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# On stability of integral delay systems

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## Abstract

The exponential stability of a class of integral delay systems is investigated by using the Lyapunov-Krasovskii functional approach. Sufficient delay-dependent stability conditions and exponential estimates for the solutions are derived.

*Key words:* Integral delay systems, exponential stability, Lyapunov-Krasovskii functionals

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## 1. Introduction

Integral delay systems play an important role in several stability problems of time-delay systems. This class of systems is found as delay approximations of the partial differential equations for describing the propagation phenomena in excitable media [12], in the stability analysis of additional dynamics introduced by some system transformations [2, 4, 5, 6], in the internal stability problem of controllers used for finite spectrum assignment of time-delay systems [10], as well as in the stability analysis of some difference operators in neutral type functional differential equations [3, 7].

In the forthcoming paper [9], we provide Lyapunov type stability conditions and a converse result guaranteeing the existence of Lyapunov functionals for some classes of integral delay systems. General expressions of quadratic functionals with a given time derivative are provided. The functionals are shown to be useful in solutions of such problems as the estimations of robustness bounds and calculations of exponential estimates for the solutions of exponentially stable integral delay systems. However, some open

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problems associated with the positivity check of such functionals actually prevent their application to the stability analysis of practical systems.

In the present paper, we continue our study on the exponential stability of integral delay systems by using the Lyapunov-Krasovskii functional approach. Our goal is to demonstrate that, based on the general expressions of Lyapunov functionals presented in [9], various reduced type functionals can be constructed to obtain stability conditions formulated directly in terms of the coefficients of integral delay systems.

The paper is structured as follows: Section 2 introduces the class of integral delay systems to be considered. In section 3, we present some preliminary results. Basic facts about the solutions are given and Lyapunov stability conditions are introduced. The main results are given in section 4. First, we consider a general case of integral delay systems for which delay-dependent conditions for exponential stability are derived. Then, a particular case of integral delay systems with multiple delays and constant system matrices is addressed. In this case, exponential estimates and delay-dependent conditions for exponential stability are expressed in terms of linear matrix inequalities. Examples illustrating the results are provided in section 5. Several concluding remarks end the paper.

## 2. Integral delay systems

Consider the following class of integral delay systems:

$$x(t) = \int_{-h}^0 G(\theta)x(t + \theta)d\theta, t \geq 0, \quad (1)$$

where  $h$  is a positive constant and the matrix function  $G(\theta)$  has piecewise continuous bounded elements defined for  $\theta \in [-h, 0]$ .

As mentioned in the introduction, integral delay systems of the form of (1) naturally arise in several stability problems of time-delay systems, see the references provided therein for a detailed presentation of the problems in which the stability analysis of such class of systems is involved.

Here we only recall that a study of the asymptotic behavior of the solutions of (1) is essential for understanding the asymptotic behavior of solutions of linear functional differential equations of the form

$$\frac{d}{dt} \left( x(t) - \int_{-h}^0 G(\theta)x(t + \theta)d\theta \right) = Lx_t, \quad (2)$$

where  $L$  is a bounded linear operator mapping the space of continuous vector functions  $\mathbb{C}([-h, 0], \mathbb{R}^n)$  to  $\mathbb{R}^n$ . Indeed, stability of (1) is a necessary condition for stability of (2), see [3].

In this context, stability of linear functional equations of the form of (1) has been studied widely in [3], [7] and, in particular, from a positive systems approach in [11].

Nevertheless, with exception of [9], one does not find contributions addressing the stability of integral systems of the form of (1) in terms of Lyapunov functionals. Here, we derive sufficient conditions for the exponential stability of (1) by means of Lyapunov functionals.

Throughout this paper we will use the Euclidean norm for vectors and the induced norm for matrices, both denoted by  $\|\cdot\|$ . We denote by  $A^T$  the transpose of  $A$ ,  $I$  stands for the identity matrix, while  $\lambda_{\min}(A)$  and  $\lambda_{\max}(A)$  denote the smallest and largest eigenvalues of a symmetric matrix  $A$ , respectively.

