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Global Trajectory Tracking Through Static Feedback for Robot Manipulators With Bounded Inputs

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Abstract

In this work, two globally stabilizing bounded control schemes for the tracking control of robot manipulators with saturating inputs are proposed. They may be seen as extensions of the so-called PD+ algorithm to the bounded input case. With respect to previous works on the topic, the proposed approaches give a global solution to the problem through static feedback. Moreover, they are not defined using a specific sigmoidal function, but any one on a set of *saturation* functions. Consequently, each of the proposed schemes actually constitutes a family of globally stabilizing bounded controllers. Furthermore, the bound of such saturation functions is explicitly considered in their definition. Hence, the control gains are not tied to satisfy any *saturation-avoidance* inequality and may consequently take any positive value, which may be considered beneficial for performance-adjustment/improvement purposes. Further, a class of *desired trajectories* that may be globally tracked avoiding input saturation is completely characterized. For both proposed control laws, global uniform asymptotic stabilization of the closed-loop system solutions towards the pre-specified desired trajectory is proved through a strict Lyapunov function. The efficiency of the proposed schemes is corroborated through experimental results.

I. INTRODUCTION

A fundamental scheme for the global trajectory tracking of n -degree-of-freedom (n -DOF) robot manipulators is the well-known PD+ control law proposed in [11]. Such an algorithm considers a continuous calculation of a special form of the robot dynamics, where the current position vector is considered at every of its terms (gravity, inertial, and centrifugal and Coriolis calculated force vectors), the desired acceleration vector is involved in the computed inertial force vector, and both the current and desired velocity vectors are considered in the Coriolis

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and centrifugal calculated force vector. This gives rise to a strategic closed loop form wherefrom it is clear that the desired trajectory is a solution of the closed-loop system. But such terms do not guarantee, by themselves, the stabilization towards the desired trajectory. This is achieved through the additional consideration of position-error (P) and velocity-error (D) linear correction terms. Nevertheless, because of the linearity of such P (*proportional* on the position error) and D (proportional on the *derivative* of the position error) terms and that of the computed Coriolis and centrifugal force vector on the current velocity vector, the PD+ controller turns out to be unbounded. As a consequence, when such an algorithm is implemented in an actual application, the resulting control signals may try to force the actuators to go beyond their natural capabilities, undergoing the well-known phenomenon of *saturation*. Unfortunately, this may give rise to undesirable effects, as pointed out for instance in [4], [7], and [15, §5.2].

In order to avoid the above-mentioned problem, a bounded dynamical extension of the PD+ algorithm has been proposed in [8]. To begin with, the current velocity vector is replaced by the desired velocity trajectory in the computed Coriolis and centrifugal force vector. Hence, by considering twice continuously differentiable desired position trajectories whose 1st and 2nd time-derivative (*i.e.* velocity and acceleration) vectors are bounded, the computed (special form of the) system dynamics turns out to be bounded. Further, the P and D gains are applied to sigmoidal functions —specifically, the hyperbolic tangent— of the closed loop error variables, giving rise to bounded nonlinear P and D terms. Moreover, an auxiliary (internal) dynamical subsystem is considered for the asymptotic estimation of the system velocity error variables. Consequently, only position measurements are involved in the developed algorithm. In a frictionless setting, such a control scheme was proven to semi-globally stabilize the closed-loop system.

By considering viscous friction in the open-loop dynamics, a globally stabilizing version of the control law in [8] was achieved in [13]. The developed scheme keeps the structure of the controller in [8], but the viscous friction force vector is added to the computed robot dynamics, replacing the current velocity vector by the desired velocity trajectory. Under such considerations, global tracking is achieved for suitable trajectories.

Two alternative dynamical approaches were proposed in [4]. Both consider P and D correction terms where the hyperbolic tangent of the tracking error and filtered tracking error variables, respectively, are involved. The first one includes a bounded adaptive compensation of the robot dynamics involving position and velocity measurements. The second one, on the contrary, is free of velocity measurements, keeping a *Computed-Torque*-like structure (see for instance [5, Chapter 10]). It considers the same form of the gravity, viscous friction, and Coriolis and centrifugal calculated force vectors used in [13], but a special form of inertial (complemented) force vector where the bounded nonlinear P and D terms are included. Semi-global tracking is achieved by both controllers.

More recently, revisited versions of the controller in [13] have been developed in [9] and [10]. In the first of these works, [9], gains scaling the argument of the hyperbolic tangents are incorporated. In the second one, [10], the hyperbolic tangents are replaced by a more general class of saturating functions. In both works, local exponential stability was proved through singular perturbation theory. Contrarily to the previously mentioned works, the developed algorithms were experimentally tested and compared to other bounded and unbounded schemes (being

[10] more exhaustive at this point).

Let us note that by the way the bounded nonlinear P and D terms are defined in the previous works, the P and D gains are tied to satisfy a *saturation-avoidance* inequality (since these define the bounds of the P and D terms). Consequently, such control gains cannot take *any* (positive) value, which restricts their *performance-adjustment* natural role. Let us further note that the above-cited works do not completely characterize the class of *desired trajectories* that may be globally tracked (through their proposed algorithms) avoiding input saturation.

When velocity measurements are unavailable or highly noisy, the algorithms in [8], [13], [9], [10], and the second one in [4] may be considered to give an appropriate solution to the tracking problem. On the contrary, the first algorithm in [4] may be suitably applied when the system parameters are uncertain. Nevertheless, none of the above-cited works solve the global tracking problem through a static controller involving all the system states (positions and velocities) and parameters. The design of such a scheme does not only represent an analytical challenge, but its implementation would give rise to faster closed-loop responses. Indeed, involving dynamic estimations of some states or parameters in the control system generally adds inertial effects that commonly slow down the stabilization time and give rise to oscillating transient responses. From this point of view, static controllers expressed in terms of the whole system data (states and parameters) remain an important choice when acceptable estimations of such information are available.

In this work, two globally stabilizing bounded control schemes for the trajectory tracking of robot manipulators with saturating inputs are proposed. They may be seen as extensions of the PD+ algorithm to the bounded input case. With respect to the above-mentioned previous works, they (both) give a global solution to the formulated problem through static feedback. Moreover, they are not defined using a specific sigmoidal function, but any one on a set of *saturation* functions. Consequently, each of the proposed schemes actually constitutes a family of globally stabilizing bounded controllers. Furthermore, the bound of such saturation functions is explicitly considered in their definition. These are consequently applied to the whole linear P and D expressions, giving the P and D gains the liberty to adopt any positive value. Such a freedom to select any combination of control gains together with the *generalized* saturation function formulation give rise to an infinite variety of possibilities to adjust or improve the closed-loop performance. The first of the proposed algorithms, denoted SP-SD+, considers (at every link) each of the P and D correction terms, separately, within a saturation function; this may be considered a basic structure since all the above-mentioned previous works keep separated saturating-proportional and saturating-derivative actions. The second proposed scheme, denoted SPD+, involves both P and D terms within a single saturation function; such a structure turns out to be more beneficial (with respect to the SP-SD+ one) for performance purposes, according to the arguments given in [16] in this connection. Further, a class of desired trajectories that may be globally tracked (by both proposed schemes) avoiding input saturation is completely characterized. For both proposed control laws, global uniform asymptotic stabilization of the closed-loop system solutions towards the pre-specified desired trajectory is proved through a strict Lyapunov function. The efficiency of both proposed schemes is corroborated through experimental tests on a 2-DOF robot manipulator.

The work is organized as follows. Section II states the general n -DOF serial rigid robot manipulator open-loop

dynamics and some of its main properties, as well as considerations and definitions that are involved throughout the study. In Section III, the proposed controllers are presented. Section IV states the main results, where the stability analyses are developed and the control objective is proved to be achieved (for both proposed controllers). Experimental results are presented in Section V. Finally, conclusions are given in Section VI.

II. PRELIMINARIES

The following notation is used throughout the paper. \mathbb{R}_+ denotes the set of nonnegative real numbers, and \mathbb{R}_+^n represents the set of n -dimensional vectors whose elements are nonnegative real numbers. We denote 0_n the origin of \mathbb{R}^n , and I_n the $n \times n$ identity matrix. Let $x \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times m}$. x_i represents the i^{th} element of x , and a_{ij} stands for the element in row i and column j of matrix A . $\|\cdot\|$ denotes the standard Euclidean vector norm and induced matrix norm, i.e. $\|x\| = [\sum_{i=1}^n x_i^2]^{1/2}$ and $\|A\| = [\lambda_{\max}(A^T A)]^{1/2}$, where $\lambda_{\max}(A^T A)$ represents the maximum eigenvalue of $A^T A$. Let \mathcal{A} and \mathcal{E} be subsets (with non-empty interior) of some vector spaces \mathbb{A} and \mathbb{E} respectively. We denote $\mathcal{C}^m(\mathcal{A}; \mathcal{E})$ the set of m -times continuously differentiable functions from \mathcal{A} to \mathcal{E} (with differentiability at any point on the boundary of \mathcal{A} , when included in the set, meant as the limit from the interior of \mathcal{A}). Consider a continuous-time function $h \in \mathcal{C}^2(\mathbb{R}_+; \mathcal{E})$. The time-derivative and second-time-derivative of h are respectively represented as \dot{h} and \ddot{h} , i.e. $\dot{h} : t \mapsto \frac{d}{dt}h$ and $\ddot{h} : t \mapsto \frac{d^2}{dt^2}h$.

