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Robust sliding mode-based extremum-seeking controller for reaction systems via uncertainty estimation approach

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SUMMARY

This paper deals with the design of a robust sliding mode-based extremum-seeking controller aimed at the online optimization of a class of uncertain reaction systems. The design methodology is based on an input–output linearizing method with variable-structure feedback, such that the closed-loop system converges to a neighborhood of the optimal set point with sliding mode motion. In contrast with previous extremum-seeking control algorithms, the control scheme includes a dynamic modelling-error estimator to compensate for unknown terms related with model uncertainties and unmeasured disturbances. The proposed online optimization scheme does not make use of a dither signal or a gradient-based optimization algorithm. Practical stabilizability for the closed-loop system around to the unknown optimal set point is analyzed. Numerical experiments for two nonlinear processes illustrate the effectiveness of the proposed robust control scheme.

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KEY WORDS: extremum-seeking control; (bio)chemical reactors; sliding mode control; uncertain reaction systems

1. INTRODUCTION

Reaction systems are characterized by high nonlinearity and great uncertainty in their mathematical description, thus often the conventional linear control schemes do not give satisfactory responses. Most control schemes for this kind of systems are designed for regulation to known set point or tracking reference trajectories. However, in many applications, the control objective is to optimize a cost criterion that can be a function of unknown parameters in order to keep a performance variable at its optimal value [1]. In practice, the explicit form of the performance function is very often unavailable or is highly uncertain, for example, the kinetic models in chemical and biological reactors. Additionally, the performance function can be subject to bounded time-varying disturbances because of the effect of variation in the environmental variables as the temperature, composition and flow of the influent, pH, and so on. [1, 2]. The perturbation-based and model-based extremum-seeking controls are two methods to handle these kinds of online optimization problems [1, 3]. The goal of the extremum-seeking schemes is to find operating set points, a priori unknown, such that an objective function (subject to uncertainties) reaches its extremum value [4, 5].

The extremum-seeking control (ESC) schemes have been an active research area with distinct application issues including, for example, the adjustment of radio telescope antennas in order to

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maximize the received signal; blade adjustment in water turbines or wind mills to maximize the generated power, and in anti-lock braking system (ABS) control to lead the maximal value of the tire/road friction force to be reached during emergency braking [6–10]. The most popular approach known as nonmodel dither-based ESC methods has been applicable to diverse control systems with local minimum (or maximum) that defines their optimal operating condition [2, 11]. These approaches consider one dynamic feedback composed of a high-frequency perturbation signal (dither signal) combined with an adaptive extremum searching to find an unknown optimal operating condition of the plant [11]. The design of ESC algorithms has also been studied considering a known plant structure with uncertain parameters. In this framework, [12] proposes a combination scheme of different types of gradient-based optimization methods with a parameter estimation algorithm. An intensive research activity has been devoted to adaptive model-based ESC schemes applied to (bio)-reaction systems (e.g., [1–5, 13, 14]). The adaptive extremum schemes are based on parameter learning laws for the estimation of the unknown parameters and a dither signal to ensure the convergence to a neighborhood of the optimal value [1, 2]. However, the model-based adaptive extremum-seeking algorithms for reaction systems require prior information about (a) the explicit form of the kinetic expressions and (b) bounds of the parameters values which, in most (bio)-chemical processes, may be hard to obtain from available data [1–3]. Besides, the extremum-seeking control problem has also been studied in the sliding mode control framework [6–8, 10, 15, 16]. The application of the sliding mode concepts in ESC can be found in [10]. More recently, sliding mode-based ESC schemes applied to automotive systems have been proposed [6–8, 15]. In these contributions, the main idea is to ensure that a desired output follows an increasing time function as close as possible to its extremal value via discontinuous feedback with sliding mode motion [9, 17]. In this framework, [3] proposes a sliding mode-based ESC controller for a class of nonlinear systems with arbitrary relative degree. However, the estimation scheme requires a high-gain observer for the estimation the time derivatives of the output signal and a normal observer for the plant states. Nevertheless, according to the authors' knowledge, there is no ESC scheme based on sliding mode techniques for real-time optimization of uncertain reaction systems.

The aim of this work is to describe the design and application of a robust sliding mode-based ESC scheme to achieve the online optimization of a class of uncertain reaction systems. The ESC strategy proposed is based on the sliding mode techniques with a modelling-error estimator to compensate for unknown terms related with model uncertainties and unmeasured disturbances. The design methodology is divided in two steps: First, an ideal controller is designed from the input–output linearizing control method for a class of minimum-phase systems and by using the ESC with sliding mode techniques [9, 17]. The ideal controller allows to reach the extremum of the performance function and converges to a neighborhood of the optimal value with sliding mode motion. In the second step, an observer-based uncertainty estimator is used to reconstruct the unknown terms in the ideal control law. The stabilization on the neighborhood (practical stabilizability) of the unknown optimal set point is ensured when the estimator scheme and controller are coupled. Unlike the previous ESC schemes, the proposed approach is based upon sliding mode techniques without using gradient-based optimization methods or dither functions and comprises a dynamic uncertain estimation scheme that computes the unknown terms and external disturbances. The main difference with respect to [18] is that the ESC strategy proposed in this work does not require a high-gain observer for the estimation of the time derivatives for the output and a normal observer for the plant states; all uncertainties are lumped and estimated by using modelling-error compensation techniques. The paper is organized as follows. Section 2 introduces the class of nonlinear systems considered in this work and the control problem formulation. Section 3 deals with the design of the robust extremum-seeking controller and the closed-loop convergence analysis. The modifications to the basic control law to handle model uncertainties and unmeasured disturbances are also discussed. In Section 4, two distinct processes are used as numerical examples to illustrate the effectiveness of the robust control scheme. The first model considers a fed-batch culture process, while the second model considers a non-isothermal chemical reactor.

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2. PROBLEM STATEMENT

Consider the following nonlinear system

$$\begin{aligned}\dot{x} &= f(x) + g(x)u \\ y &= h(x)\end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the dynamical state, $u \in \mathbb{R}$ is the control input, and $y \in \mathbb{R}$ an output signal. The functions $f : \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}^n \to \mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}$ are smooth on a domain $\Omega \subset \mathbb{R}^n$. In order to formulate the specific control problem, we consider the following assumptions.

Assumption 1
The system (1) has a well posed relative degree $r < n$ with respect to the output signal y and is minimum-phase in the usual sense of the stability of the zero dynamics [19].

Assumption 2
There exists a static performance map from the output signal y to the global cost criterion ξ, represented by

$$\xi = F(y) \tag{2}$$

which is locally smooth and has a unique maximum at $y = y^*$, such that $F(y^*) = \bar{\xi}$ with $\bar{\xi}$ as the maximum static global cost criterion value.

