI [?] and PII ^{2,?} are designed to stabilize the bed inclination angle by the formulations of two control approaches. The text is organized as follows, in Section 2 the experimental setup is presented. Section 3 describes the vision system of measuring the bed inclination angle. The results are explained in Section 4, and finally, the conclusions are presented in the last section.

Experimental Setup

The experimental protocol consisted of measurements of bed inclination angles using computer vision. The objective is to identify the bed motions in distinct angular velocities and calculate the values of the bed inclination angle that correspond them. For this purpose, two experiments were made which consisted in rotating the drum to different angular velocities during periods of 5 minutes each velocity. The experiments were recorded with a SONY HXR-NX70N Full HD camera.

In the first experiment, an industrial rotary drum was used, which was made of steel with 2.4 meters of length, 1.6 meters of diameter and 3 mm of thickness. It has 6 rows of 4 baffles each, whose dimensions are 16 inches of length, 1 inch of width and 0.5 inches of thickness. The drum was driven by an AC motor with 7.5 HP and controlled by the frequency converter to obtain constant angular velocities in a interval of 0.76 - 7 revolutions per minute (rpm). In this experiment, 350kg of particles were used which is approximately the 21.6 filling percentage of the drum.

In the second experiment, the used rotary drum had the same characteristics as the one used in the first experiment, the only difference is that the latter had no baffles. The AC motor for this rotary drum is the same type that was used in the first experiment, but its frequency converter reached constant angular velocities in a interval of 1 - 8 revolutions per minute (rpm). In this experiment, 460 kg of particles were used which is the 23.5 filling percentage of the drum. The particles in both experiments had the same characteristics.

Theoretical fundament

Computer vision system

The images obtained from the recordings of the two experiments were analyzed. The original image had a resolution of 1280x720 pixels and a color depth of 24-bit 3-channel BGR (blue, green and red). The original 3-channel image was converted into 1-channel 8-bit, a gray scale image. Then, it was "smoothed" with a Gaussian filter to reduce the noise which affects the edge detection. In the next step, the smoothed image was processed by the Canny edge detector,[?] the resulted image was a binary image with 255 values for edge pixels and 0 for the rest of the image. See Figure 1.

In this work only a portion of the image was used and it corresponds to the particles bed, the information that was not needed was removed by a mask. We applied an AND operation between the mask and the original image. Figure 2 shows the image with the area of interest highlighted in white.

The Hough method was applied to the image for line detection.[?] In this method, the lines are expressed in the polar coordinate system. The equation of the line in polar coordinates can be expressed as:

$$y = \left(-\frac{\cos \theta}{\sin \theta}\right)x + \left(\frac{r}{\sin \theta}\right) \quad (1)$$

In the Hesse normal form:

$$r = x \cos \theta + y \sin \theta \quad (2)$$

where r is the distance from the origin to the closest point on the straight line, and θ is the angle between the x axis and the line connecting the origin with the closest point. See Figure 3.

According to the Hough method, a line can be detected by finding the number of intersections between curves. As more curves intersect as more points belong to the same line.

We can defined the minimum number of intersections in order to declare a line, in this case, we defined five intersections. The obtained lines were filtered by angle (only lines with angles between 0 and 60 degrees are taken into account), so that the line angle was calculated with the equation :

$$\theta = \arctan(2(y_1 - y_0, x_1 - x_0)) \frac{180}{\pi} \quad (3)$$

Being x_0, y_0, x_1, y_1 the endpoints of each line.