Let us consider the general n -DOF serial rigid robot manipulator dynamics with viscous friction [14, §6.2], [2, §2.1]:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = \tau \quad (1)$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity and acceleration vectors, $D(q) \in \mathbb{R}^{n \times n}$ is the inertia matrix, and $C(q, \dot{q})\dot{q}, F\dot{q}, g(q), \tau \in \mathbb{R}^n$ are, respectively, the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with F being a constant, positive definite, diagonal (viscous friction coefficient) matrix, i.e. $F = \text{diag}[f_1, \dots, f_n]$, with $f_i > 0, \forall i \in \{1, \dots, n\}$. The terms of such a dynamical model satisfy some well-known properties (see for instance [5, Chapter 4]). Some of them are recalled here (in particular, Property 5 below may be corroborated in [4]).

Property 1: The inertia matrix $D(q)$ is a positive definite symmetric matrix satisfying $d_m I \leq D(q) \leq d_M I, \forall q \in \mathbb{R}^n$, for some positive constants $d_m \leq d_M$.

Property 2: The Coriolis matrix $C(q, \dot{q})$ satisfies:

- 2.1. $x^T \left[\frac{1}{2} \dot{D}(q, \dot{q}) - C(q, \dot{q}) \right] x = 0, \forall x, q, \dot{q} \in \mathbb{R}^n$;
- 2.2. $\dot{D}(q, \dot{q}) = C(q, \dot{q}) + C^T(q, \dot{q}), \forall q, \dot{q} \in \mathbb{R}^n$;
- 2.3. $C(w, x + y)z = C(w, x)z + C(w, y)z, \forall w, x, y, z \in \mathbb{R}^n$;
- 2.4. $C(x, y)z = C(x, z)y, \forall x, y, z \in \mathbb{R}^n$;
- 2.5. $\|C(x, y)z\| \leq k_c \|y\| \|z\|, \forall x, y, z \in \mathbb{R}^n$, for some constant $k_c \geq 0$.

Property 3: The gravity vector satisfies $\|g(q)\| \leq \gamma, \forall q \in \mathbb{R}^n$, for some positive constant γ , or equivalently, every element of the gravity vector, $g_i(q), i = 1, \dots, n$, satisfies $|g_i(q)| \leq \gamma_i, \forall q \in \mathbb{R}^n$, for some positive constants $\gamma_i, i = 1, \dots, n$.

Property 4: The viscous friction coefficient matrix satisfies $f_m \|x\|^2 \leq x^T F x \leq f_M \|x\|^2, \forall x \in \mathbb{R}^n$, where $0 < f_m \triangleq \min_i \{f_i\} \leq \max_i \{f_i\} \triangleq f_M$.

Property 5: The left-hand side of the robot dynamic model in (1) may be rewritten as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = Y(q, \dot{q}, \ddot{q})\theta$$

where $\theta \in \mathbb{R}^p$, for some integer $p \geq 1$, is a constant vector whose elements are defined exclusively in terms of the robot parameters, and $Y : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times p}$ is an $n \times p$ matrix whose elements depend exclusively on q, \dot{q} , and \ddot{q} and do not involve any of the robot parameters.

Let us suppose that the absolute value of each input τ_i is constrained to be smaller than a given saturation bound $T_i > 0$, i.e. $|\tau_i| \leq T_i, i = 1, \dots, n$. In other words, if u_i represents the control signal (controller output) relative to the i^{th} DOF, then

$$\tau_i = T_i \text{sat} \left(\frac{u_i}{T_i} \right) \quad (2)$$

$i = 1, \dots, n$, where $\text{sat}(\cdot)$ is the standard saturation function, i.e. $\text{sat}(\varsigma) = \text{sign}(\varsigma) \min\{|\varsigma|, 1\}$.

The control scheme proposed in this work involves a special type of (saturation) functions satisfying the following definition.

Definition 1: Given a positive constant M , a function $\sigma : \mathbb{R} \rightarrow \mathbb{R} : \varsigma \mapsto \sigma(\varsigma)$ is said to be a **generalized saturation** with bound M , if it is locally Lipschitz, nondecreasing, and satisfies:

- 1) $\varsigma \sigma(\varsigma) > 0, \forall \varsigma \neq 0$;
- 2) $|\sigma(\varsigma)| \leq M, \forall \varsigma \in \mathbb{R}$.

A strictly increasing continuously differentiable function fulfilling Definition 1 has the following properties.

Lemma 1: Let $\sigma : \mathbb{R} \rightarrow \mathbb{R} : \varsigma \mapsto \sigma(\varsigma)$ be a strictly increasing continuously differentiable generalized saturation function with bound M , k and \bar{k} be positive constants, and $\sigma' : \varsigma \mapsto \frac{d\sigma}{d\varsigma}$. Then

- 1) $y[\sigma(x+y) - \sigma(x)] > 0, \forall y \neq 0, \forall x \in \mathbb{R}$;
- 2) $\lim_{|\varsigma| \rightarrow \infty} \sigma'(\varsigma) = 0$;
- 3) $\sigma'(\varsigma)$ is positive and bounded, i.e. there exists a constant $\sigma'_M \in (0, \infty)$ such that $0 < \sigma'(\varsigma) \leq \sigma'_M, \forall \varsigma \in \mathbb{R}$;
- 4) $\frac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma'_M \varsigma^2}{2}, \forall \varsigma \in \mathbb{R}$;
- 5) $\int_0^\varsigma \sigma(kr)dr > 0, \forall \varsigma \neq 0$;
- 6) $\int_0^\varsigma \sigma(kr)dr \rightarrow \infty$ as $|\varsigma| \rightarrow \infty$;
- 7) $|\sigma(kx + \bar{k}y) - \sigma(\bar{k}y)| \leq \sigma'_M k|x|, \forall x, y \in \mathbb{R}$.
- 8) $|\sigma(kx)| \leq \sigma'_M k|x|, \forall x \in \mathbb{R}$.

Proof: See [1]. ■

We state the **control objective** as the global uniform asymptotic stabilization of the robot configuration vector variable, q , towards a desired trajectory vector, $q_d(t)$, through bounded control signals avoiding input saturations (i.e. such that $|\tau_i(t)| = |u_i(t)| < T_i, i = 1, \dots, n, \forall t \geq 0$; see (2)).

III. PROPOSED CONTROLLERS

The following assumption turns out to be crucial within the analytical setting considered in this work:

Assumption 1: $T_i > \gamma_i, \forall i \in \{1, \dots, n\}$.

Further, in order to guarantee the achievement of the stated control objective, the proposed scheme is restricted to desired trajectory vectors meeting the following:

Assumption 2: The desired trajectory vector $q_d(t)$ is a twice continuously differentiable function —i.e. $q_d \in \mathcal{C}^2(\mathbb{R}_+; \mathbb{R}^n)$ — satisfying

$$\sup_{t \geq 0} \|\dot{q}_d(t)\| \leq B_{dv} \quad (3a)$$

and

$$\sup_{t \geq 0} \|\ddot{q}_d(t)\| \leq B_{da} \quad (3b)$$

for some (velocity and acceleration vector) bounds such that

$$(B_{dv}, B_{da}) \in \mathcal{B}_1 \cup \mathcal{B}_2 \quad (4)$$

where

$$\mathcal{B}_i \triangleq \{(\xi, \zeta) \in \mathbb{R}_+^2 \mid \xi < B_{vi}, \zeta < B_{ai}\} \quad (5)$$