Assumption 3
The global cost criterion ξ and the output signal y are available for online measurement. However, the static map (2) is not known analytically, and consequently, the optimal value y^* is uncertain.

Thus, the control objective is to design a robust extremum-seeking scheme to achieve the practical stabilization of the system (1) around the optimum output value y^* (a priori unknown or highly uncertain) while maximizing the global cost criterion given by (2), despite uncertainty in the dynamical model and unmeasured disturbances.

3. CONTROLLER DESIGN

The design of the robust sliding mode-based ESC scheme will be performed in two steps. In the first design step, an "ideal" extremum-seeking controller is developed by assuming perfect knowledge of the system (1).

Because the system (1) has a well defined relative degree and is minimum phase (Assumption 1), let us introduce the following change of variables $z = \Phi(x)$ and with $z_i = \phi_i(x) = L_f^{i-1}h(x)$ for $1 \leqslant i \leqslant r$ and $L_g\phi_j(x) = 0$ for $r+1 \leqslant j \leqslant n$, where $L_f h(x)$ and $L_g\phi(x)$ represent the Lie derivatives of the maps h and ϕ with respect to the vector fields f and g, respectively, such that the nonlinear system (1) can be rewritten in the following canonical form [19]:

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= b(z,\varsigma) + a(z,\varsigma)u \\ \dot{z}_{r+1} &= \varsigma_{r+1}(z) \\ &\vdots \\ \dot{z}_n &= \varsigma_n(z)\end{aligned} \tag{3}$$

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where $z = [z_1 \dots z_r] \in \mathbb{R}^r$, $\varsigma = [z_{r+1} \dots z_n] \in \mathbb{R}^{n-r}$, $a(z,\varsigma) = L_g L_f^{r-1} h(\Phi^{-1}(z,\varsigma))$ and $b(z,\varsigma) = L_f^r h(\Phi^{-1}(z,\varsigma))$. Once in the earlier canonical form, the following result can be emphasized.

Theorem 1 ([19]*)*
The following input–output linearizing feedback control law

$$u = \frac{1}{L_g L_f^{r-1} h(x)} \left[-L_f^r h(x) + \upsilon \right]$$

will make the system (1) linear and controllable of the form

$$\begin{aligned} \dot{z}_i &= z_{i+1} i = 1, \dots, r-1 \\ \dot{z}_r &= \upsilon \\ \dot{\varsigma} &= \zeta(z,\varsigma) \end{aligned}$$

where $\dot{\varsigma} = \zeta(z,\varsigma)$ represents the zero dynamics and υ is an external reference input. Let us now give a brief review of the sliding mode-based ESC methods proposed in [9, 17]. Consider the system of the form (1) satisfying the Assumptions 2 and 3. A switching function $s(t)$ is defined as

$$s(t) = \xi(t) - g_s(t)$$

where $g_s(t)$ is an increasing signal satisfying $\dot{g}_s(t) = \rho > 0$. The following variable structure feedback is proposed

$$\upsilon = \lambda sgn\left(\sin(\frac{\pi}{\alpha}s)\right); \lambda, \alpha > 0 \tag{4}$$

Then the derivative of the switching function is

$$\frac{ds(t)}{dt} = \dot{\xi}(t) - \dot{g}_s(t) = F'(y)\dot{y} - \rho$$

where $F'(y) = \frac{\partial F(y)}{\partial y}$ denotes the partial derivative of the global cost criterion $\xi = F(y)$.

If there exists a constant c such that the so-called reaching condition holds

$$(s(t) - c)\frac{d}{dt}(s(t) - c) < 0 \tag{5}$$

then the sliding mode takes place at $s(t) = c$ [9]. On the sliding mode $s(t) = c$, the cost criterion $\xi(t)$ increases with the increase of the reference signal $g_s(t)$, because $y(t) = g_s(t) + c \Rightarrow \dot{y}(t) = \dot{g}_s(t)$, and the system moves towards the critical point y^*. In [9], it is shown that there exists a series of $c's$ that guarantee the convergence of the aforementioned sliding mode reaching, while the system trajectory is outside of a vicinity defined as

$$Y_\delta = \{y \in \mathbb{R} | y^* - \delta \leqslant y \leqslant y^* + \delta\} \tag{6}$$

where δ is a positive constant. Additionally, if the system enters the vicinity where the reaching condition does not hold, then

1. It either converges to the maximum inside the vicinity (6) with oscillations,
2. Or it moves through the vicinity, goes outside, and switching among sliding modes, $s(t) = c'$.

The convergence of the sliding mode-based ESC proposed in [9, 17] can be summarized in the following result.

Theorem 2 ([9]*)*
Consider the system (1) with the cost criterion (2) satisfying Assumptions 2 and 3, and the output dynamics governed by the variable structure feedback (4); then the closed-loop system converges to the vicinity (6) in a finite time, if the parameters of the controller are chosen to satisfy $\frac{\rho}{\lambda} < \frac{\alpha}{2\delta}$.
Based on the aforementioned results, the follows variable-structure controller is proposed such that the system (1) converges close to the optimal set-point y^*.

Proposition 1
Consider the system (1) and suppose that Assumptions 1–3 are satisfied. Then the variable-structure control law

$$u = \frac{1}{L_g L_f^{r-1} h(x)} \left[-L_f^r h(x) + \lambda sgn \left(\sin(\frac{\pi}{\alpha} s) \right) \right]; \lambda, \alpha > 0 \tag{7}$$

where $s(t) = \xi(t) - g_s(t)$ with $\dot{g}_s(t) = \rho > 0$, will converge to the vicinity (6) in a finite time, while $\frac{\rho}{\lambda} < \frac{\alpha}{2\delta}$.

Proof
Replacing u from (7) in (3) yields

$$\dot{z}_r = \lambda sgn \left[\sin(\frac{\pi}{\alpha} s(t)) \right]$$

Because the zero dynamics is asymptotically stable (Assumption 1), then from Theorems 1 and 2, and considering the fact that the system (3) is in cascade form, it follows that the control law (7) achieves the convergence of the output y at the δ-vicinity Y_δ in a finite time interval. □

3.1. *Extremum-seeking under uncertain vector fields*

The aforementioned linearizing ESC feedback cannot be implemented directly, because (7) requires perfect knowledge of the uncertain terms associated with the modelling errors and the external disturbances of the system. In what follows, the ideal control law is modified to account these uncertainties. To this end, let us define a function for lumping the uncertain terms as follows: $\eta \equiv \Theta(z, \varsigma, u) = a(z, \varsigma) + b(z, \varsigma)u$. We make the following assumption with respect to the uncertain function.