### 3. Preliminaries

#### 3.1. Solutions and stability concept

In order to define a particular solution of (1) an initial vector function  $\varphi(\theta)$ ,  $\theta \in [-h, 0)$  should be given. We assume that  $\varphi$  belongs to the space of piecewise continuous bounded functions  $\mathbb{C}^0([-h, 0], \mathbb{R}^n)$ , equipped with the uniform norm  $\|\varphi\|_h = \sup_{\theta \in [-h, 0)} \|\varphi(\theta)\|$ .

Existence and uniqueness of solutions to (1) can be concluded from the well-known step-by-step method for computing solutions of time-delayed systems and from an appropriate representation of (1) as a Volterra integral equation of the second kind. Thus, from (1) we have

$$x(t) = w(t) + \int_0^t G(\xi - t)x(\xi)d\xi, \forall t \in [0, h], \quad (3)$$

where

$$w(t) = \int_{t-h}^0 G(\xi - t)\varphi(\xi)d\xi.$$

Since  $t \rightarrow w(t)$  is a continuous function in the interval  $[0, h]$ , equation (3) is a particular class of Volterra integral equations of the second kind. Therefore, an application of the successive approximations method allows to conclude the existence of a unique solution  $x(t)$  of (3), defined for all  $t \in [0, h]$ . Upon

obtaining  $x(t)$  for  $t \in [0, h]$ , we can calculate  $x(t)$  for  $t \in [h, 2h]$  similarly, that is

$$x(t) = w(t) + \int_h^t G(\xi - t)x(\xi)d\xi, \forall t \in [h, 2h], \quad (4)$$

where

$$w(t) = \int_{t-h}^h G(\xi - t)x(\xi)d\xi.$$

Continuing this process we obtain the existence of a unique solution  $x(t, \varphi)$  of (1) which is defined for all  $t \in [0, \infty)$ . This solution is continuous and differentiable for all  $t \in [0, \infty)$ , at  $t = 0$  a right-hand side derivative is assumed, and it suffers a jump discontinuity given by

$$\Delta x(0, \varphi) = x(0, \varphi) - x(-0, \varphi) = \int_{-h}^0 G(\theta)\varphi(\theta)d\theta - \varphi(-0).$$

Moreover, the solution  $x(t, \varphi)$  becomes smoother with increasing values of  $t$ . In this sense, (1) belongs to the class of retarded type delay systems.

**Definition 1.** *System (1) is said to be exponentially stable if there exist  $\mu > 0$  and  $\alpha > 0$  such that every solution of (1) satisfies the inequality*

$$\|x(t, \varphi)\| \leq \mu \|\varphi\|_h e^{-\alpha t}, \quad t \geq 0. \quad (5)$$

From now on, we shall use this concept of stability.

### 3.2. Lyapunov stability conditions

We present here Lyapunov type stability conditions for (1). Essentially, the conditions are those presented in [9] for the case when  $G(\theta)$  is a continuous and differentiable matrix function.

In order to make our presentation self-contained we give here the Lyapunov stability conditions along with the proof of the result.

For any  $t \geq 0$  we denote the restriction of the solution  $x(t, \varphi)$  on the interval  $[t - h, t)$  by  $x_t(\varphi) = \{x(t + \theta, \varphi), \theta \in [-h, 0)\}$ . When the initial function is irrelevant, we simply write  $x(t)$  and  $x_t$  instead of  $x(t, \varphi)$  and  $x_t(\varphi)$ .

A simple inspection shows that for  $t \in [0, h)$ ,  $x_t(\varphi)$  belongs to  $\mathbb{C}^0([-h, 0), \mathbb{R}^n)$ , and for  $t \geq h$ ,  $x_t(\varphi)$  belongs to  $\mathbb{C}([-h, 0), \mathbb{R}^n)$ .