$i = 1, 2,$

$$B_{v1} \triangleq \begin{cases} \min \left\{ \frac{f_m}{k_c}, B_{11} \right\} & \text{if } k_c > 0 \\ B_{10} & \text{if } k_c = 0 \end{cases} \quad (6a)$$

$$B_{a1} \triangleq \frac{\Delta_m - k_c B_{dv}^2 - f_M B_{dv}}{d_M} \quad (6b)$$

$$B_{11} \triangleq -\frac{f_M}{2k_c} + \sqrt{\left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m}{k_c}} \quad (6c)$$

$$B_{10} \triangleq \frac{\Delta_m}{f_M} \quad (6d)$$

and

$$B_{a2} \triangleq \frac{\Delta_m}{d_M} \quad (7a)$$

$$B_{v2} \triangleq \begin{cases} \min \left\{ \frac{f_m}{k_c}, B_{21} \right\} & \text{if } k_c > 0 \\ B_{20} & \text{if } k_c = 0 \end{cases} \quad (7b)$$

$$B_{21} \triangleq -\frac{f_M}{2k_c} + \sqrt{\left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m - d_M B_{da}}{k_c}} \quad (7c)$$

$$B_{20} \triangleq \frac{\Delta_m - d_M B_{da}}{f_M} \quad (7d)$$

with

$$\Delta_m \triangleq \min_i \{T_i - \gamma_i\} \quad (8)$$

Under Assumptions 1 and 2, we propose an **SP-SD+** control scheme of the form

$$u = -s_2(K_2\bar{q}) - s_1(K_1\dot{\bar{q}}) + \tau_c(q, \dot{q}_d, \ddot{q}_d) \quad (9)$$

and an **SPD+** control law of the form

$$u = -s_0(K_2\bar{q} + K_1\dot{\bar{q}}) + \tau_c(q, \dot{q}_d, \ddot{q}_d) \quad (10)$$

where

$$\tau_c(q, \dot{q}_d, \ddot{q}_d) = D(q)\ddot{q}_d + C(q, \dot{q}_d)\dot{q}_d + F\dot{q}_d + g(q) \quad (11)$$

$\bar{q} \triangleq q - q_d(t)$; K_1 and K_2 are positive definite diagonal matrices, *i.e.* $K_1 = \text{diag}[k_{11}, \dots, k_{1n}]$ and $K_2 = \text{diag}[k_{21}, \dots, k_{2n}]$ with $k_{1i} > 0$ and $k_{2i} > 0$ for all $i = 1, \dots, n$; and

$$\begin{aligned} s_j &: \mathbb{R}^n \rightarrow \mathbb{R}^n \\ x &\mapsto s_j(x) = \left(\sigma_{j1}(x_1) \dots \sigma_{jn}(x_n) \right)^T, \quad j = 0, 1, 2 \end{aligned} \quad (12)$$

with $\sigma_{ji}(\cdot)$, $i = 1, \dots, n$, being **strictly increasing continuously differentiable generalized saturation functions** with bounds M_{ji} satisfying

$$M_{1i} + M_{2i} \leq T_i - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} - \gamma_i \quad (13)$$

(see Properties 1, 2.5, 3, and 4), $\forall i = 1, \dots, n$, in the SP-SD+ case (controller (9)), and

$$M_{0i} \leq T_i - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} - \gamma_i \quad (14)$$

$\forall i = 1, \dots, n$, in the SPD+ case (controller (10)).

Remark 1: Observe that $s_1(\cdot)$ and $s_2(\cdot)$ in (9), as well as $s_0(\cdot)$ in (10), are *decoupled* vector functions (see (12)) whose elements are strictly increasing continuously differentiable scalar functions $\sigma_{ji}(\cdot)$ ($j = 1, 2, 3$, $i = 1, \dots, n$) satisfying Definition 1. As a matter of fact, any strictly increasing continuously differentiable function from the whole set of scalar functions satisfying Definition 1 may be chosen for every $\sigma_{ij}(\cdot)$ involved in each of the proposed schemes. The elements of u in (9) are thus given by $u_i = -\sigma_{2i}(k_{2i}\bar{q}_i) - \sigma_{1i}(k_{1i}\dot{\bar{q}}_i) + \tau_{ci}(q, \dot{q}_d, \ddot{q}_d)$, and in (10) by $u_i = -\sigma_{0i}(k_{2i}\bar{q}_i + k_{1i}\dot{\bar{q}}_i) + \tau_{ci}(q, \dot{q}_d, \ddot{q}_d)$, $i = 1, \dots, n$. Such *generalized saturation* functions $\sigma_{ji}(\cdot)$ have the required properties for the achievement of the formulated control objective: *passivity* (they keep the sign of their argument), boundedness, and monotonicity (observe that apart from boundedness, these are actually the properties possessed by the identity function which would give rise to the linear P and D correction terms that are conventionally used in the PD+ controller for the solution of the global tracking problem in the unbounded input context). Thus, for instance in (9), the SP and SD terms play the same *qualitative* role as the P and D actions in the linear unbounded case: $-s_2(K_2\bar{q})$ opposes to displacement away from the desired trajectory $q_d(t)$, and $-s_1(K_1\dot{\bar{q}})$ opposes to motion relative to that generated by the manipulator over $\dot{q}_d(t)$; but they do in a nonlinear bounded fashion. In other words, at every link, the SP term $-\sigma_{2i}(k_{2i}\bar{q}_i)$ acts like a virtual spring that tries to restore the

concerned link to the desired trajectory, and the SD action $-\sigma_{1i}(k_{1i}\dot{\bar{q}}_i)$ acts like a virtual damper that dissipates the motion generated away from that characterized by the desired velocity; moreover, they both act by generating suitable forms of nonlinear bounded (generalized) forces. In the case of (10), the same scenario is observed but with different forms of nonlinear bounded forces. Indeed, note that, at every link, the control expression can be rewritten as $u_i = -[\sigma_{0i}(k_{2i}\bar{q}_i + k_{1i}\dot{\bar{q}}_i) - \sigma_{0i}(k_{2i}\bar{q}_i)] - \sigma_{0i}(k_{2i}\bar{q}_i) + \tau_{ci}(q, \dot{q}_d, \ddot{q}_d)$. Observe, on the one hand, that this equivalent expression has an SP term identical to the one in controller (9), and on the other that, from item 1 of Lemma 1, the first term in its right hand side, $-\sigma_{0i}(k_{2i}\bar{q}_i + k_{1i}\dot{\bar{q}}_i) - \sigma_{0i}(k_{2i}\bar{q}_i)$, has the opposite sign of $\dot{\bar{q}}_i$, and consequently opposes to motion away from that characterized by the desired velocity.

Remark 2: Under inequalities (13) and (14), input saturation is avoided globally in time, as will be shown in Section IV below. In this direction, let us note that the satisfaction of Assumption 2 guarantees the existence of positive values M_{1i} and M_{2i} fulfilling (13) and M_{0i} meeting (14). In turn, Assumption 1 renders possible the *tractable* desired trajectory characterization stated by Assumption 2. Indeed, observe on the one hand that, under the satisfaction of Assumption 1, we have $\Delta_m > 0$ (see (8)), which implies $B_{11} > 0$ (see (6c)) and $B_{10} > 0$ (see (6d)), which in turn entail $B_{v1} > 0$ (see (6a)), which renders possible to state some positive value $B_{dv} < B_{v1}$. With such a value of B_{dv} , we have, if $k_c > 0$: $B_{dv} < -\frac{f_M}{2k_c} + \sqrt{\left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m}{k_c}} \implies \left(B_{dv} + \frac{f_M}{2k_c}\right)^2 < \left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m}{k_c} \implies B_{dv}^2 + \frac{f_M B_{dv}}{k_c} < \frac{\Delta_m}{k_c} \implies \frac{\Delta_m - k_c B_{dv}^2 - f_M B_{dv}}{d_M} > 0$, i.e. $B_{a1} > 0$ (see (6b)), or similarly, if $k_c = 0$: $B_{dv} < \frac{\Delta_m}{f_M} \implies \frac{\Delta_m - f_M B_{dv}}{d_M} > 0$, or equivalently, for any $k_c \geq 0$: $\frac{\Delta_m - k_c B_{dv}^2 - f_M B_{dv}}{d_M} > 0$, i.e. $B_{a1} > 0$, which makes possible to state some positive value $B_{da} < B_{a1}$, by virtue of which \mathcal{B}_1 is non-empty. On the other hand, observe that, under the satisfaction of Assumption 1, we have $\Delta_m > 0$ (see (8)), which implies $B_{a2} > 0$ (see (7a)), which renders possible to state some positive value $B_{da} < B_{a2}$. With such a value of B_{da} , we have $B_{da} < \frac{\Delta_m}{d_M} \implies \Delta_m - d_M B_{da} > 0$, which implies $B_{20} > 0$ (see (7d)) and $B_{21} > 0$ (see (7c)), which in turn entail $B_{v2} > 0$ (see (7b)), which makes possible to state some positive value $B_{dv} < B_{v2}$, by virtue of which \mathcal{B}_2 is non-empty. Thus, the satisfaction of Assumption 1 renders possible to choose a desired trajectory q_d fulfilling Assumption 2. Further, observe that with a value of $B_{da} < B_{a1}$, we have $B_{da} < \frac{\Delta_m - k_c B_{dv}^2 - f_M B_{dv}}{d_M} \implies \min_i \{T_i - \gamma_i\} - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} > 0 \implies T_i - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} - \gamma_i > 0, \forall i = 1, \dots, n$, ensuring positivity of the right-hand-side expression of inequalities (13) and (14), while with a value of $B_{dv} < B_{v2}$ we have, if $k_c > 0$: $B_{dv} < -\frac{f_M}{2k_c} + \sqrt{\left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m - d_M B_{da}}{k_c}} \implies \left(B_{dv} + \frac{f_M}{2k_c}\right)^2 < \left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m - d_M B_{da}}{k_c} \implies B_{dv}^2 + \frac{f_M B_{dv}}{k_c} < \frac{\Delta_m - d_M B_{da}}{k_c} \implies \min_i \{T_i - \gamma_i\} - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} > 0 \implies T_i - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} - \gamma_i > 0, \forall i = 1, \dots, n$, or similarly if $k_c = 0$: $B_{dv} < \frac{\Delta_m - d_M B_{da}}{f_M} \implies \min_i \{T_i - \gamma_i\} - d_M B_{da} - f_M B_{dv} > 0 \implies T_i - d_M B_{da} - f_M B_{dv} - \gamma_i > 0, \forall i = 1, \dots, n$, or equivalently, for any $k_c \geq 0$: $T_i - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} - \gamma_i > 0, \forall i = 1, \dots, n$, ensuring positivity of the right-hand-side expression of inequalities (13) and (14) in this case too. Thus, the satisfaction of Assumption 2 indeed guarantees the existence of positive values M_{1i} and M_{2i} fulfilling (13) and M_{0i} meeting (14).