Assumption 4
The function $\eta \equiv \Theta(z, \varsigma, u)$ is locally smooth bounded, and its time derivative denoted by $\Xi(z, \eta, \varsigma, u)$ is also bounded. Then once the uncertain function η is defined, it is possible to rewrite the system (3) in the following extended state-space representation:

$$\begin{aligned} \dot{z}_i &= z_{i+1} i = 1, \ldots, r-1 \\ \dot{z}_r &= \eta \\ \dot{\eta} &= \Xi(z, \eta, \varsigma, u) \\ \dot{\varsigma} &= \zeta(z, \varsigma) \end{aligned} \tag{8}$$

where η is interpreted as an augmented state whose dynamics can be reconstructed from measurements of the input and the output signals [20, 21]. (a) It can be proved that the solution of system (3) is a projection of the solution of system (8); (b) A feature of system (8) is that the uncertainties have been lumped into an uncertain function $\Theta(z, \varsigma, u)$ that can be estimated by an unmeasurable but observable state η [20, 21]. Thus, if system (8) is stabilized, then system (3) will be also stabilized as well as its equivalent system (1). Some works have focused on the robust stabilization of nonlinear systems via output feedback (e.g., [20, 22, 23]), where the main idea consists in designing a high-gain Luenberger-like observer as an uncertainty estimator. By following this idea, because $z = [z_1, \ldots, z_r]' \in \mathbb{R}^r$ represents the observable states [19], the problem of estimating z can be addressed by using a high-gain observer. Thus, the dynamics of the states (z, η) can be reconstructed from the measurements of the output signal $y = h(x) = z_1$ in the following way [20, 21]

$$\begin{aligned} \dot{\hat{z}}_i &= \hat{z}_{i+1} + \Gamma^i \kappa_i (z_1 - \hat{z}_1) i = 1, \ldots, r-1 \\ \dot{\hat{z}}_r &= \hat{\eta} + \Gamma^r \kappa_r (z_1 - \hat{z}_1) \\ \dot{\hat{\eta}} &= \Gamma^{r+1} \kappa_{r+1} (z_1 - \hat{z}_1) \end{aligned} \tag{9}$$

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where $(\hat{z}, \hat{\eta})$ denotes the estimate for z and the lumped uncertainty state η, respectively. The observer parameters $\kappa_i' s$ are chosen such that the polynomial $\kappa_{r+1} p^r + \kappa_{r-2} p^{r-1} + \ldots + \kappa_1 = 0$ is Hurwitz, and $\Gamma > 0$ is a positive parameter (high-gain observer). Then the following variable-structure control law can drive the trajectories of the system (1) arbitrarily close to the optimal set point y^*:

$$u = \frac{1}{\gamma} \left[-\hat{\eta} + \lambda sgn \left(\sin(\frac{\pi}{\alpha} s) \right) \right] \tag{10}$$

where γ represents the well known terms in the Lie derivatives defined in (7). In this work, we assume that γ is bounded away from zero. Here, note that the control parameters $\lambda, \rho > 0$ are associated with the speed of convergence of the robust ESC taking into account the restriction $\frac{\rho}{\lambda} < \frac{\alpha}{2\delta}$ [9, 17]. In this way, the resulting robust ESC scheme comprises the variable structure feedback (10), where the uncertain terms lumped in η are computed by the dynamic uncertain estimator (9). The following result is based on stability results of feedback linearizable uncertain systems under the action of a high-gain observer-based uncertain estimator [21–23]. Let us consider the following definition related with the practical stability concept [22, 23].

Definition 1 ([22]*)*
One system is practically stabilizable at x^*, if for any set $\Sigma \subset M$ containing x^* and such that its closure $cl(\Sigma)$ is compact, there exists a control law (practical stabilizer) which renders this set the stable attractor of the system within some neighborhood $\delta(\Sigma)$ of Σ, and moreover, any system trajectory beginning in $\delta(\Sigma)$ enters Σ in a finite time interval.

Proposition 2
Consider the uncertain nonlinear system (1) and suppose (i) the Assumptions 1–4 are satisfied; (ii) $\kappa_i' s$ is chosen such that the polynomial $\kappa_{r+1} p^r + \kappa_{r-2} p^{r-1} + \ldots + \kappa_1 = 0$ is Hurwitz; (iii) $\Gamma > 0$ is a positive parameter (high-gain observer); and (iv) $\frac{\rho}{\lambda} < \frac{\alpha}{2\delta}$ holds. Then the variable-structure control scheme (9–10) achieves the practical stabilization around to the optimal set point y^*.

Proof
From Proposition 1, it follows that the control law (7) achieves the convergence of output signal $y(t)$ at the δ-vicinity $Y_\delta = \{y \in \mathbb{R} | y^* - \delta \leqslant y \leqslant y^* + \delta\}$ and consequently achieves the output stabilization around the optimal set point y^*. Now, with respect to the high-gain observer-based uncertain estimator (9), we have the following. Let us consider the following estimation error vector $e \in \mathbb{R}^{r+1}$ whose components are defined by

$$\begin{aligned} e_i(t) &= \Gamma^{r+1-i}(z_i - \hat{z}_i) 1 \leqslant i \leqslant r \\ e_{r+1}(t) &= \eta - \hat{\eta} \end{aligned}$$

By replacing $(\dot{z}, \dot{\eta})$ and $(\dot{\hat{z}}, \dot{\hat{\eta}})$ from (8), (9), respectively, and from straightforward algebraic manipulation, it follows that the dynamics of the estimation error is governed by

$$\dot{e}(t) = \Gamma A(\kappa) e(t) + B \Phi_2(z, \eta, \varsigma, N(\Gamma^{-1}) e, u)$$

where $A(\kappa) = \begin{pmatrix} -\kappa_1 & 1 \; 0 \cdots 0 \\ -\kappa_2 & 0 \; 1 \cdots 0 \\ \vdots & \vdots \; \vdots \; \ddots \; \vdots \\ -\kappa_r & 0 \; 0 \cdots 1 \\ -\kappa_{r+1} & 0 \; 0 \cdots 0 \end{pmatrix}$ and $B = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^{r+1}$,
$N(L) = diag\left[\Gamma^{-r}, \ldots, \Gamma^{-1}, 1\right]$ and $\Phi_2(z, \eta, \varsigma, N(\Gamma^{-1}) e, u)$ is a continuous and bounded function. Now, with respect to the function Φ_2, it is clear that if $\Phi_2(\cdot) = 0$ the estimation error system reduces to the nominal dynamics of the estimation

$$\dot{e}(t) = \Gamma A(\kappa) e(t)$$

For $\Gamma > 0$ and because $\kappa_i' s$ is chosen such that $A(\kappa)$ is Hurwitz, the system $\dot{e}(t) = \Gamma A(\kappa) e(t)$ is asymptotically stable ($\lim_{t \to \infty} e(t) = 0$). From the robust observer Lemma about the stability of

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feedback linearizable uncertain systems under the action of a high-gain observer-based uncertain estimator [22, 23], it is well known that there exists a positive number Γ^*, depending mainly on the magnitude of $\Xi(z, \eta, \varsigma, u)$, such that this nonlinear term Φ_2 will be negligible with respect to the nominal dynamics of the estimation error. Then for a given compact set of the initial condition $I \subset \mathbb{R}^{r+1}$ and for all $\Gamma > \Gamma^*$, there exists a region of attraction $\aleph_\epsilon(y^*)$ around y^*, with $\epsilon > 0$ for the closed-loop system (9–10). Hence, the controller (9–10) drives the trajectories of the system (1) arbitrarily close (practical output stabilization) to the δ-vicinity Y_δ. □

4. NUMERICAL VERIFICATION

In this section, the aforementioned ESC design methodology is illustrated via numerical simulations for two uncertain reaction systems.