**Theorem 1.** *System (1) is exponentially stable if there exists a functional  $v : \mathbb{C}^0([-h, 0], \mathbb{R}^n) \rightarrow \mathbb{R}$  such that  $t \rightarrow v(x_t(\varphi))$  is differentiable on  $\mathbb{R}_+$  and the following conditions hold:*

1.  $\alpha_1 \int_{-h}^0 \|\varphi(\theta)\|^2 d\theta \leq v(\varphi) \leq \alpha_2 \int_{-h}^0 \|\varphi(\theta)\|^2 d\theta$ , for some  $0 < \alpha_1 \leq \alpha_2$ ,
2.  $\frac{d}{dt}v(x_t(\varphi)) \leq -\beta \int_{-h}^0 \|x(t + \theta, \varphi)\|^2 d\theta$ , for some  $\beta > 0$ .

**Proof.** Given any  $\varphi \in \mathbb{C}^0([-h, 0], \mathbb{R}^n)$  it follows from the conditions of the theorem that for  $2\alpha = \beta\alpha_2^{-1} > 0$  the following inequality holds:

$$\frac{d}{dt}v(x_t(\varphi)) + 2\alpha v(x_t(\varphi)) \leq 0, \quad t \geq 0.$$

Thus, on one hand, it follows that

$$v(x_t(\varphi)) \leq e^{-2\alpha t} v(\varphi) \leq \alpha_2 e^{-2\alpha t} \int_{-h}^0 \|\varphi(\theta)\|^2 d\theta \leq h\alpha_2 e^{-2\alpha t} \|\varphi\|_h^2, t \geq 0.$$

On the other hand, one gets

$$\|x(t, \varphi)\|^2 \leq \left( m_g \int_{-h}^0 \|x(t + \theta, \varphi)\| d\theta \right)^2 \leq m_g^2 h \int_{-h}^0 \|x(t + \theta, \varphi)\|^2 d\theta, t \geq 0,$$

where

$$m_g = \sup_{\theta \in [-h, 0]} \|G(\theta)\|.$$

We therefore have the following exponential upper bound:

$$\|x(t, \varphi)\| \leq \mu \|\varphi\|_h e^{-\alpha t}, \quad t \geq 0,$$

where

$$\mu = m_g h \sqrt{\frac{\alpha_2}{\alpha_1}}.$$

■

**Remark 1.** *Note that if there exists a functional  $v : \mathbb{C}^0([-h, 0], \mathbb{R}^n) \rightarrow \mathbb{R}$  satisfying the conditions of the theorem such that the positive constants  $\alpha_1, \alpha_2$  and  $\beta$  can be explicitly calculated, then an exponential upper bound for the solutions of (1) can be explicitly determined.*

## 4. Main Results

In this section, we construct some particular functionals satisfying the conditions of the Theorem 1 for the exponential stability of (1).

### 4.1. General case

We begin with the general case for which a simple-to-check delay-dependent stability condition is derived.

Consider the functional

$$v(\varphi) = \int_{-h}^0 \varphi^T(\theta) [P + (\theta + h)Q] \varphi(\theta) d\theta, \quad (6)$$

where  $P$  and  $Q$  are positive definite matrices.

The functional (6) satisfies the following inequalities:

$$\lambda_{\min}(P) \int_{-h}^0 \|\varphi(\theta)\|^2 d\theta \leq v(\varphi) \leq \lambda_{\max}(P + hQ) \int_{-h}^0 \|\varphi(\theta)\|^2 d\theta.$$

The time-derivative of (6) along the solutions of (1) is

$$\begin{aligned} \frac{d}{dt}v(x_t) &= x^T(t) [P + hQ] x(t) - x^T(t-h) P x(t-h) \\ &\quad - \int_{-h}^0 x^T(t+\theta) Q x(t+\theta) d\theta \\ &= \left( \int_{-h}^0 G(\theta) x(t+\theta) d\theta \right)^T [P + hQ] \left( \int_{-h}^0 G(\theta) x(t+\theta) d\theta \right) \\ &\quad - x^T(t-h) P x(t-h) - \int_{-h}^0 x^T(t+\theta) Q x(t+\theta) d\theta. \end{aligned}$$