Remark 3: Observe that the *tractable* trajectory characterization stated through Assumption 2 restricts the desired velocity and acceleration vectors but not the location of the desired task. That is, the desired trajectory may be

defined anywhere on the configuration space as long as it gives rise to sufficiently slow motions. Tasks that may be characterized by sufficiently slow desired trajectories (according to the criterion stated in Assumption 2) are achievable through the control schemes in (9) and (10).

IV. MAIN RESULTS

Proposition 1: Consider the system (1)–(2) with the control law (9) under Assumptions 1 and 2 and the satisfaction of inequalities (13). For any positive definite diagonal control gain matrices K_1 and K_2 , global uniform asymptotic stabilization of the closed-loop system solutions $q(t)$ towards the desired trajectory vector $q_d(t)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \dots, n$, $\forall t \geq 0$.

Proof: From (9), (13), Properties 1, 2.5, 3 and 4, and the strictly increasing character of the involved generalized functions, one sees that $|u_i(t)| < M_{1i} + M_{2i} + d_M B_{da} + k_c B_{dv}^2 + f_M B_{dv} + \gamma_i \leq T_i$, $i = 1, \dots, n$, $\forall t \geq 0$. From this and (2), it follows that $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \dots, n$, $\forall t \geq 0$. We now focus on the stability analysis. The closed-loop dynamics takes the form

$$D(q)\ddot{\bar{q}} + [C(q, \dot{q}) + C(q, \dot{q}_d(t))]\dot{\bar{q}} + F\dot{\bar{q}} + s_1(K_1\dot{\bar{q}}) + s_2(K_2\dot{\bar{q}}) = 0_n \quad (15)$$

where Property 2.4 has been used (observe from the definition of \bar{q} , stated in Section III, that $q = \bar{q} + q_d(t)$ and $\dot{q} = \dot{\bar{q}} + \dot{q}_d(t)$). Let us define the scalar function¹

$$V_1(t, \bar{q}, \dot{\bar{q}}) = \frac{1}{2}\dot{\bar{q}}^T D(\bar{q} + q_d(t))\dot{\bar{q}} + \int_{0_n}^{\bar{q}} s_2^T(K_2 r) dr + \varepsilon_1 s_2^T(K_2 \bar{q}) D(\bar{q} + q_d(t))\dot{\bar{q}} \quad (16)$$

where

$$\int_{0_n}^{\bar{q}} s_2^T(K_2 r) dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{2i}(k_{2i} r_i) dr_i \quad (17)$$

and ε_1 is a positive constant satisfying²

$$\varepsilon_1 < \min \left\{ \frac{f_m - k_c B_{dv}}{k_c B_{2M} + d_M k_{2M} \sigma'_{2M} + \left(k_c B_{dv} + \frac{\sigma'_{1M} k_{1M} + f_M}{2} \right)^2}, \sqrt{\frac{d_m}{2d_M^2 k_{2M} \sigma'_{2M}}} \right\} \quad (18)$$

with $\sigma'_{jM} \triangleq \max_i \{\sigma'_{jiM}\}$ (see item 3 of Lemma 1) and $k_{jM} \triangleq \max_i \{k_{ji}\}$, $j = 1, 2$, and $B_{2M} \triangleq [\sum_{i=1}^n M_{2i}^2]^{1/2}$.

Let us note, from Property 1 and items 4 and 8 of Lemma 1, that

$$W_{11}(\bar{q}, \dot{\bar{q}}) \leq V_1(t, \bar{q}, \dot{\bar{q}}) \leq W_{12}(\bar{q}, \dot{\bar{q}}) \quad (19)$$

¹Observe that the first term in the right-hand side of (16) is a quadratic form, $\frac{1}{2}\dot{\bar{q}}^T D(q)\dot{\bar{q}} = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n d_{ij}(q) \dot{\bar{q}}_i \dot{\bar{q}}_j$; the second term is the (definite) integral of a differential form generated from the internal product among the position-error-variable-dependent vector function $s_2(K_2 \bar{q}) = (\sigma_{21}(k_{21} \bar{q}_1) \cdots \sigma_{2n}(k_{2n} \bar{q}_n))^T$ and the position-error-variable differential vector $d\bar{q} = (d\bar{q}_1 \cdots d\bar{q}_n)^T$, i.e. $s_2^T(K_2 \bar{q}) d\bar{q} = \sum_{i=1}^n \sigma_{2i}(k_{2i} \bar{q}_i) d\bar{q}_i$, whose integral from 0_n to \bar{q} gives rise to the scalar expression in (17); and the third term is a bilinear form (in $s_2(K_2 \bar{q})$ and $\dot{\bar{q}}$), $\varepsilon_1 s_2^T(K_2 \bar{q}) D(q)\dot{\bar{q}} = \varepsilon_1 \sum_{i=1}^n \sum_{j=1}^n d_{ij}(q) \sigma_{2i}(k_{2i} \bar{q}_i) \dot{\bar{q}}_j$.

²Observe that the satisfaction of Assumption 2 guarantees positivity of the first term within the braces in (18), consequently ensuring the existence of a positive ε_1 fulfilling inequality (18). Indeed, note that for a desired trajectory with velocity vector bound such that $B_{dv} < B_{v1}$ or $B_{dv} < B_{v2}$, we have, if $k_c > 0$: $B_{dv} < \frac{f_m}{k_c} \implies f_m - k_c B_{dv} > 0$. From this and the consideration of Property 4, we have, for any $k_c \geq 0$: $f_m - k_c B_{dv} > 0$, wherefrom positivity of the first term within the braces in (18) is guaranteed.

with

$$W_{11}(\bar{q}, \dot{\bar{q}}) \triangleq \frac{d_m}{2} \|\dot{\bar{q}}\|^2 + \frac{1}{2} \int_{0_n}^{\bar{q}} s_2^T(K_2 r) dr + \frac{\|s_2(K_2 \bar{q})\|^2}{4k_{2M}\sigma'_{2M}} - \varepsilon_1 d_M \|\dot{\bar{q}}\| \|s_2(K_2 \bar{q})\|$$

and

$$W_{12}(\bar{q}, \dot{\bar{q}}) \triangleq \frac{d_M}{2} \|\dot{\bar{q}}\|^2 + \frac{k_{2M}\sigma'_{2M}\|\bar{q}\|^2}{2} + \varepsilon_1 d_M k_{2M}\sigma'_{2M} \|\bar{q}\| \|\dot{\bar{q}}\|$$

Moreover, notice that $W_{11}(\bar{q}, \dot{\bar{q}})$ and $W_{12}(\bar{q}, \dot{\bar{q}})$ may be rewritten as

$$W_{11}(\bar{q}, \dot{\bar{q}}) = \frac{1}{2} \int_{0_n}^{\bar{q}} s_2^T(K_2 r) dr + \frac{1}{2} \begin{pmatrix} \|s_2(K_2 \bar{q})\| \\ \|\dot{\bar{q}}\| \end{pmatrix}^T P_{11} \begin{pmatrix} \|s_2(K_2 \bar{q})\| \\ \|\dot{\bar{q}}\| \end{pmatrix} \quad (20)$$

and

$$W_{12}(\bar{q}, \dot{\bar{q}}) = \frac{1}{2} \begin{pmatrix} \|\bar{q}\| \\ \|\dot{\bar{q}}\| \end{pmatrix}^T P_{12} \begin{pmatrix} \|\bar{q}\| \\ \|\dot{\bar{q}}\| \end{pmatrix}$$

where

$$P_{11} = \begin{pmatrix} \frac{1}{2k_{2M}\sigma'_{2M}} & -\varepsilon_1 d_M \\ -\varepsilon_1 d_M & d_m \end{pmatrix}$$

and

$$P_{12} = \begin{pmatrix} k_{2M}\sigma'_{2M} & \varepsilon_1 d_M k_{2M}\sigma'_{2M} \\ \varepsilon_1 d_M k_{2M}\sigma'_{2M} & d_M \end{pmatrix}$$