Finite Impulse Response

An FIR (finite impulse response) filter is designed by finding the coefficients and filter order that meet certain specifications, which can be in the time-domain and/or the frequency domain (most common). Matched filters perform a cross-correlation between the input signal and a known pulse-shape. The FIR convolution is a cross-correlation between the input signal and a time-reversed copy of the impulse-response. Therefore, the matched-filter's impulse response is designed by sampling the known pulse-shape and using those samples in reverse order as the coefficients of the filter. Window design is one of the common methods to build a FIR filter.[?] In the window design method, one first designs an ideal FIR filter and then truncates the infinite impulse response by multiplying it with a finite length window function. Multiplying the infinite impulse by the window function in the time domain results in the frequency response of the FIR being convolved with the Fourier transform of the window function. The convolution theorem states that correlation becomes an element-wise multiplication in the Fourier domain. Therefore, by using the \cdot symbol to explicitly denote element-wise multiplication, the correlation takes the form:

$$G = x \cdot w(n) \quad (4)$$

where x is the signal and $w(n)$, in this case, is the Hann window filter, defined as $w(n) =$

$0.5(1 - \cos(\frac{2\pi n}{n-1}))$, with $0 \leq n \leq N-1$, N is number of samples. The data of two experiments were filtered with the Hann window filter. Figure 4 shows the filtered data; in which a considerable noise reduction is observed. The methodology described in this section was implemented in C++.

Results

The main objective in this paper regards to stabilize the bed inclination angle by using images under changes on the angular velocity of a drum. The results in this contribution have been derived by stepping on the following: (i) To calculate the bed inclination angles for each angular velocity, (ii) to obtain an input-output model using the calculated data and (iii) to design controllers to stabilize the bed inclination angle. In this context, a scheme to stabilize the bed inclination angle by using computer vision in industrial applications is showed in Figure 5-A, which shows the following measurement methodology; that is, the digital camera captures the image of the bed while the drum is rotating, the computer board computes the captured image and calculates the bed inclination angle (see Figure 5-B) next sends it to the Programmable Logic Controller(PLC) in which the controller is programmed. This controller calculates the frequency of the AC motor such that the angle of reference is tracked.

Experiment 1: drum with baffles

The first experiment was realized using a industrial rotary drum with baffles, the filling percentage of the drum was 21.6 %. In this experiment the angular velocities varied between 0.76 rpm and 7 rpm. The motions were classified according to the regime observed in the bed and the bed inclination angles were calculated in each of the angular velocities, these results are showed in Table 1. Note that, the slumping motion was found in an angular velocity of 0.76 rpm and the inclination angles interval from 32° to 49.5° . Whereas, the rolling motion

was found in the interval from 2.56 rpm to 5.26 rpm and the inclination angles interval from 35.2° to 37.2°.

Identification of the input-output model

For control purposes, we considered the data of the slumping motion to obtain a model that represents the plant. We used the filtered data (see Figure 4-A) to obtain some input-output models using the *System Identification Toolbox* by MatLab[®], where the drum angular velocity was defined as the input and the bed inclination angle was defined as the output. The best identified model had a percentage of fit estimation data of 66.69 %. In Figure 6-A, the identified model was compared with the experimental data. The identified model adequately represents the dynamics of the slumping motion.

The transfer function that represents the identified model is the following:

$$P_1(s) = \frac{\theta_1(s)}{\omega_1(s)} = \frac{num_{p_1}(s)}{den_{p_1}(s)} \quad (5)$$

where $\omega_1(s)$ is the transfer function input that represents the drum angular velocity and $\theta_1(s)$ is the transfer function output that represents the bed inclination angle. With $num_{p_1}(s) = -4.3 \times 10^{-6}s^6 + 6.8 \times 10^{-5}s^5 + 0.0022s^4 + 3.4 \times 10^{-6}s^3 + 6.5 \times 10^{-6}s^2 + 2.15 \times 10^{-8}s + 3.8 \times 10^{-9}$ and $den_{p_1}(s) = s^7 + 0.09s^6 + 0.0056s^5 + 3.8 \times 10^{-4}s^4 + 9.7 \times 10^{-6}s^3 + 4.3 \times 10^{-7}s^2 + 5.1 \times 10^{-9}s + 7.7 \times 10^{-11}$.

Stabilization of the bed inclination angle

In the coating process, it is desirable that the bed inclination angle must be maintained in the rolling motion, thereby ensuring a uniform coating. Figure 7 shows a block diagram representing the control problem, where θ_r is the angle of reference, e is the error, ω is the drum angular velocity and θ is the bed inclination angle which is obtained with a computer vision system. The plant is defined by equation 5.