Taking into account the fact that by the Cauchy-Schwarz inequality in  $L^2([-h, 0], \mathbb{R})$  we have

$$\left( \int_{-h}^0 \|\varphi(\theta)\| d\theta \right)^2 \leq h \int_{-h}^0 \|\varphi(\theta)\|^2 d\theta,$$

the inequality

$$\begin{aligned} &\left( \int_{-h}^0 G(\theta) x(t+\theta) d\theta \right)^T [P + hQ] \left( \int_{-h}^0 G(\theta) x(t+\theta) d\theta \right) \\ &\leq \lambda_{\max}(P + hQ) \left( \sup_{\theta \in [-h, 0]} \|G(\theta)\| \right)^2 h \int_{-h}^0 \|x(t+\theta)\|^2 d\theta, \end{aligned}$$

holds. As a consequence we obtain the following upper bound for the derivative:

$$\begin{aligned} \frac{d}{dt}v(x_t) &\leq \lambda_{\max}(P + hQ) \left( \sup_{\theta \in [-h, 0]} \|G(\theta)\| \right)^2 h \int_{-h}^0 \|x(t + \theta)\|^2 d\theta \\ &\quad - x^T(t - h)Px(t - h) - \lambda_{\min}(Q) \int_{-h}^0 \|x(t + \theta)\|^2 d\theta \\ &\leq -\beta \int_{-h}^0 \|x(t + \theta)\|^2 d\theta, \end{aligned}$$

where

$$\beta = \lambda_{\min}(Q) - \lambda_{\max}(P + hQ) \left( \sup_{\theta \in [-h, 0]} \|G(\theta)\| \right)^2 h.$$

Thus, if

$$\left( \sup_{\theta \in [-h, 0]} \|G(\theta)\| \right)^2 h < \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P + hQ)},$$

then the exponential stability of (1) follows.

Selecting  $Q = I$  and  $P = \varepsilon I$ ,  $\varepsilon > 0$  sufficiently small, the right-side of the above inequality tends to  $\frac{1}{h}$  when  $\varepsilon \rightarrow +0$ .

The above is summarized in the following proposition.

**Proposition 2.** *System (1) is exponentially stable if*

$$\left( \sup_{\theta \in [-h, 0]} \|G(\theta)\| \right) h < 1. \quad (7)$$

**Remark 2.** *When  $h = 0$ , (1) becomes the trivial system  $x(t) \equiv 0$ , which is obviously exponentially stable. Thus, the inequality (7) provides an upper bound for the delay value at which (1) remains exponentially stable.*

#### 4.2. A particular case

Now consider the following class of integral delay systems with multiple delays:

$$x(t) = \sum_{j=1}^m G_j \int_{-h_j}^0 x(t + \theta) d\theta, \quad (8)$$



where  $0 < h_1 < h_2 < \dots < h_m = h$  and  $G_j \in \mathbb{R}^{n \times n}$ ,  $j = 1, 2, \dots, m$ .

System (8) is a particular class of (1), which can be found in the stability analysis of the additional dynamics introduced by some system transformations, see [4, 5, 6]. Indeed, (8) is obtained from (1) for

$$G(\theta) = \begin{cases} G_m, & \theta \in [-h_m, -h_{m-1}), \\ G_m + G_{m-1}, & \theta \in [-h_{m-1}, -h_{m-2}), \\ \vdots & \vdots \\ \sum_{j=0}^{m-1} G_{m-j}, & \theta \in [-h_1, 0). \end{cases}$$

In the sequel we will make use of the following inequalities, see e.g. [1]:

**Lemma 3.** (*Jensen Inequality*) For any constant matrix  $M \in \mathbb{R}^{n \times n}$ ,  $M = M^T > 0$ , vectors  $\xi_j \in \mathbb{R}^n$ ,  $j = 1, 2, \dots, m$ , scalar  $\gamma > 0$  and vector function  $\omega : [0, \gamma] \rightarrow \mathbb{R}^n$  such that the integrations concerned are well defined, then

$$\gamma \int_0^\gamma \omega^T(\beta) M \omega(\beta) d\beta \geq \left( \int_0^\gamma \omega(\beta) d\beta \right)^T M \left( \int_0^\gamma \omega(\beta) d\beta \right), \quad (9)$$

$$m \sum_{j=1}^m \xi_j^T M \xi_j \geq \left( \sum_{j=1}^m \xi_j^T \right) M \left( \sum_{j=1}^m \xi_j \right). \quad (10)$$