Further, since $\varepsilon_1 < \sqrt{\frac{d_m}{2d_M^2 k_{2M}\sigma'_{2M}}}$ (see (18)), one can verify (after several basic developments) that P_{11} and P_{12} are positive definite symmetric matrices. From this and items 5 and 6 of Lemma 1, one sees that $V_1(t, \bar{q}, \dot{\bar{q}})$ is positive definite,³ radially unbounded,⁴ and decrescent. Its derivative along the system trajectories is given by

$$\begin{aligned} \dot{V}_1(t, \bar{q}, \dot{\bar{q}}) &= -\dot{\bar{q}}^T C(\bar{q} + q_d(t), \dot{q}_d(t)) \dot{\bar{q}} - \dot{\bar{q}}^T F \dot{\bar{q}} - \dot{\bar{q}}^T s_1(K_1 \dot{\bar{q}}) - \varepsilon_1 s_2^T(K_2 \bar{q}) C(\bar{q} + q_d(t), \dot{q}_d(t)) \dot{\bar{q}} \\ &\quad - \varepsilon_1 s_2^T(K_2 \bar{q}) F \dot{\bar{q}} - \varepsilon_1 s_2^T(K_2 \bar{q}) s_1(K_1 \dot{\bar{q}}) - \varepsilon_1 s_2^T(K_2 \bar{q}) s_2(K_2 \bar{q}) + \varepsilon_1 \dot{\bar{q}}^T C(\bar{q} + q_d(t), \dot{q}_d(t)) s_2(K_2 \bar{q}) \\ &\quad + \varepsilon_1 \dot{\bar{q}}^T C(\bar{q} + q_d(t), \dot{q}_d(t)) s_2(K_2 \bar{q}) + \varepsilon_1 \dot{\bar{q}}^T D(\bar{q} + q_d(t)) s_2'(K_2 \bar{q}) K_2 \dot{\bar{q}} \end{aligned}$$

with $s_2'(K_2 \bar{q}) = \text{diag}[\sigma'_{21}(k_{21}\bar{q}_1), \dots, \sigma'_{2n}(k_{2n}\bar{q}_n)]$, where $D(\bar{q} + q_d(t))\dot{\bar{q}}$ has been replaced by its equivalent expression from the closed-loop dynamics (15), and Properties 2.1–2.3 have been used. From Properties 1, 2.5

³ $V_1(t, \bar{q}, \dot{\bar{q}})$ is said to be positive definite if $W_{11}(\bar{q}, \dot{\bar{q}})$ in (19) is positive definite. Since, under the satisfaction of (18), P_{11} is a positive definite matrix, and from item 1 of Definition 1 —according to which generalized saturations are zero if and only if their argument is zero, wherefrom we have that $s_2(K_2 \bar{q}) = 0_n \iff \bar{q} = 0_n$ —, the second term in the right-hand side of (20) is positive definite (in $(\bar{q}, \dot{\bar{q}})$). Furthermore, observe from (17) and item 5 of Lemma 1 that the integral term in the right-hand side of (20) is zero if $\bar{q} = 0_n$ and positive for any $(\bar{q}, \dot{\bar{q}})$ such that $\bar{q} \neq 0_n$. Therefore, $W_{11}(\bar{q}, \dot{\bar{q}})$ is positive definite.

⁴ $V_1(t, \bar{q}, \dot{\bar{q}})$ is said to be radially unbounded if $W_{11}(\bar{q}, \dot{\bar{q}})$ in (19) is radially unbounded. From the positive definite quadratic form in the right-hand side of (20), one easily sees that $W_{11}(\bar{q}, \dot{\bar{q}})$ grows to infinity as any component of $\dot{\bar{q}}$ does. Furthermore, from (17) and item 6 of Lemma 1, one sees that the integral term in the right-hand side of (20) diverges as any component of \bar{q} does. Thus, $W_{11}(\bar{q}, \dot{\bar{q}})$ is radially unbounded.

and 4, item 8 of Lemma 1, and the satisfaction of inequalities (3), we have that

$$\dot{V}_1(t, \bar{q}, \dot{\bar{q}}) \leq W_{13}(\bar{q}, \dot{\bar{q}}) \quad (21)$$

with

$$\begin{aligned} W_{13}(\bar{q}, \dot{\bar{q}}) &\triangleq k_c B_{dv} \|\dot{\bar{q}}\|^2 - f_m \|\dot{\bar{q}}\|^2 - \dot{\bar{q}}^T s_1(K_1 \dot{\bar{q}}) + \varepsilon_1 k_c B_{dv} \|\dot{\bar{q}}\| \|s_2(K_2 \bar{q})\| + \varepsilon_1 f_M \|\dot{\bar{q}}\| \|s_2(K_2 \bar{q})\| \\ &\quad + \varepsilon_1 k_{1M} \sigma'_{1M} \|\dot{\bar{q}}\| \|s_2(K_2 \bar{q})\| - \varepsilon_1 \|s_2(K_2 \bar{q})\|^2 + \varepsilon_1 k_c B_{2M} \|\dot{\bar{q}}\|^2 + \varepsilon_1 k_c B_{dv} \|\dot{\bar{q}}\| \|s_2(K_2 \bar{q})\| \\ &\quad + \varepsilon_1 d_M k_{2M} \sigma'_{2M} \|\dot{\bar{q}}\|^2 \end{aligned}$$

where the facts that $\|s_2(K_2 \bar{q})\| \leq [\sum_{i=1}^n M_{2i}^2]^{1/2} \triangleq B_{2M}$ and $\|s'_2(K_2 \bar{q})\| \leq \max_i \{\sigma'_{2iM}\} \triangleq \sigma'_{2M}$, $\forall \bar{q} \in \mathbb{R}^n$, have been considered. Notice that $W_{13}(\bar{q}, \dot{\bar{q}})$ may be rewritten as

$$W_{13}(\bar{q}, \dot{\bar{q}}) = -\dot{\bar{q}}^T s_1(K_1 \dot{\bar{q}}) - \begin{pmatrix} \|s_2(K_2 \bar{q})\| \\ \|\dot{\bar{q}}\| \end{pmatrix}^T Q_1 \begin{pmatrix} \|s_2(K_2 \bar{q})\| \\ \|\dot{\bar{q}}\| \end{pmatrix} \quad (22)$$

where

$$Q_1 = \begin{pmatrix} \varepsilon_1 & -\varepsilon_1 \left(k_c B_{dv} + \frac{\sigma'_{1M} k_{1M} + f_M}{2} \right) \\ -\varepsilon_1 \left(k_c B_{dv} + \frac{\sigma'_{1M} k_{1M} + f_M}{2} \right) & f_m - k_c B_{dv} - \varepsilon_1 (k_c B_{2M} + d_M k_{2M} \sigma'_{2M}) \end{pmatrix}$$

Further, since $\varepsilon_1 < \frac{f_m - k_c B_{dv}}{k_c B_{2M} + d_M k_{2M} \sigma'_{2M} + \left(k_c B_{dv} + \frac{\sigma'_{1M} k_{1M} + f_M}{2} \right)^2}$ (see (18)), one can verify (after several basic developments) that Q_1 is a positive definite symmetric matrix. From this and item 1 of Definition 1, one sees that $\dot{V}_1(t, \bar{q}, \dot{\bar{q}})$ is negative definite.⁵ Thus, from Lyapunov's stability theory (applied to non-autonomous systems; see for instance [6, Theorem 4.9]), the proposition follows. ■

Proposition 2: Consider the system (1)–(2) with the control law (10) under Assumptions 1 and 2 and the satisfaction of inequalities (14). For any positive definite diagonal control gain matrices K_1 and K_2 , global uniform asymptotic stabilization of the closed-loop system solutions $q(t)$ towards the desired trajectory vector $q_d(t)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \dots, n$, $\forall t \geq 0$.