4.1. Fed-batch culture process

In the first example, we consider a biomass culture process occurring within a fed-batch bioreactor, where the microorganisms X grow by consuming a substrate S. Such a bioreaction can be written as follows:

$$S \overset{\mu X}{\looparrowright} X$$

The growth rate is denoted by μX, where $\mu : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a smooth function of the substrate and the symbol $\looparrowright$ indicates that the biomass X is an autocatalyst. The following dynamical model can be obtained from a mass balance in the bioreactor:

$$\dot{S} = \frac{u}{V}(S_f - S) - k_1\mu(S)X \tag{11}$$

$$\dot{X} = \mu(S)X - \frac{u}{V}X \tag{12}$$

$$\dot{V} = u \tag{13}$$

The state vector $x = [S, X, V]^T \in \mathbb{R}^3_+$ whose components represent the concentrations of substrate, biomass, and volume of the culture medium in the vessel, respectively; the inflow rate is the control input u; S_f denotes the inlet substrate concentration; and $k_1 > 0$ is a yield coefficient. Let us consider the following properties on the specific growth rate μ:

Property 1
$\mu \in C^\infty(\Sigma)$, where $\Sigma = \left\{S \in \mathbb{R} \middle| 0 \leqslant S \leqslant S_m\right\}$ with $S_m < \infty$; and there exists a value $S^* \in \Sigma$ such that $\mu \leqslant \mu(S^*) \triangleq \overline{\mu} \in \mathbb{R}\ \forall\, S \in \Sigma$, with $\overline{\mu} < \infty$ as the upper bound of μ.

Property 2 (Concavity property)
The first derivative of μ with respect to S, denoted by μ', satisfies as follows: (a) $\mu' > 0 \forall \tilde{S} < S^*$; $\mu' = 0$ at $\tilde{S} = S^*$ and (b) $\mu' < 0\ \forall \tilde{S} > S^*$, where $\tilde{S} \in \Sigma$.

A major difficulty in the monitoring and control of bioprocesses is the lack of reliable and simple sensors for following the evolution of the key state variables and parameters such as biomass and specific growth rate. In fact, a typical situation in bioreactor applications is when the biomass concentration is not available for online measurement while the product gaseous outflows rate (e.g., CO_2 in fermentation processes) is easier to measure online [1]. Now, it is well known that the product outflow rate can be modeled as follows [24]:

$$Q = k_2\mu(S)X \tag{14}$$

where Q is the product gaseous outflow rate and $k_2 > 0$ is a yield coefficient. We assume that only the product gaseous outflow rate and the substrate concentration are available for online monitoring, that is, the specific growth rate μ and the biomass concentration X are not available for online measurement.

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Hence, the control objective is to design of a control law with u as the control action such that the biomass production VX achieves its maximum at the end of the fed-batch operation. It is well known from [24] that the maximum biomass production will be achieved if the specific growth rate is kept at the optimum value S^* (Property 2). In many biotechnological processes, the inlet substrate concentration S_f can be subject to load disturbances; then we need to design a robust ESC despite uncertainties in the growth kinetics and load disturbances in the inlet composition. The bioreactor system can be rewritten in the affine control form (1), where

$$f(x) = \begin{bmatrix} -k_1\mu(S)X \\ \mu(S)X \\ 0 \end{bmatrix}$$

$$g(x) = \begin{bmatrix} \frac{S_f - S}{V} \\ -\frac{X}{V} \\ 1 \end{bmatrix}$$

and $h(x) = S$. It is well known that in conventional operating conditions ($S(t) \neq S_f$), the system has a well defined relative degree $r = 1$. Now, we can define a function for lumped the uncertainties of the bioreaction system as follows:

$$\eta(t) \triangleq -k_1\mu X + u(S_f - \bar{S_f})$$

where $\bar{S_f}$ is a constant such that it provides a nominal value of the inlet substrate concentration S_f ($\bar{S_f} \neq 0 \in \mathbb{R}$). Thus, S_f will be a piecewise constant and uncertain function varying around the nominal value $\bar{S_f}$, that is, $S_f = \bar{S_f} + \Delta S_f$.

Hence, a robust ESC is proposed as

$$\begin{aligned} \dot{\hat{S}} &= \hat{\eta} + \frac{u}{V}(\bar{S_f} - \hat{S}) + \Gamma\kappa_1(S - \hat{S}) \\ \dot{\hat{\eta}} &= \Gamma^2\kappa_2(S - \hat{S}) \\ u &= \frac{V}{\bar{S_f}}\left[-\hat{\eta} + \lambda sgn\left(\sin\left(\frac{\pi}{\alpha}s\right)\right)\right] \end{aligned} \tag{15}$$

where $s(t) = Q(t) - g_s(t)$ with $\dot{g}_s = \rho > 0$.

The simulation results are shown in Figures 1–3, considering the nominal values from [5]. The controller parameters are set to the following values: $\lambda = 0.5$, $\kappa_1 = 1$, $\kappa_2 = 2$, $\Gamma = 2.5$, $\alpha = 0.5$, and $\rho = 0.5$. Global performance of the robust controller (15) for load disturbances in the inlet substrate concentration S_f is shown in Figure 1. In the same figure, we can see that the substrate concentration converges to the unknown optimal set-point, while the specific growth rate lies on its maximum value $\bar{\mu}$. Figure 1(c) shows the corresponding control input profile where we can see a large control effort for last 50 h. Many biotechnological processes can be subject to adaptive mechanisms by microorganisms, as response to variations in the environmental variables such as dissolved oxygen, temperature, and pH. This behavior is illustrated here by small changes in the kinetic parameters around the nominal values reported in [5]. In Figure 2, the controller performance is shown for the corresponding changes in the unknown optimal value for substrate concentration S^* each 20 h (dashed line). The corresponding control input behavior is presented in Figure 2(c) where a large control effort is shown close to 90–100 h. In these simulations, the ability of the ESC scheme (15) to adapt the control action is shown in order to track the true optimal set point. Finally, in Figure 3, the asymptotic convergence of the high-gain observer-based uncertainty estimator 3.1 is shown.

