

**Este artículo puede ser usado únicamente para uso personal o académico.
Cualquier otro uso requiere permiso del autor.**

May 10, 2004

Feedback induction of limit cycle in a bioreactor: controlling towards scale-down

V. Ibarra-Junquera, R. Femat*

Matemáticas Aplicadas y Sistemas Computacionales, IPICYT

Apdo. Postal 3-90, 78231, Tanagamanga

corresponding author, rfemat@ipicyt.edu.mx

Abstract

The feedback stabilization of periodic orbits (induction of limit cycle) via PI-like control is proposed as plausible tool for scale-down studies. An isothermal continuous stirred tank bioreactor (CSTB), with nonideal mixing, is studied. Kinetics is assumed to be governed by Haldane law. The Ready-to-use equations for selecting the control gains are given. Thus, oscillatory behavior with arbitrary frequency and amplitude can be induced into the PI-controlled CSTB.

I. Introduction

Dynamical analysis of reacting systems is now a classical topic in chemical engineering.¹⁴ From its study, oscillatory behavior has been found in reactions which has been attributed to kinetics¹⁸ or the reactor configuration.^{2,19} Concerning kinetics, among others, results have been reported on reactions isothermal forced,²⁰ autocatalytic (self-oscillating)^{18,3,21} and electrochemical.⁹ In regard reactors, the most studied has been the non-isothermal stirred-tank (see results in^{14,2,19,4,6,13}). Oscillations in reactors has already been explained in terms of feedback^{6,13} or recycling⁷ interconnections.

Currently, the reasearch goal of studying oscillatory behavior in reacting systems is about industry applications. In particular, scientific efforts are directed onto scale-down analysis of biological reactors. The underlying idea is to reproduce phenomena from production-scale to laboratory- or pilot-salce reactors. Several studies have been done in this sense but two research directions can be identified in open literature:

- *Construct novel bioreactor configuration.* This can be performed in mixing designs (see¹⁵ for an experimental setup) or connected bioreactors (see¹² for numerical simulation). First case, mixing designs, aims to emulate conditions in production-scale reactors (fluctuating conditions). Whereas the connected reactors confuguration search to different conditions (each in a respective reactor corresponding, for example, to oxygen-rich and oxygen-poor conditions). In both mixing-design and copupled-reactors, the prupose is to emulate fluctating conditions species concentration, related to mixing-time-dependent phenomena, and scale-down is suitably studied by modifying stirres. The drawback of constructing reactors configurations is to perform experimental runs with unpredictable results (increasing cost of scale-down studies).
- *Control structure.* Studies are devoted to (i) tracking of oscillatory (arbitrary) signals and (ii) analysis of oscillations induction due to existence of homoclinic orbits (see¹³ and references therein). Since proportional-integral controller (PI) is the mostly used in industry to control reactors, the efforts are devoted to study PI-like controlled reactors. Two drawbacks are found in tracking. On the one hand, tracking does not exploit the dynamic nature of the reactor, and as a consequence, since the idea in tracking is to compensate (destroy) the reactor dynamics, large control effort is often required to track the oscillatory signal. The second disadvantage is that a PI-like controller is, by its nature, restricted to track high frequency oscillations. Concerning oscillation induction, the attention is payed on existence of chaotic attractors⁶ or bifurcation.¹⁶ Teh problem in thescale-down context is to propose algorithms to induce periodic oscillations with different ferquencies and amplitudes. Actually, chaotic behavior cannot be exploited in this direction because of choas implies wide-band oscillatory behavior. Additionally, bifurcation análisis cannot also be used due to it involves

existence of oscillatory behavior. That is, ready-to-use equations towards scale-down analysis cannot currently be found either chaos or bifurcation.

Substrate gradients can be found in production-scale bioreactors,¹ and can be seen as substrate fluctuations around cells into the bioreactor. Henceforth, the induction of oscillatory behavior into bioreactor in lab- or pilot-scale bioreactor can be used to understand kinetic effect (physiological responses) from such fluctuations.⁵ In this contribution, ready-to-use equations towards scale-down are derived for a class of biological reactor. The underlying idea is, by exploiting dynamical properties of a PI-like controlled bioreactor, to induce periodic oscillatory behavior with arbitrary frequency and amplitude. The PI-controller does not perform tracking; indeed bioreactor stabilizability is used. Thus, the substrate oscillation into the bioreactor lies in the physical domain of the PI-controlled bioreactor. The class of system includes Cholette's reactors (i.e., an isothermal CSTB) with nonideal mixing and Haldane reaction rate. Haldane kinetics involves inhibitory effects at high substrate concentration. Some studies on Cholette's reactor show that: (i) it can be stabilized by PI-like control¹⁷ and (ii) it is affected by nonideal mixing.¹¹ Then, the contribution raises from practical problem: *scale-down analysis*.

The problem is addressed from theory of dynamical system.⁸ The analysis is done on a 2-dimensional smooth system (the PI-controlled bioreactor) with form $\dot{x} = f(x, u)$, where x is a real scalar that represents the substrate concentration and u is the control command. The purpose is to study the existence of periodic orbits at domain - of the pair $(x, u) \in R_+ \times R$ near equilibrium the point $(x^*, u^*) \in -$. The control gains are found such that oscillatory behavior is induced into the PI-controlled bioreactor by using its dynamical properties. In this manner, a guideline is provided to study oscillatory phenomena at bioreactor in the context of the scale-down analysis.