Proof: From (10), (14), Properties 1, 2.5, 3 and 4, and the strictly increasing character of the involved generalized functions, one sees that $|u_i(t)| < M_{0i} + d_M B_{da} + k_c B_{dv}^2 + f_M B_{dv} + \gamma_i \leq T_i$, $i = 1, \dots, n$, $\forall t \geq 0$. From this and (2), it follows that $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \dots, n$, $\forall t \geq 0$. The stability analysis is now developed. The closed-loop dynamics takes the form

$$D(q)\ddot{\bar{q}} + [C(q, \dot{q}) + C(q, \dot{q}_d(t))]\dot{\bar{q}} + F\dot{\bar{q}} + s_0(K_1 \dot{\bar{q}} + K_2 \bar{q}) = 0_n \quad (23)$$

⁵ $\dot{V}_1(t, \bar{q}, \dot{\bar{q}})$ is said to be negative definite if $W_{13}(\bar{q}, \dot{\bar{q}})$ in (21) is negative definite. Since, under the satisfaction of (18), Q_1 is a positive definite matrix, and from item 1 of Definition 1 —according to which $s_2(K_2 \bar{q}) = 0_n \iff \bar{q} = 0_n$ —, the second term in the right-hand side of (22) is negative definite (in $(\bar{q}, \dot{\bar{q}})$). Furthermore, observe that $-\dot{\bar{q}}^T s_1(K_1 \dot{\bar{q}}) = -\sum_{i=1}^n \dot{q}_i \sigma_{1i}(k_{1i} \dot{q}_i)$. From this and item 1 of Definition 1, one sees that the first term in the right-hand side of (22) is zero if $\dot{\bar{q}} = 0_n$ and negative for any $(\bar{q}, \dot{\bar{q}})$ such that $\dot{\bar{q}} \neq 0_n$. Therefore, $W_{13}(\bar{q}, \dot{\bar{q}})$ is negative definite.

where Property 2.4 has been used (recall that $q = \bar{q} + q_d(t)$ and $\dot{q} = \dot{\bar{q}} + \dot{q}_d(t)$). Let us define the scalar function

$$V_2(t, \bar{q}, \dot{\bar{q}}) = \frac{1}{2} \dot{\bar{q}}^T D(\bar{q} + q_d(t)) \dot{\bar{q}} + \int_{0_n}^{\bar{q}} s_0^T(K_2 r) dr + \varepsilon_2 s_0^T(K_2 \bar{q}) D(\bar{q} + q_d(t)) \dot{\bar{q}} \quad (24)$$

where

$$\int_{0_n}^{\bar{q}} s_0^T(K_2 r) dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{0i}(k_{2i} r_i) dr_i$$

and ε_2 is a positive constant satisfying

$$\varepsilon_2 < \min \left\{ \frac{f_m - k_c B_{dv}}{k_c B_{0M} + d_M k_{2M} \sigma'_{0M} + \left(k_c B_{dv} + \frac{\sigma'_{0M} k_{1M} + f_M}{2} \right)^2}, \sqrt{\frac{d_m}{2d_M^2 k_{2M} \sigma'_{0M}}} \right\} \quad (25)$$

where $\sigma'_{0M} \triangleq \max_i \{\sigma'_{0iM}\}$ and $k_{jM} \triangleq \max_i \{k_{ji}\}$, $j = 1, 2$, and $B_{0M} \triangleq [\sum_{i=1}^n M_{0i}^2]^{1/2}$. Notice that $V_2(t, \bar{q}, \dot{\bar{q}})$ in (24) adopts the same form of $V_1(t, \bar{q}, \dot{\bar{q}})$ in (16) (by simply replacing s_2 in V_1 by s_0 ; hence, analog observations to those pointed out in Footnotes 1, 3, and 4 in relation to V_1 in (16) apply to V_2 in (24)). Thus, following a procedure analog to the one developed for $V_1(t, \bar{q}, \dot{\bar{q}})$ in the proof of Proposition 1, we get

$$W_{21}(\bar{q}, \dot{\bar{q}}) \leq V_2(t, \bar{q}, \dot{\bar{q}}) \leq W_{22}(\bar{q}, \dot{\bar{q}})$$

with

$$W_{21}(\bar{q}, \dot{\bar{q}}) = \frac{1}{2} \int_{0_n}^{\bar{q}} s_0^T(K_2 r) dr + \frac{1}{2} \begin{pmatrix} \|s_0(K_2 \bar{q})\| \\ \|\dot{\bar{q}}\| \end{pmatrix}^T P_{21} \begin{pmatrix} \|s_0(K_2 \bar{q})\| \\ \|\dot{\bar{q}}\| \end{pmatrix}$$

and

$$W_{22}(\bar{q}, \dot{\bar{q}}) = \frac{1}{2} \begin{pmatrix} \|\bar{q}\| \\ \|\dot{\bar{q}}\| \end{pmatrix}^T P_{22} \begin{pmatrix} \|\bar{q}\| \\ \|\dot{\bar{q}}\| \end{pmatrix}$$

where

$$P_{21} = \begin{pmatrix} \frac{1}{2k_{2M} \sigma'_{0M}} & -\varepsilon_2 d_M \\ -\varepsilon_2 d_M & d_m \end{pmatrix}$$

and

$$P_{22} = \begin{pmatrix} k_{2M} \sigma'_{0M} & \varepsilon_2 d_M k_{2M} \sigma'_{0M} \\ \varepsilon_2 d_M k_{2M} \sigma'_{0M} & d_M \end{pmatrix}$$

Further, since $\varepsilon_2 < \sqrt{\frac{d_m}{2d_M^2 k_{2M} \sigma'_{0M}}}$ (see (25)), one can verify (after several basic developments) that P_{21} and P_{22} are positive definite symmetric matrices. From this and items 5 and 6 of Lemma 1, one sees that $V_2(t, \bar{q}, \dot{\bar{q}})$ is positive definite, radially unbounded, and decrescent. Its derivative along the system trajectories is given by

$$\begin{aligned} \dot{V}_2(t, \bar{q}, \dot{\bar{q}}) &= -\dot{\bar{q}}^T C(\bar{q} + q_d(t), \dot{q}_d(t)) \dot{\bar{q}} - \dot{\bar{q}}^T F \dot{\bar{q}} - \dot{\bar{q}}^T [s_0(K_1 \dot{\bar{q}} + K_2 \bar{q}) - s_0(K_2 \bar{q})] \\ &\quad - \varepsilon_2 s_0^T(K_2 \bar{q}) C(\bar{q} + q_d(t), \dot{q}_d(t)) \dot{\bar{q}} - \varepsilon_2 s_0^T(K_2 \bar{q}) F \dot{\bar{q}} - \varepsilon_2 s_0^T(K_2 \bar{q}) [s_0(K_1 \dot{\bar{q}} + K_2 \bar{q}) - s_0(K_2 \bar{q})] \\ &\quad - \varepsilon_2 s_0^T(K_2 \bar{q}) s_0(K_2 \bar{q}) + \varepsilon_2 \dot{\bar{q}}^T C(\bar{q} + q_d(t), \dot{q}_d(t)) s_0(K_2 \bar{q}) + \varepsilon_2 \dot{\bar{q}}^T C(\bar{q} + q_d(t), \dot{q}_d(t)) s_0(K_2 \bar{q}) \\ &\quad + \varepsilon_2 \dot{\bar{q}}^T D(\bar{q} + q_d(t)) s_0'(K_2 \bar{q}) K_2 \dot{\bar{q}} \end{aligned}$$

with $s'_0(K_2\bar{q}) = \text{diag}[\sigma'_{01}(k_{21}\bar{q}_1), \dots, \sigma'_{0n}(k_{2n}\bar{q}_n)]$, where $D(\bar{q} + q_d(t))\ddot{q}$ has been replaced by its equivalent expression from the closed-loop dynamics (23), and Properties 2.1–2.3 have been used. From Properties 1, 2.5 and 4, item 7 of Lemma 1, and the satisfaction of inequalities (3), we have that

$$\dot{V}_2(t, \bar{q}, \dot{\bar{q}}) \leq W_{23}(\bar{q}, \dot{\bar{q}}) \quad (26)$$

with

$$\begin{aligned} W_{23}(\bar{q}, \dot{\bar{q}}) \triangleq & k_c B_{dv} \|\dot{\bar{q}}\|^2 - f_m \|\dot{\bar{q}}\|^2 - \dot{\bar{q}}^T [s_0(K_1\dot{\bar{q}} + K_2\bar{q}) - s_0(K_2\bar{q})] + \varepsilon_2 k_c B_{dv} \|\dot{\bar{q}}\| \|s_0(K_2\bar{q})\| \\ & + \varepsilon_2 f_M \|\dot{\bar{q}}\| \|s_0(K_2\bar{q})\| + \varepsilon_2 k_{1M} \sigma'_{0M} \|\dot{\bar{q}}\| \|s_0(K_2\bar{q})\| - \varepsilon_2 \|s_0(K_2\bar{q})\|^2 + \varepsilon_2 k_c B_{0M} \|\dot{\bar{q}}\|^2 \\ & + \varepsilon_2 k_c B_{dv} \|\dot{\bar{q}}\| \|s_0(K_2\bar{q})\| + \varepsilon_2 d_M k_{2M} \sigma'_{0M} \|\dot{\bar{q}}\|^2 \end{aligned}$$

where the facts that $\|s_0(K_2\bar{q})\| \leq [\sum_{i=1}^n M_{0i}^2]^{1/2} \triangleq B_{0M}$ and $\|s'_0(K_2\bar{q})\| \leq \max_i \{\sigma'_{0iM}\} \triangleq \sigma'_{0M}$, $\forall \bar{q} \in \mathbb{R}^n$, have been considered. Notice that $W_{23}(\bar{q}, \dot{\bar{q}})$ may be rewritten as

$$W_{23}(\bar{q}, \dot{\bar{q}}) = -\dot{\bar{q}}^T [s_0(K_1\dot{\bar{q}} + K_2\bar{q}) - s_0(K_2\bar{q})] - \begin{pmatrix} \|s_0(K_2\bar{q})\| \\ \|\dot{\bar{q}}\| \end{pmatrix}^T Q_2 \begin{pmatrix} \|s_0(K_2\bar{q})\| \\ \|\dot{\bar{q}}\| \end{pmatrix} \quad (27)$$

where

$$Q_2 = \begin{pmatrix} \varepsilon_2 & -\varepsilon_2 \left(k_c B_{dv} + \frac{\sigma'_{0M} k_{1M} + f_M}{2} \right) \\ -\varepsilon_2 \left(k_c B_{dv} + \frac{\sigma'_{0M} k_{1M} + f_M}{2} \right) & f_m - k_c B_{dv} - \varepsilon_2 (k_c B_{0M} + d_M k_{2M} \sigma'_{0M}) \end{pmatrix}$$