4.2. *Nonisothermal chemical reactor*

As the second example, we consider the well known van de Vusse reaction system occurring into a nonisothermal continuous stirred tank reactor (CSTR) [25]. The reaction network for this reaction

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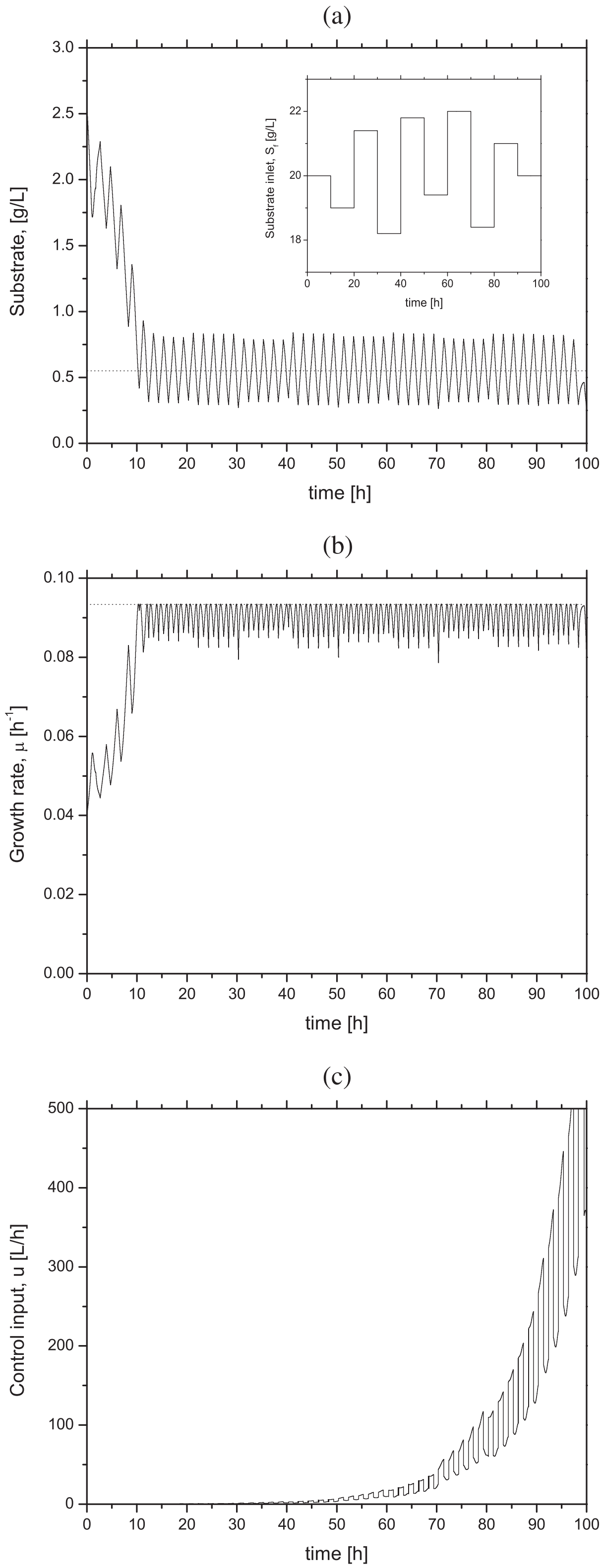

Figure 1. Closed-loop response for the fed-batch culture with load disturbances in the inlet substrate concentration S_f.

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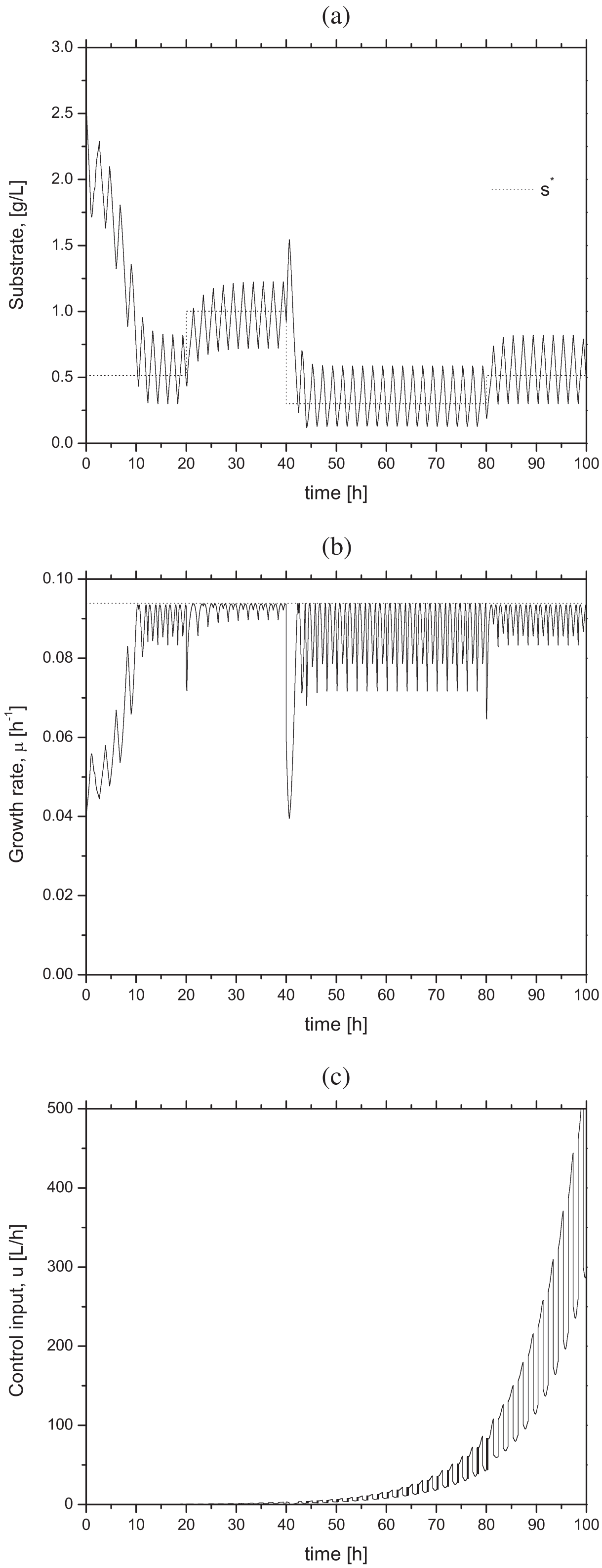

Figure 2. Closed-loop response for the fed-batch culture for different nominal kinetic parameters with the corresponding changes in the optimal set point S^* every 20 h (dashed line in (a)).