A PI controller is used for the stabilization of the bed inclination angle. We have formulated the regulation and tracking problems in such way that the bed inclination angle is stabilized for the rolling and slumping motions, respectively. In the regulation approach, the angle of reference is the angle constant value 37.2° , which is within the rolling motion interval. The idea is to maintain the particles bed in this motion. We obtained satisfactory results with the PI controller; Figure 8-A shows that the closed-loop output reaches the angle of reference approximately on the 40^{th} second which for coating purposes is an adequate time. We used the PII^2 feedback controller with the same angle of reference. The results are showed in Figure 8-A, note that, the PII^2 controller reaches the angle of reference in less time than the PI controller. Likewise the control signals of both controllers are presented in Figure 8-B. The results of both controllers are similar, the control signals are approximately 0.76 rpm which indicate that the rotary drum should have an angular velocity of 0.76 rpm to maintain the particles bed in the rolling motion.

Next, by using the tracking approach, both PI and PII^2 were also tested. Note that, the tracking approach concerns the slumping motion, which contained the largest inclination angle interval. The results for both controllers are similar, the closed-loop output reaches the reference approximately on the 40^{th} second (see Figure 9-A) and the control signals are approximately 0.76 rpm like in the regulation approach, see Figure 9-B. Table 3 presents the used parameters values in the controllers design. Notice that, the values are the same in both approaches which indicate that for the regulation and tracking approaches we can use the PI controller reaching the control objective; in the same way for the PII^2 controller, the control objective is reached using this controller in both approaches.

Experiment 2: drum without baffles

The second experiment was realized using a industrial rotary drum without baffles, the filling percentage of the drum was 23.5 %. In this experiment the angular velocities varied between 1 rpm and 8 rpm. Like in the first experiment, the motions were classified according to the

regime observed in the bed and the bed inclination angles were calculated in each of the angular velocities, these results are presented in Table 2. Note that, the slumping motion was found in angular velocity of 1 rpm and the inclination angles interval from 35.5° to 40.5° . While, the rolling motion was found in the interval from 3 rpm to 5 rpm and the inclination angles interval from 38° to 39.5° . The values of inclination angles in the second experiment are higher than those at the one. In addition, the filling percentage of drum of the second experiment increased 1.9%.

Identification of the plant

The same methodology for the first experiment was used for the second experiment. A filtered data set was used to obtain an input-output model with the drum angular velocity defined as the input and the bed inclination angle defined as the output. The bed inclination angles in the second experiment changed with more frequency (see Figure 4-B). We obtained an identified model with a percentage of fit estimation data of 52.25 %. Figure 6-B shows the identified model and the captured data that corresponds to the slumping motion. Note that, the identified model adequately represents this motion.

The transfer function that represents the identified model is the following:

$$P_2(s) = \frac{\theta_2(s)}{\omega_2(s)} = \frac{num_{p_2}(s)}{den_{p_2}(s)} \quad (6)$$

where $\omega_2(s)$ is the transfer function input that represents the drum angular velocity and $\theta_2(s)$ is the transfer function output that represents the bed inclination angle. With $num_{p_2}(s) = 0.1061s^6 + 0.4158s^5 - 0.0057s^4 + 0.0084s^3 + 5.2 \times 10^{-5}s^2 + 3.0 \times 10^{-5}s + 8.52 \times 10^{-8}$ and $den_{p_2}(s) = s^7 + 0.227s^6 + 0.04s^5 + 4.6 \times 10^{-3}s^4 + 4.1 \times 10^{-4}s^3 + 2.1 \times 10^{-5}s^2 + 7.55 \times 10^{-7}s + 21.8 \times 10^{-10}$.