Further, since $\varepsilon_2 < \frac{f_m - k_c B_{dv}}{k_c B_{0M} + d_M k_{2M} \sigma'_{0M} + \left(k_c B_{dv} + \frac{\sigma'_{0M} k_{1M} + f_M}{2} \right)^2}$ (see (25)), one can verify (after several basic developments) that Q_2 is a positive definite symmetric matrix. From this and item 1 of Lemma 1, one sees that $\dot{V}_2(t, \bar{q}, \dot{\bar{q}})$ is negative definite.⁶ Thus, from Lyapunov's stability theory (applied to non-autonomous systems; see for instance [6, Theorem 4.9]), the proposition follows. ■

Remark 4: Observe that, according to Property 5, τ_c in (11) may be rewritten as $\tau_c(q, \dot{q}_d, \ddot{q}_d; \theta) = Y(q, \dot{q}_d, \ddot{q}_d)\theta$, and note that, in view of Properties 1, 2.5, and 3, and Assumption 2, we have that $\|Y(q, \dot{q}_d, \ddot{q}_d)\| \leq B_Y$, for some positive constant B_Y dependent on B_{dv} and B_{da} . Furthermore, note that, through the consideration of $\tau_c(q, \dot{q}_d, \ddot{q}_d; \theta)$ in (9) and (10), we have implicitly assumed the exact knowledge of the system parameters. Let us consider the more realistic case in which $\tau_c(q, \dot{q}_d, \ddot{q}_d; \hat{\theta})$ is rather considered in (9) and (10), where $\hat{\theta}$ represents a vector of *estimated* parameters which is not necessarily equal to θ . Moreover, let us also take into account other type of model imprecisions whose consideration gives rise to additional bounded nonlinear terms in the robot dynamics, such as the omission of Coulomb or static friction forces. Such additional terms will be subsequently (all together)

⁶By analog arguments to those given in Footnote 5, the second term in the right-hand side of (27) turns out to be negative definite (in $(\bar{q}, \dot{\bar{q}})$). Furthermore, observe that $-\dot{\bar{q}}^T [s_0(K_1\dot{\bar{q}} + K_2\bar{q}) - s_0(K_2\bar{q})] = -\sum_{i=1}^n \dot{\bar{q}}_i [\sigma_{0i}(k_{1i}\dot{\bar{q}}_i + k_{2i}\bar{q}_i) - \sigma_{0i}(k_{2i}\bar{q}_i)]$. From this and item 1 of Lemma 1, one sees that the first term in the right-hand side of (27) is zero if $\dot{\bar{q}} = 0_n$ and negative for any $(\bar{q}, \dot{\bar{q}})$ such that $\dot{\bar{q}} \neq 0_n$. Therefore, $W_{23}(\bar{q}, \dot{\bar{q}})$ is negative definite, wherefrom, according to (26), negative definiteness of $\dot{V}_2(t, \bar{q}, \dot{\bar{q}})$ is concluded.

represented as $\phi(q, \dot{q})$, and (in view of their assumed boundedness) will be considered to satisfy $\|\phi(q, \dot{q})\| \leq B_\phi$, $\forall (q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, for some positive constant B_ϕ .⁷ Thus, the *perturbed* closed loop dynamics takes the form

$$D(q)\ddot{\bar{q}} + [C(q, \dot{q}) + C(q, \dot{q}_d(t))]\dot{\bar{q}} + F\dot{\bar{q}} + u_s(\bar{q}, \dot{\bar{q}}) = Y(q, \dot{q}_d, \ddot{q}_d)(\hat{\theta} - \theta) - \phi(q, \dot{q}) \quad (28)$$

where $u_s(\bar{q}, \dot{\bar{q}}) = s_1(K_1\dot{\bar{q}}) + s_2(K_2\bar{q})$ in the case of (9), and $u_s(\bar{q}, \dot{\bar{q}}) = s_0(K_1\dot{\bar{q}} + K_2\bar{q})$ in the case of (10). Let us define $z \triangleq (\bar{q}^T, \dot{\bar{q}}^T)^T$, and let $\dot{z} = f(t, z) + h(t, z)$ denote the consequent closed loop state-space representation, with $\dot{z} = f(t, z)$ representing the *nominal* closed-loop system, *i.e.* under the consideration of $\hat{\theta} = \theta$ and $\phi(q, \dot{q}) \equiv 0_n$, and $h(t, z)$ accounting for the model imprecisions, *i.e.* accounting for the right-hand-side terms of (28). Observe that, by the assumptions made, $\|h(t, z)\| \leq B_h$ for some positive constant B_h whose value is directly influenced by $\|\hat{\theta} - \theta\|$ and B_ϕ . Hence, according to Lemma 9.3 in [6], for any $z(t_0) \in \mathbb{R}^{2n}$, with t_0 representing the solution initial time, there exists a nonnegative constant T such that $\|z(t)\| \leq \rho(B_h)$, $\forall t \geq t_0 + T$, for some class \mathcal{K} function ρ . Thus, the closed-loop system response $z(t)$ gets into a positively invariant ball around the origin after a finite time, whose radius is directly related to $\|\hat{\theta} - \theta\|$ and B_ϕ . Therefore, certain degree of robustness against *sufficiently small* model imprecisions (like biased parameter estimations or unmodelled friction forces) is concluded.

V. EXPERIMENTAL RESULTS

Aiming at verifying the effectiveness of the proposed controllers, real-time experiments were carried out on a well-identified two-axis planar robot arm. This manipulator was built keeping the same mechanical structure of the one described and used in [12]. The actuators are direct-drive brushless motors operated in torque mode, so they act as torque sources and accept an analog voltage as a reference of torque signal. The control algorithm is executed at a 2.5 msec sampling period in a control board (based on a DSP 32-bit floating point microprocessor) mounted on a PC host computer.

For the considered experimental robot, the various terms characterizing the system dynamics in (1) are given by

$$D(q) = \begin{bmatrix} 2.351 + 0.168 \cos q_2 & 0.102 + 0.084 \cos q_2 \\ 0.102 + 0.084 \cos q_2 & 0.102 \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} -0.084\dot{q}_2 \sin q_2 & -0.084(\dot{q}_1 + \dot{q}_2) \sin q_2 \\ 0.084\dot{q}_1 \sin q_2 & 0 \end{bmatrix}$$

$$g(q) = \begin{bmatrix} 38.465 \sin q_1 + 1.825 \sin(q_1 + q_2) \\ 1.825 \sin(q_1 + q_2) \end{bmatrix}$$

⁷Except for viscous friction, which is explicitly considered in the general n -DOF robot manipulator dynamic model (1), the rest of the friction force components are bounded, as may be corroborated for instance in [3]. Other (typical) imprecisions, like those due to measurement inaccuracies and noise, are generally bounded too. The experimental results presented in Section V corroborate the considerations made as well as the observations claimed and proved in Remark 4.

and

$$F = \begin{bmatrix} 2.288 & 0 \\ 0 & 0.175 \end{bmatrix}$$

Thus, Properties 1–4 are satisfied with $d_m = 0.07 \text{ kg m}^2$, $d_M = 2.5 \text{ kg m}^2$, $k_c = 0.1422 \text{ kg m}^2$, $\gamma_1 = 40.29 \text{ Nm}$, $\gamma_2 = 1.83 \text{ Nm}$, $f_m = 0.175 \text{ kg m}^2/\text{sec}$, and $f_M = 2.288 \text{ kg m}^2/\text{sec}$. The maximum torques allowed are $T_1 = 150 \text{ Nm}$ and $T_2 = 15 \text{ Nm}$ for the first and second links respectively. Observe that Assumption 1 is fulfilled.

For every proposed algorithm, the control parameter values were selected from numerous combinations of them that were experimentally tested with the aim at searching for the best closed-loop performance. More precisely, the shortest possible response times that could be obtained avoiding overshoots, or giving rise to negligible ones, were aimed for every tested controller.