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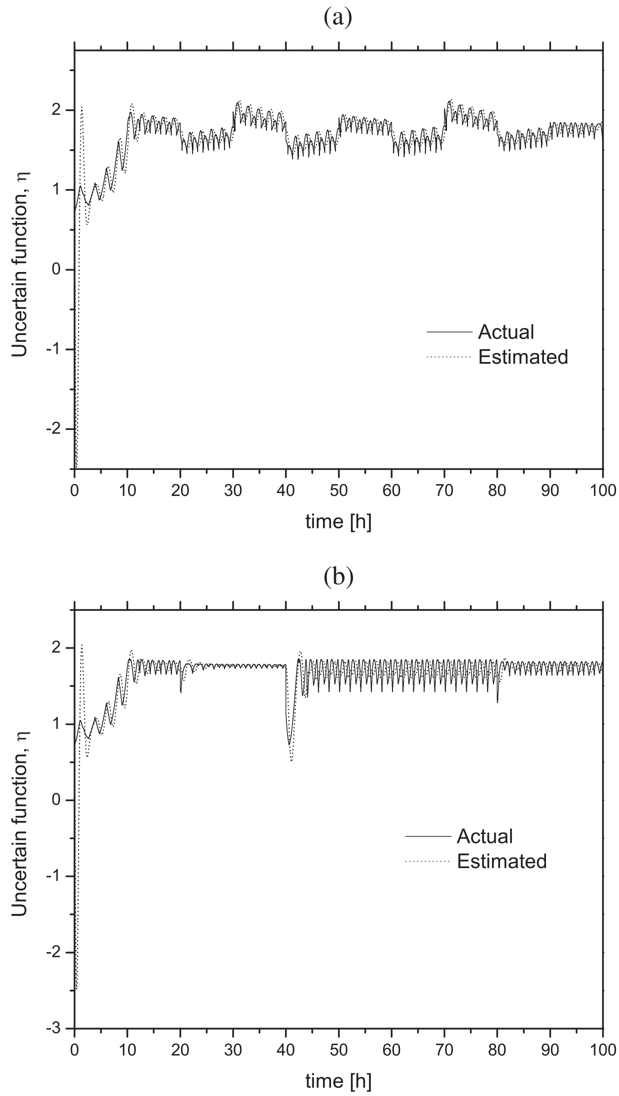

Figure 3. Performance of the uncertain dynamical estimator for (a) load disturbances in the inlet substrate concentration S_f; (b) different nominal kinetic parameters with the corresponding changes in the optimal set point S^*.

system is given by

$$A \to B \to C$$
$$2A \to D$$

From mass and energy balance in the CSTR, we can formulate the following dynamical model for the system

$$\begin{aligned} \dot{C}_A &= D(C_{Af} - C_A) - k_1(T)C_A - k_3(T)C_A^2 \\ \dot{C}_B &= -DC_B + k_1(T)C_A - k_2(T)C_B \\ \dot{T} &= -\frac{1}{\rho_m C_P} H_R(C_A, C_B, T) + D(T_f - T) + \frac{u}{\rho_m C_P V} \end{aligned} \tag{16}$$

where $H_R(C_A, C_B, T) = k_1(T)\triangle H_1 C_A + k_2(T)\triangle H_2 C_B + k_3(T)\triangle H_3 C_A^2$ and $k_i(T) = k_{io}exp\left[-\frac{E_i}{T+273}\right]$ for $i = 1, 2, 3$. If we focus on the isothermal operation of the reactor, that is,

$\dot{\bar{T}} = 0$, we can calculate the equilibrium coordinate $\bar{C_B}$ as a function of the reactor temperature as

$$\bar{C_B} = \frac{k_1(\bar{T})\bar{C_A}}{D + k_2(\bar{T})}$$

Now, by solving $\dot{\bar{C_A}} = 0$ for $\bar{C_A}$ and replacing in the aforementioned expression, we have the following ‡:

$$\bar{C_B} = \frac{k_1(T)}{D + k_2(T)} \left[-\frac{D + k_1(T)}{2k_3(T)} + \frac{\sqrt{(k_1(T) + D)^2 + 4k_3(T)DC_{Af}}}{2k_3(T)} \right] \tag{17}$$

In previous work [25], [25], it has been shown that the static equilibrium profile of the desired product B as a function of the reactor temperature exhibits a unique maximum for one specific-temperature value T^* (Figure 7). In this work, we consider that the concentration of the desired product C_B and the reactor temperature T are available for online measurement; and the flux-heat is the control action. Thus, the control objective is to design a robust ESC to regulating the reactor temperature around to the optimal and uncertain setpoint T^*, despite unknown chemical reaction kinetics and load disturbances in the inlet flow. Then the system (16) can be written in the control affine form (1), where the state vector is given by $x = [C_A, C_B, T]' \in \Omega \subset \mathbb{R}^3_+$ with

$$f(x) = \begin{bmatrix} D(C_{Af} - C_A) - k_1(T)C_A - k_3(T)C_A^2 \\ -DC_B + k_1(T)C_A - k_2(T)C_B \\ -\frac{1}{\rho_m C_P}\left[k_1(T)\triangle H_1 C_A + k_2(T)\triangle H_2 C_B + k_3(T)\triangle H_3 C_A^2\right] + D(T_f - T) \end{bmatrix}$$

$$g(x) = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{\rho_m C_P V} \end{bmatrix}$$

and $h(x) = T$. The relative degree of the CSTR (16) with respect to the output signal $y = T$ is $r = 1$; then the system satisfies the Assumptions 1–4. Now, we can define the uncertain function as

$$\eta = -\frac{1}{\rho_m C_P}\left[k_1(T)\triangle H_1 C_A + k_2(T)\triangle H_2 C_B + k_3(T)\triangle H_3 C_A^2\right] + D(T_f - T)$$

Hence, a robust ESC is proposed as

$$\begin{aligned} \dot{\hat{T}} &= \hat{\eta} + \frac{u}{\rho C_P V} + \Gamma\kappa_1(T - \hat{T}) \\ \dot{\hat{\eta}} &= \Gamma^2\kappa_2(T - \hat{T}) \\ u &= \rho_m C_P V\left[-\hat{\eta} + \lambda sgn\left(\sin\left(\frac{\pi}{\alpha}s\right)\right)\right] \end{aligned} \tag{18}$$

where $s(t) = C_B(t) - g_s(t)$ with $\dot{g}_s = \rho > 0$.

The simulation results are shown in Figures 4–7, considering the nominal values from [25]. The controller parameters are set to the following values: $\lambda = 12.5$, $\kappa_1 = 1$, $\kappa_2 = 2$, $\Gamma = 2.5$, $\alpha = 5$, and $\rho = 1.5$. The performance of the closed-loop system relative to the unknown optimal set point for distinct initial conditions is shown in Figure 7. The controller performs very well for different initial conditions and is able to lead the trajectories close to optimal set point. The closed-loop profile for the concentration of the component and the reactor temperature are shown in Figure 4. It is clear that the closed-loop dynamics is close to the desired optimal set point. The required control action is given in Figure 4(c). Figure 5 shows the controller performance for load disturbances in the influent flow. We can see that the closed-loop system is able to reach the optimal profile despite external disturbances. So as to illustrate the performance of the dynamical estimator described in 3.1, in

‡Note that the system displays multiple equilibria, but the one of interest will be the positive equilibrium point.