Stabilization of the bed inclination angle

In the second experiment, two controllers were designed by the *PI* and *PII*² techniques for the regulation and tracking approaches under the same conditions of the first experiment

but with the plant defined by equation 6. In the regulation approach, the angle of reference is the constant angle value 40.5° , which is in the rolling motion interval. Figure 10-A shows that the closed-loop output reaches the angle of reference approximately on the 80^{th} second. We used the PII^2 feedback controller with the same angle of reference. The results are showed in Figure 10-A, note that, PII^2 controller reaches the angle of reference in less time than the PI controller (approximately on the 50^{th} second). Likewise, the control signals of both controllers are presented in Figure 10-B. The results of both controllers are similar, the control signals indicate that the drum should have an angular velocity of 1 rpm to maintain the particles bed in the rolling motion.

Next, by using the tracking approach, both the PI and PII^2 controllers were also tested. Note in Figure 11-A that the reference is not tracked, however, it is possible to maintain the bed inclination angle in the rolling motion. The control signals are showed in Figure 11-B. Table 4 presents the used parameters values in the controllers design. Similar to the first experiment, the values are the same in both approaches which indicates that for regulation and tracking approaches we can use PI controller reaching the control objective; in the same way for PII^2 controller, the control objective is reached using this controller in both approaches. Even more, the values of the parameters are equal to those of the first experiment, except for τ_c , which represents close-loop time constant. Since the filling percentage of the drum influences the bed motion, it is to be expected that the closed-loop time constant is different in the second experiment 2 where the filling percentage of the drum increased 1.9% respect to the first experiment. This facts allow us to show the robustness of two designed controllers under plant parameters changes.

Conclusions

The contribution of this article is about the use of a computer vision system for the control of an industrial application. This system has allowed us to measure the bed inclination angle

in rotary drums. Furthermore, this system was used for two experiments, the first one with a drum with baffles and 21.6 filling percentage of drum and the second one with a drum without baffles and 23.5 filling percentage of drum. Moreover, with the measurements it was possible to obtain two models that represent the slumping motion and also to establish the approach of the control problem considering the bed inclination angle as the measured variable. Two control techniques (PI and PII^2) were used to obtain controllers for the regulation and tracking approaches. The results show that the parameters values are the same for PI controller in both experiments, i.e, this controller can be used in both approaches and even more in both experiments. In the same way for the PII^2 , the parameters values are equal for the two experiments except for τ_c which is attributed to the filling percentage of the drum which is 1.9% greater in the second experiment. This facts allow us to show the robustness of two designed controllers under plant parameters changes. Stabilization of the rolling motion within the drum is reached using both controllers (PI and PII^2), in such way that the bed inclination angle allows to hold condition for homogeneity by mixture. In summary, our contribution lies in the usage of computer vision control for a process involving potential thermodynamical phenomena. These phenomena are involved the industrial rotary drum when the system is exploited during the coating process. A different challenge to incorporate thermodynamical issues to the control problem of the stabilization of the bed inclination angle. Results in this direction are under progress and will be reported elsewhere.

Acknowledgement

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Table 3 Values of the parameters in the controllers design for the first experiment.

Table 4 Values of the parameters in the controllers design for the second experiment.

Table 1: Data of the first experiment: drum angular velocity, values of the bed inclination angle for each motion considering velocities between 0.76 and 7 rpm.

| Drum angular velocity (rpm) | Motion | Bed inclination angle ($^{\circ}$) |
|-----------------------------|-----------|--------------------------------------|
| 0.76 | slumping | 32 - 49.50 |
| 2.56 | rolling | 35.72 |
| 3.45 | rolling | 36.37 |
| 4.34 | rolling | 36.76 |
| 5.26 | rolling | 37.21 |
| 7 | cascading | 37.5 - 38.5 |

Table 2: Data of the second experiment: drum angular velocity, values of the bed inclination angle for each motion considering velocities between 1 and 8 rpm.

| Drum angular velocity (rpm) | Motion | Bed inclination angle ($^{\circ}$) |
|-----------------------------|-----------|--------------------------------------|
| 1 | slumping | 35.5-40.5 |
| 3 | rolling | 38 |
| 4 | rolling | 38.5 |
| 5 | rolling | 39.3 |
| 7 | cascading | 41 - 43 |
| 8 | cascading | 41 - 43 |