The model and basic assumptions are shown in Section 2. The stability of the PI-controlled bioreactor is also discussed. In Section 3, we give few basic preliminaries on the dynamical systems theory; which are used in main results. The main results are contained in Section 4 and, finally, the text is closed with concluding remarks.

II. Model and stabilization

A. Reactor dynamics

III. Research Bibliography

- ¹Bylund, F., Collet, E., Enfors S.O., Larsson, G., Substrate gradients formation in the large-scale bioreactor lower cell yield and increases by-product formation. *Bioproc. Eng.* 2000, 18, 171-180.
- ²Chen, C-C., Fu, C-C., Tsai, C-H., Stabilized chaotic dynamics of coupled non-isothermal CSTR's, *Chem. Eng., Sci.*, 1996, 51, 5159-5169.
- ³Dolnik, M., Epstein, I.R., Coupled chaotic chemical oscillations, *Phys. Rev. E*, 1996, 54, 3361-3368.
- ⁴Elnashaie, S.S.E.H., Abashar, M.E., Chaotic behavior of periodically forced fluidized-bed catalytic reactors with consecutive exothermic chemical reactions, *Chem. Eng. Sci.*, 1994, 49
- ⁵Enfors, S.O., Jahic, M., Rozkiv, A., Xu, B., Hecker, M., Jürgen, B., Krüger, E., Schweder, T., Hamer, G., O'Beirne, D., Noisommit-Rizzi, N., Reuss, M., Boone, L., Hewitt, C., McFarlane, C., Nienow, A., Kovacs, T., Trägårdh, C., Fuchs, L., Revstedt, J., Friberg, P.C., Hjertager, B., Blomten, G., Skogman, H., Hjort, S., Hoeks, F., Lin, H-Y., Nuebauer, P., van der Lans, R., Luyben, K., Vrabel, P., Maneluis, Å., Physiological responses to mixing in large-scale bioreactors, *J. Biotech.*, 2001, 85, 175-185.
- ⁶Femat, R., Chaos in a class of reacting systems induced by robust asymptotic feedback, *Physica D*, 2000, 136, 193-204
- ⁷Femat, R., Méndez-Acosta, H.O., Steyer, J.P., González-Alvarez, V., Temperature oscillations in a biological reactor with recycle, *Chaos Solitons and Fractals*, 2004, 19, 875-889.
- ⁸Gordillo, F., Salas, F., Ortega, R., Aracil, J., Hopf bifurcation in indirect field-oriented control of induction motors, *Automatica*, 2002, 38, 829-835.
- ⁹González-García, R., Rico-Matrinez, R., Wolf, W., Lübke, W., Eiswirth, E., Anderson, J.S., Kevrekidis, I.G., Characterization of a two-parameters mixed-mode electrochemical behavior regime using neural networks, *Physica D*, 2001, 151, 27-43, 2483-2498.
- ¹⁰Kumar, V.R., Kulkarni, B.D., On the operation of a bistable CSTR: a strategy employing stochastic resonance, *Chem. Eng. Sci.*, 1994, 49, 2709-2713.
- ¹¹Lious, C.Y., Chien, Y.S., The effect of nonideal mixing on input multiplicity in a CSTR. *Chem. Eng. Sci.*, 1991, 46, 2113-2116

- ¹²Ozbek, B., Mathematical modeling and simulation studies of scale-down fermentation systems, *Chem Eng. Tech.*, 1997, 20, 259-267.
- ¹³Pérez, M., Albertos, P., Self-oscillating and chaotic behavior of a PI-controlled CSTR with control valve saturation, *J. Proc. Control*, 2004, 14, 51-59.
- ¹⁴Razón, L.F., Schmitz, R.A., Multiplicities and instabilities in chemically reacting systems- a review, *Chem. Eng. Sci.*, 1987, 42, 1005-1047.
- ¹⁵Schilling B.M., Pfefferle, W., Bachmann, B., Leuchtenberger, W., Decker W.D., A special reactor design for investigations of mixing time effects in a scaled-down industrial L-Lysine fed-batch fermentation process, 1999, 64 (5) 599-606.
- ¹⁶Serra, S., Tablino-Possio, C., Analytical analysis of the gavrillov-guckenheimer bifurcation unfolding in the case of a proportional-Integral Controlled CSTR, *SIAM J. Appl. Math.*, 1999, 59, 1716-1744.
- ¹⁷Sree, R.P., Chidambaram, M.A., Simple method of tuning PI controllers for unstable systems with a zero, *Chem Biochem, Eng.*, 2003, 17, 207-212.
- ¹⁸Steinfeld, J.I., Francisco, J.S., Hase, W.L., *Chemical Kinetics and dynamics*, Prentice Hall, 1989, N.J.
- ¹⁹Svoronos, S., Aris, R., Stephanopoulos, G., On the behavior of two stirred tanks in series, *Chem. Eng. Sci.*, 1982, 37, 357-366.
- ²⁰Tambe S.S., Kulkarni, B.D., Intermittency route to chaos in a periodically forced model reaction systems, *Chem. Eng. Sci.*, 1993, 48, 2817-2821.
- ²¹Vanag, V.K., Yang, L., Dolnik, M., Zhatinsky A.M., Oscillatory cluster patterns in a homogeneous chemical systems with global feedback, *Nature*, 200, 406, 389-391.