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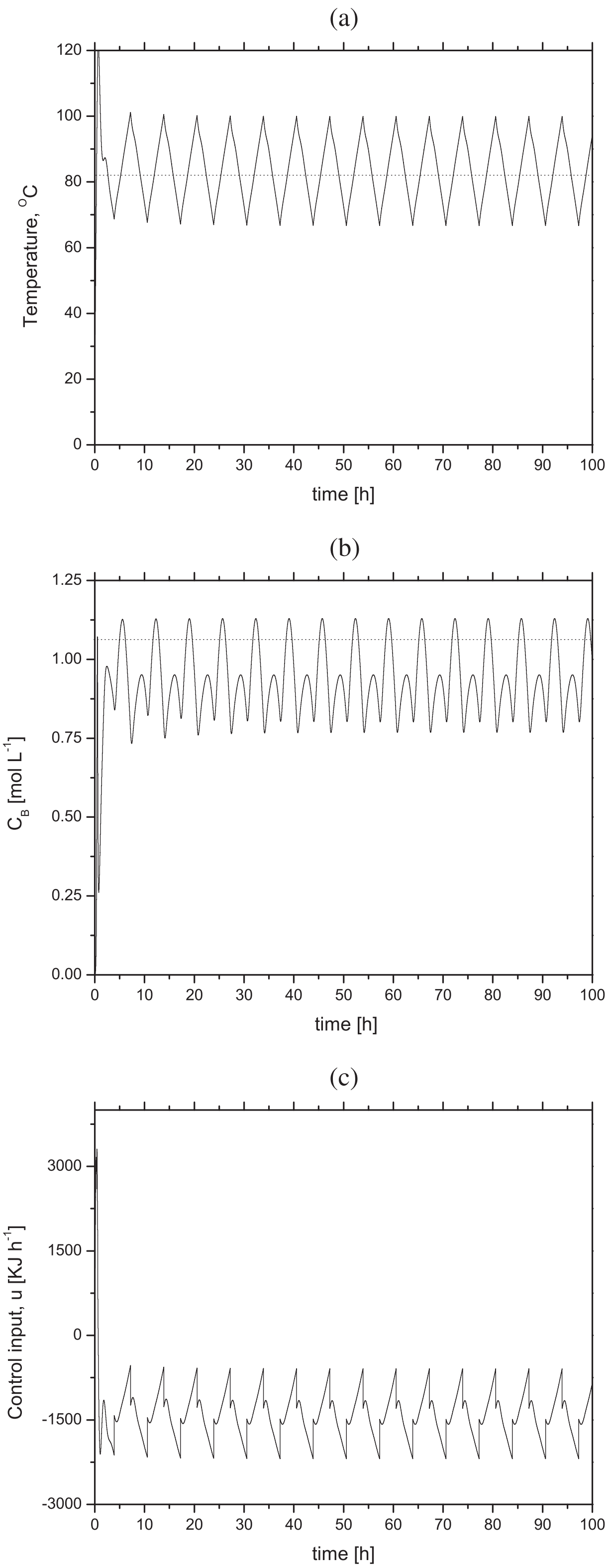

Figure 4. Closed-loop response for the van de Vusse reactor. The dotted lines represent the optimal set point.

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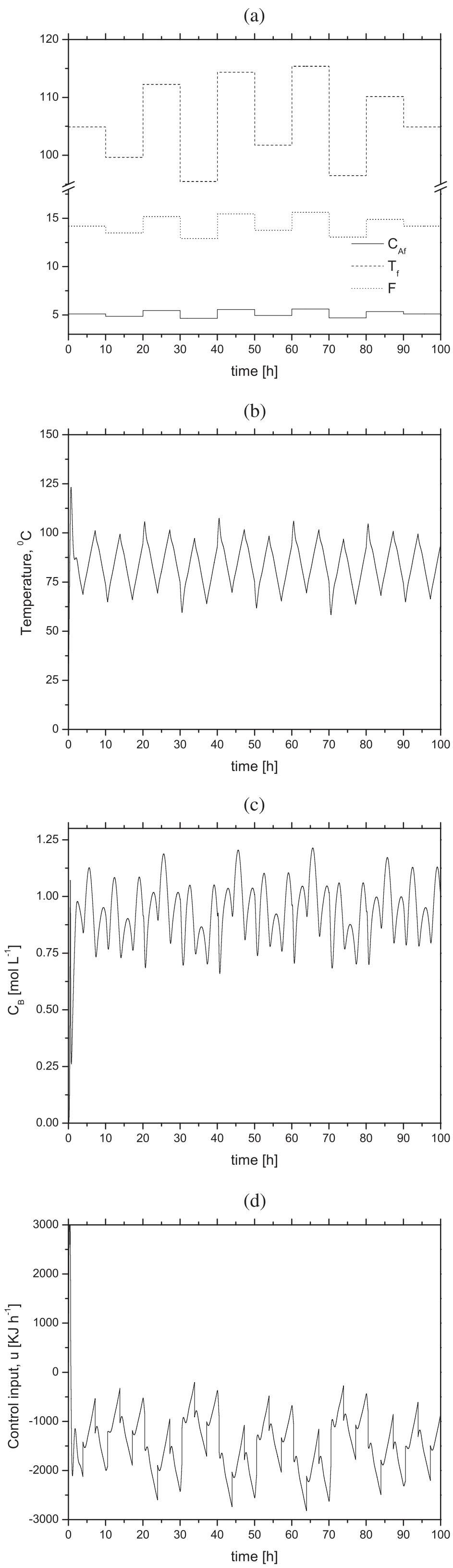

Figure 5. Closed-loop response for the van de Vusse reactor with load disturbances in the inlet flow (T_f, C_{Af} and F) (a).