Table 3: Values of the parameters in the controllers design for the first experiment.

| Parameter | Value (<i>PI</i>) | Value (<i>PII</i> ²) |
|-----------|------------------------|--------------------------------------|
| K_p | 0.0016 | 0.7522 |
| τ | 9.6e-05 | 65.92 |
| τ_c | N/A | 58 |
| L | N/A | 0.0092 |

Table 4: Values of the parameters in the controllers design for the second experiment.

| Parameter | Value (<i>PI</i>) | Value (<i>PII</i> ²) |
|-----------|------------------------|--------------------------------------|
| K_p | 0.0016 | 0.7522 |
| τ | 9.6e-05 | 65.92 |
| τ_c | N/A | 63 |
| L | N/A | 0.0092 |

Figure Captions

Figure 1 caption: Image after applying a Gaussian filter and the Canny edge detector.

Figure 2 caption: Image after applying AND operation, the area of interest corresponding to the particles bed is represented in white.

Figure 3 caption: In (A) the result of the Hough method for line detection is showed and in (B) a graphic description of r which is the distance from the origin to the closest point on the straight line.

Figure 4 caption: Comparison between captured data and filtered data with the Hann Window Filter. In (A) data of the first experiment and in (B) data of the second experiment.

Figure 5 caption: A) Elements for the stabilization of a bed inclination angle by using computer vision in industrial rotary drum applications. The instrumentation is conformed by a Programmable Logic Controller (PLC) and a Variable Frequency Drive (VFD). The controller is programmed into the PLC to send the frequency signal to the AC motor in order to stabilize the bed inclination angle. In this sense, the digital camera is only used as transducer into the closed-loop. B) Captured image of the first experiment.

Figure 6 caption: Comparison identified model vs measured data. By using filtered data with the Hann Window Filter. In (A) identified model of the first experiment had a percentage of fit estimation data of 66.69 % and in (B) identified model of the second experiment had a percentage of fit estimation data of 52.25 %.

Figure 7 caption: A Block diagram representing the control problem for the stabilization of a bed inclination angle.

Figure 8 caption: First experiment: regulation approach. A) PII^2 controller (dash-dot line) reached the angle of reference before the controller PI (dash line). B) The control signal value, to maintain the bed inclination angle in the rolling motion interval, is 0.76 rpm in both controllers.

Figure 9 caption: First experiment: tracking approach. A) The two controllers (PI and PII^2) track the reference (slumping motion). B) Control signals permit to maintain the bed inclination angle in the rolling motion interval.

Figure 10 caption: Second experiment: regulation approach. A) The two controllers (PI and PII^2) track the angle of reference which is 40.5° in this experiment. B) Control signals indicate that the drum should have an angular velocity of 1 rpm to maintain the bed inclination angle in the rolling motion interval.

Figure 11 caption: Second experiment: tracking approach. A) The reference is not tracked, however, the bed inclination angle remains in the rolling motion interval. B) Control signals indicate that the drum should have an angular velocity of 1 rpm to maintain the bed inclination angle in the rolling motion interval.

Figures

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Graphical TOC Entry

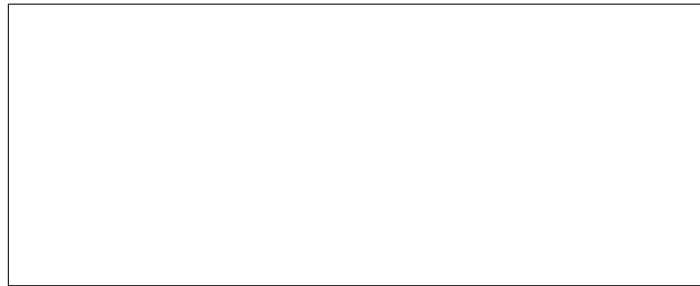


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