**Proposition 4.** System (8) is exponentially stable if there exist positive definite matrices  $P, Q_j$ ,  $j = 1, 2, \dots, m$  such that

$$Q_j - m h_j G_j^T \left[ P + \sum_{j=1}^m h_j Q_j \right] G_j > 0, j = 1, 2, \dots, m. \quad (11)$$

**Proof.** Consider the functional

$$v(\varphi) = \int_{-h}^0 \varphi^T(\theta) P \varphi(\theta) d\theta + \sum_{j=1}^m \int_{-h_j}^0 \varphi^T(\theta) [(\theta + h_j) Q_j] \varphi(\theta) d\theta, \quad (12)$$

where  $P, Q_j$ ,  $j = 1, 2, \dots, m$  are positive definite matrices.

From (12) we get the following inequalities for the functional  $v(\varphi)$ :

$$\alpha_1 \int_{-h}^0 \|\varphi(\theta)\|^2 \leq v(\varphi) \leq \alpha_2 \int_{-h}^0 \|\varphi(\theta)\|^2,$$

where

$$\alpha_1 = \lambda_{\min}(P), \quad (13)$$

$$\alpha_2 = \lambda_{\max}(P) + \sum_{j=1}^m \lambda_{\max}(h_j Q_j). \quad (14)$$

The time derivative of (12) along the solutions of (8) is

$$\begin{aligned} \frac{d}{dt}v(x_t) &= x^T(t)Px(t) - x^T(t-h)Px(t-h) \\ &+ \sum_{j=1}^m \left[ x^T(t)h_j Q_j x(t) - \int_{-h_j}^0 x^T(t+\theta)Q_j x(t+\theta)d\theta \right] \\ &= \left( \sum_{j=1}^m G_j \int_{-h_j}^0 x(t+\theta)d\theta \right)^T \left[ P + \sum_{j=1}^m h_j Q_j \right] \left( \sum_{j=1}^m G_j \int_{-h_j}^0 x(t+\theta)d\theta \right) \\ &\quad - x^T(t-h)Px(t-h) - \sum_{j=1}^m \int_{-h_j}^0 x^T(t+\theta)Q_j x(t+\theta)d\theta. \end{aligned}$$

By using the Jensen inequalities (9) and (10) the following inequality:

$$\begin{aligned} &\left( \sum_{j=1}^m G_j \int_{-h_j}^0 x(t+\theta)d\theta \right)^T \left[ P + \sum_{j=1}^m h_j Q_j \right] \left( \sum_{j=1}^m G_j \int_{-h_j}^0 x(t+\theta)d\theta \right) \\ &\leq m \sum_{j=1}^m h_j \int_{-h_j}^0 x^T(t+\theta)G_j^T \left[ P + \sum_{j=1}^m h_j Q_j \right] G_j x(t+\theta)d\theta \end{aligned}$$

holds. As a consequence we get the following estimate for the derivative

$$\frac{d}{dt}v(x_t) \leq - \sum_{j=1}^m \int_{-h_j}^0 x^T(t+\theta) \left( Q_j - mh_j G_j^T \left[ P + \sum_{j=1}^m h_j Q_j \right] G_j \right) x(t+\theta)d\theta.$$

Inequalities (11) implies that

$$\frac{d}{dt}v(x_t) \leq -\beta \int_{-h}^0 \|x(t+\theta)\|^2,$$

where

$$\beta = \lambda_{\min} \left( Q_m - mhG_m^T \left[ P + \sum_{j=1}^m h_j Q_j \right] G_m \right), \quad (15)$$

which ends the proof. ■

**Corollary 5.** *If there exist positive definite matrices  $P, Q_j, j = 1, 2, \dots, m$  such that the inequalities (11) hold, then an exponential estimate for the solutions of (8) is determined by*

$$\|x(t, \varphi)\| \leq \mu \|\varphi\|_h e^{-\alpha t}, t \geq 0,$$

where

$$\mu = \left\| \sum_{j=1}^m G_j \right\| h \sqrt{\frac{\alpha_2}{\alpha_1}} \text{ and } 2\alpha = \beta \alpha_2^{-1},$$

with  $\alpha_1, \alpha_2$  and  $\beta$  given by (13), (14) and (15), respectively.