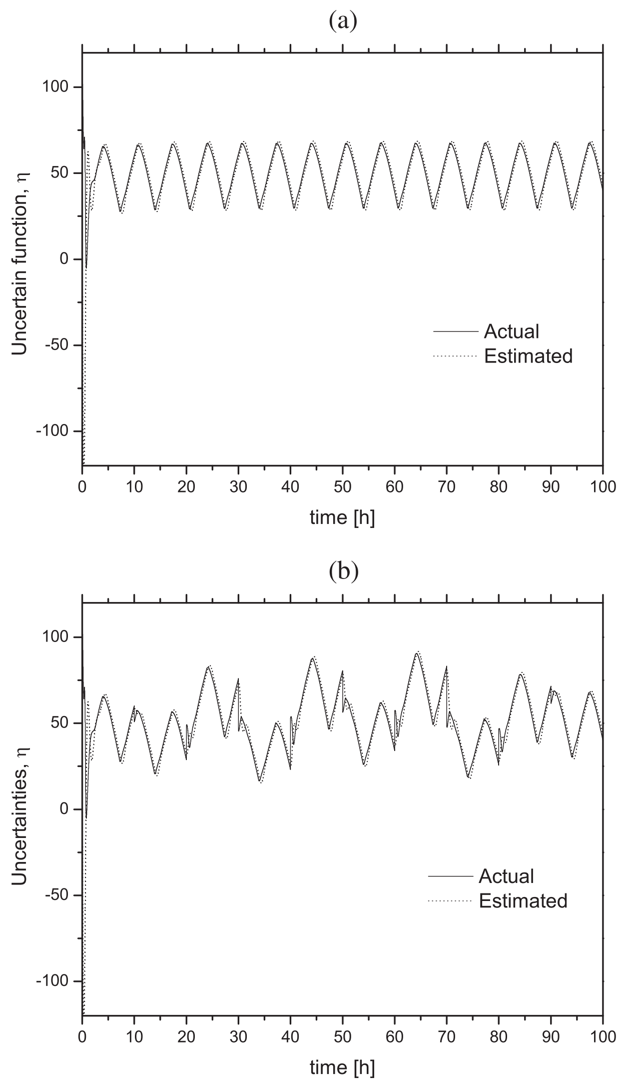

Figure 6. Performance of the uncertain dynamical estimator. The down graph corresponds with the load disturbances in the inflow rate.

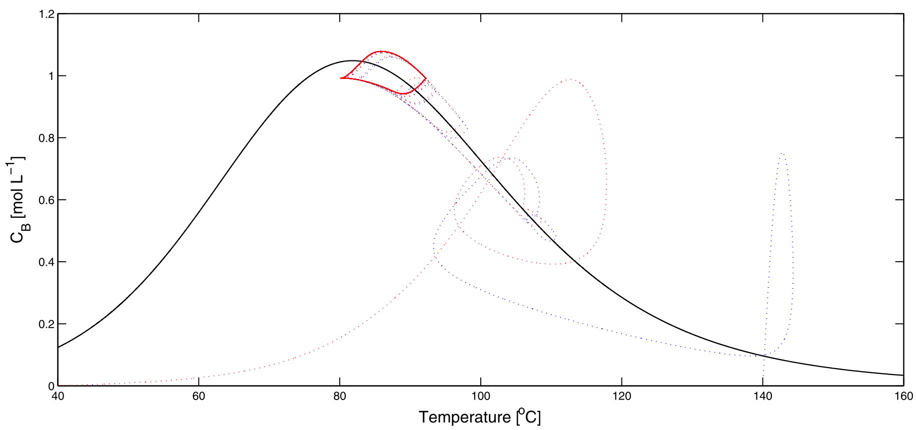

Figure 7. Closed-loop performance of the van de Vusse reactor. The solid line represents the unknown equilibrium profile; the dotted lines are the closed-loop trajectories for distinct initial conditions. [Colour figure can be viewed at wileyonlinelibrary.com]

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Figure 6 is shown the corresponding profiles. We can see that this estimation algorithm is able to predict the uncertainties associated with modelling error and external disturbances. Compared with previous contributions for extremum-seeking control for reaction systems (e.g., [25] and [5]), the proposed ESC scheme (9 and 10) is able to reach the online optimization of the plant despite high uncertainties and external disturbances, but the control action still presents a large control effort. This drawback needs to be looked into more deeply in the context of experimental implementation.

An important issue in the frame of a practical implementation of the proposed ESC scheme is the appropriate selection of the control parameters. In our case, it is important to find an adequate balance between the convergence rate and the accuracy of the ESC scheme. For instance, it is possible to increase the convergence rate to the optimal neighborhood by adjusting the parameters λ and ρ. However, these parameters also affect the size of the δ-vicinity by the restriction imposed in Theorem 2. In fact, the δ-vicinity size is restricted by $2\delta < \lambda\alpha\rho^{-1}$. On the other hand, the high-gain observer-based estimators might be highly sensitive to process noise [26]. In fact, in practical implementations, it may destabilize the closed-loop system by amplifying the process noise with a gain on the order $O(\Gamma^2)$. In a previous contribution, it has been shown that the estimation schemes based on high-gain observers have a structure of a second-order filter (see transfer function (8) in [27]). By following this idea, it is possible to attenuate the process noise by an adequate choice of the observer parameters κ_i' and Γ. However, in practical implementations, the characterization of the dominant frequency of the process disturbance with a fine tuning of the estimation parameters is required; this issue might be addressed in a future contribution.

5. CONCLUSION

In this paper, an extremum-seeking algorithm for a class of uncertain reaction systems is proposed, which is robust with respect to unmodeled dynamics and unmeasured disturbances. A design methodology is proposed by using the input–output linearizing method with sliding mode techniques, coupled with a dynamic modelling-error estimator to compensate for unknown terms. Unlike previous sliding mode-based ESC algorithms, the proposed control scheme guarantees robustness against modeling errors, parametric variations, and external perturbations. The practical stabilizability of the uncertain reactive systems around to the optimal set-point has been discussed for the closed-loop system. Numerical simulation for two different reaction systems shows that the ESC control algorithm yields good uncertain set-point tracking as well as disturbances rejection.

NOMENCLATURE

c	arbitrary constants
C_A	concentration of A
C_{Af}	concentration of A in the inlet flow
C_B	concentration of B
C_P	calorific capacity
D	dilution rate
e	estimation error
f, g	vector fields
F	performance map
g_s	increasing signal
h	output map
H_R	heat reaction
$k_i(T)$	kinetic coefficients
k_1, k_2	yield coefficients
n	order of the system
Q	product gaseous outflow rate
r	relative degree
$\mathbb{R}^n$	n-dimensional real number space
s	switching function

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S	substrate concentration
S_f	substrate concentration in inlet flow
sgn	sign function
$\bar{S_f}$	nominal value for inlet substrate concentration
t	time
T	temperature
T_f	temperature of the inlet flow
u	control input
V	volume
x	dynamic state
X	microorganisms concentration
y	output signal
y^*	critical value of y
Y_δ	δ-vicinity around of y^*
z	new coordinates
α, λ, ρ	control parameters
γ	known terms in the Lie derivatives
ϵ, δ	small positive constants
η	uncertainties
$\hat{\eta}$	estimation of η
Θ	function for lumping the uncertainties
$\kappa_i' s, \Gamma$	observer parameters
μ	specific growth rate
$\bar{\mu}$	upper bound of μ
ν	external reference input
ξ	global cost criterion
$\bar{\xi}$	maximum global cost criterion
Ξ	time derivative for Θ
ρ_m	density
Φ	coordinates transformation
Ω	domain in $\mathbb{R}^n$

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