In addition to the implementation of the SP-SD+ and SPD+ proposed algorithms, experiments were carried out considering the PD+ algorithm of [11], *i.e.*

$$u = -K_2\bar{q} - K_1\dot{\bar{q}} + D(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + g(q)$$

with control gains chosen such that the best possible closed-loop performance could be obtained avoiding input saturation. The desired trajectory vector, for all the controllers, was defined as

$$q_d(t) = \begin{pmatrix} q_{d1}(t) \\ q_{d2}(t) \end{pmatrix} \triangleq \begin{pmatrix} \frac{\pi}{3} + \sin t \\ \cos t \end{pmatrix} \quad (29)$$

For such a desired trajectory, inequalities (3) are satisfied with $B_{dv} = 1 \text{ rad/sec}$ and $B_{da} = 1 \text{ rad/sec}^2$. The initial conditions at every test were $q_i(0) = \dot{q}_i(0) = 0$, $i = 1, 2$. The generalized saturation functions were defined as

$$\sigma_{ji}(\varsigma) = \begin{cases} -L_{ji} + (M_{ji} - L_{ji}) \tanh\left(\frac{\varsigma + L_{ji}}{M_{ji} - L_{ji}}\right) & \forall \varsigma < -L_{ji} \\ \varsigma & \forall |\varsigma| \leq L_{ji} \\ L_{ji} + (M_{ji} - L_{ji}) \tanh\left(\frac{\varsigma - L_{ji}}{M_{ji} - L_{ji}}\right) & \forall \varsigma > L_{ji} \end{cases}$$

with $L_{ji} < M_{ji}$, $\forall (i, j) \in \{1, 2\} \times \{0, 1, 2\}$. The control gains and saturation function parameters were adjusted as indicated in Table I. One can easily verify that Assumption 2 as well as inequalities (13) and (14) are satisfied.

Figures 1–3 show the evolution of the shoulder and elbow joint position errors, *i.e.* $\bar{q}_1(t)$ and $\bar{q}_2(t)$, respectively for the SP-SD+, SPD+, and PD+ controllers. Observe that, among the proposed schemes, the SPD+ algorithm proves to be the one that gives rise to the smallest rising and stabilization times. On the other hand, note that while acceptable post-transient responses take place through the proposed controllers, disappointing ones are obtained through the PD+ algorithm. Observe further that tiny variations around zero can be appreciated after the transient times in the position error responses obtained with the SP-SD+ and SPD+ algorithms. They take place as a consequence of modelling imprecisions, namely: system parameter inaccurate estimations and unmodelled friction phenomena (see Remark 4). Note further that among the post-transient variations observed in Figs. 1 and 2, those related to the SP-SD+ algorithm are the smallest ones. This was actually corroborated through a Root-Mean-Square (RMS) index

TABLE I
CONTROL PARAMETER VALUES

<i>prmtr.</i>	SP-SD+	SPD+	PD+	<i>units</i>
k_{21}	1500	1000	110	Nm
k_{22}	500	200	14.5	
k_{11}	60	100	30	Nm sec
k_{12}	8.9	15	8.9	
M_{21}	50			Nm
M_{11}	40			
M_{22}	4			
M_{12}	3			
M_{01}		100		
M_{02}		8		
L_{ji}	$0.9M_{ji}$ $i = 1, 2$ $j = 1, 2 \quad \quad j = 0$			

calculated for all the position error responses from the 3rd to the 10th second of the tests, and is a consequence of the higher control gains (specially concerning the P ones) that the performed tuning method gave rise for the SP-SD+ algorithm with respect to those chosen for the SPD+ scheme. Note that in view of the considerably smaller gains tuned for the PD+ algorithm, the prominently larger post-transient variations observed in Fig. 3 find an explanation. Unfortunately, for the desired trajectory in (29), the control gains were not able to be chosen larger without saturating the actuators.

Figs. 4–6 show the applied inputs, τ_1 and τ_2 , for all the tested schemes. Observe that the control signals generated through the SP-SD+ and SPD+ controllers are clearly within the input bounds considered at every link. Further, saturation was avoided in the case of the PD+ algorithm too in view of the small gains that were selected with this intention. But, as previously noted, the consequent closed-loop performance is disappointing compared to those obtained with the proposed SP-SD+ and SPD+ schemes.

VI. CONCLUSIONS

In this work, two globally stabilizing bounded control schemes for the trajectory tracking of robot manipulators with saturating inputs were proposed. They may be seen as extensions of the so-called PD+ algorithm to the bounded input case. With respect to previous works on the topic, the proposed approaches gave a global solution to the problem through static feedback. Moreover, they were not defined using a specific sigmoidal function, but any one on a set of *saturation* functions. Consequently, each of the proposed schemes actually constitutes a family of globally stabilizing bounded controllers. Such a *generalized* formulation permitted the developed algorithms to adopt a suitable structure where the control gains were able to take any positive value, which may be considered beneficial for performance-adjustment/improvement purposes. Furthermore, a class of *desired trajectories* that may

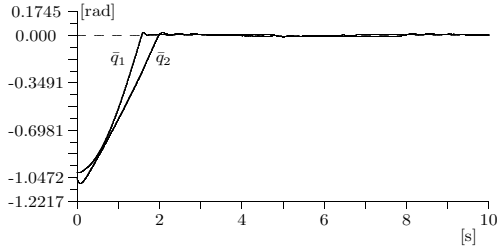


Fig. 1. Position error responses to the SP-SD+ control action

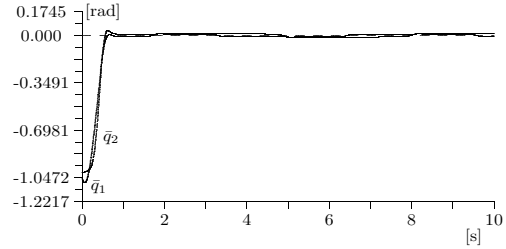


Fig. 2. Position error responses to the SPD+ control action

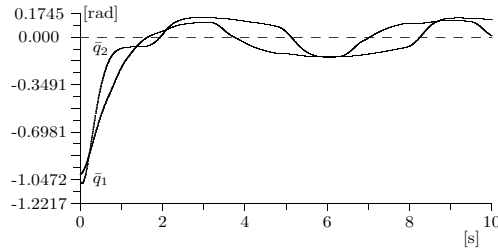


Fig. 3. Position error responses to the PD+ control action

be globally tracked avoiding input saturation was completely characterized. For both proposed control laws, global uniform asymptotic stabilization of the closed-loop system solutions towards the pre-specified desired trajectory was proved through a strict Lyapunov function.

The efficiency of the proposed schemes was corroborated through experimental tests carried out on a 2-DOF robot. Both proposed algorithms proved their ability to satisfactorily achieve the tracking control objective avoiding input saturation. On the contrary, additional implementations of the PD+ algorithm showed disappointing results.

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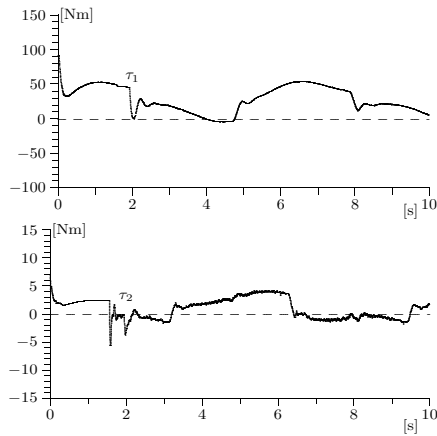


Fig. 4. SP-SD+ control torques

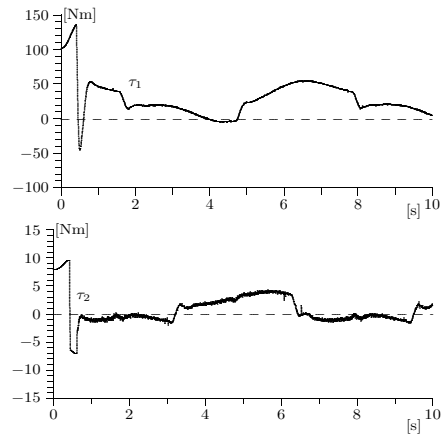


Fig. 5. SPD+ control torques

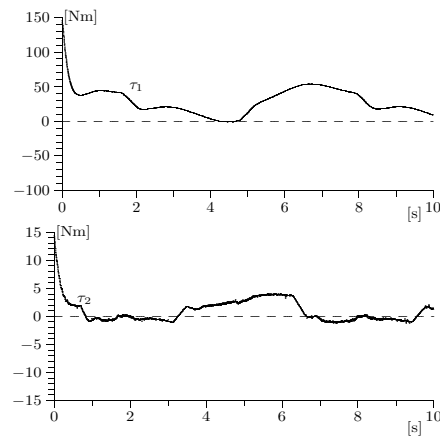


Fig. 6. PD+ control torques

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