**Proof.** Given positive definite matrices  $P, Q_j, j = 1, 2, \dots, m$  satisfying the inequalities (11), we calculate the positive constants  $\alpha_1, \alpha_2$  and  $\beta$  determined by (13), (14) and (15). Noting that

$$m_g = \sup_{\theta \in [-h, 0]} \|G(\theta)\| = \left\| \sum_{j=1}^m G_j \right\|,$$

the result follows directly from the proof of the Theorem 1. ■

## 5. Examples

Let us consider the simple integral delay system

$$x(t) = \int_{-h}^0 e^{a\theta} x(t + \theta) d\theta, \quad (16)$$

where  $a$  is an arbitrary real parameter.

System (16) could describe the internal dynamics of some controllers used to assign a finite spectrum to systems with delay in the control input, see [10] and [8]. In [11], exponential stability of positive solutions of (16) has been shown for  $a > 0$  and  $h = 1$ .

By Proposition 2, (16) is exponentially stable if

$$\left( \sup_{\theta \in [-h, 0]} e^{a\theta} \right) h < 1.$$

It follows that when  $a \geq 0$ , (16) is exponentially stable if  $h < 1$  and when  $a < 0$ , the exponential stability of (16) is assured if  $h < e^{ha}$ .

As a numerical example consider that  $a = -1$  in (16). Then, (16) is exponentially stable if

$$0 \leq h < 0.567.$$

Now let us consider the integral delay system

$$x(t) = G \int_{-h}^0 x(t + \theta) d\theta, \quad (17)$$

where

$$G = \begin{pmatrix} -4 & 1 \\ -13 & 2 \end{pmatrix}.$$

A direct application of the stability condition (7) yields the exponential stability of (17) if

$$0 \leq h < 0.0726.$$

On the other hand, in this case, the stability condition (11) becomes

$$Q - hG^T [P + hQ] G > 0. \quad (18)$$

It is found that the inequality (18) is feasible for

$$0 \leq h < 0.4473.$$

This simple example illustrates that the two stability conditions (7) and (11) are not equivalent in general.

For comparison, by using the frequency domain approaches developed in [2] and [4], the exact delay interval for which (17) remains exponentially stable is  $[0, 1.0172)$ .

Now let us compute an exponential estimate for the solutions of (17).

For  $h = 0.4$  the corresponding matrices  $P$  and  $Q$  satisfying (18) are

$$P = \begin{pmatrix} 0.0575 & -0.0031 \\ -0.0031 & 0.0478 \end{pmatrix} \text{ and } Q = \begin{pmatrix} 151.8719 & -34.2336 \\ -34.2336 & 11.6031 \end{pmatrix}.$$

With the calculated matrices,  $P$  and  $Q$ , we have from (13), (14), and (15) that  $\alpha_1 = 0.0469$ ,  $\alpha_2 = 63.9707$  and  $\beta = 0.9754$ .

Then, by Corollary 5 we derive the following exponential upper bound:

$$\|x(t, \varphi)\| \leq \mu \|\varphi\|_{0.4} e^{-\alpha t}, t \geq 0,$$

where

$$\mu \approx 203.596 \text{ and } \alpha \approx 0.0076,$$

for all solutions  $x(t, \varphi)$ ,  $\varphi \in \mathbb{C}^0([-0.4, 0], \mathbb{R}^2)$ , of (17).

## 6. Conclusions

In this paper, we continue the study of the exponential stability of integral delay systems by means of Lyapunov-Krasovskii functionals started in [9]. We show, by constructing some particular functionals of the general form given in [9], that exponential estimates and delay-dependent conditions for the exponential stability of such class of systems can be obtained.

We have restricted our presentation to the case of unperturbed systems. However, similar functionals can be constructed for the robust stability analysis of perturbed integral delay systems, but in this case the results are more involved.

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