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A natural saturating extension of the *PD-with-desired-gravity-compensation* control law for robot manipulators with bounded inputs

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***Abstract*— This work proposes a natural saturating extension of the *PD with desired gravity compensation* (PD*d*gc) control law for the global regulation of robot manipulators with bounded inputs. Compared to other algorithms previously proposed under the same analytical framework, it proves to be advantageous in several senses. First, it involves a single *saturation function* at each joint. Second, it does not need to discriminate the terms that shall be bounded since these are simply all of them included within the only saturation involved at every joint. Experimental results show the effectiveness of the proposed scheme. As far as the authors are aware, no such type of natural saturating extension (*i.e.* involving only one saturation function at each joint, where all the terms of the controller are embedded) to the bounded input case had been previously proposed for the PD*d*gc scheme. Furthermore, it is shown how the proposed approach may be conceived within the framework of the *energy shaping plus damping injection* methodology.**

***Index Terms*— Robot control, global regulation, bounded inputs, saturation functions.**

I. INTRODUCTION

PD controllers have proved to be the simplest algorithms to achieve the regulation and trajectory tracking of robot manipulators. In order to guarantee the global stabilization towards the desired position/motion, they are generally designed including terms that partially (or totally) compensate the system dynamics. In the case of global regulation, adding terms counterbalancing the gravity (generalized) forces suffices to avoid steady-state errors (at least analytically, under basic assumptions). Such a scheme gives rise to the so-called *PD with gravity compensation* (**PDgc**) control law [11]. Moreover, such a gravity counterweight needs not to be performed continuously (*i.e.* updating the gravity compensation terms at every instant). Adding constant terms accounting for the forces needed to counterbalance the gravity effects at the desired steady-state configuration is enough to guarantee the global regulation objective (under additional tuning conditions); see for instance [11]. Such a structure is frequently referred as the *PD with desired gravity compensation* (**PD*d*gc**) control law [11]. However, the original versions of such schemes were designed disregarding the inherent power supply limitations of real-life actuators. As it is well known, such limitations have a saturating effect in the signal transfer from the controller outputs to the manipulator inputs. Under such a constraint, the above mentioned controllers do not any longer ensure the achievement of the regulation objective; not at least globally. Furthermore, they may lead to undesirable effects in the closed-loop performance, as pointed out in [7], [21], [17], [5]. Thus, synthesis of control algorithms taking into account such a nonlinear saturating phenomenon turns out to be important.

Several approaches for the regulation and trajectory tracking of robot manipulators (mechanical or Euler-Lagrange systems) with bounded inputs have been proposed in the literature under the consideration of various analytical frameworks: Global regulation through full-state (position and velocity) feedback and gravity compensation is treated in [10], [17]. Solutions released from velocity measurements, but still compensating the gravity terms, are proposed in [3], [13]. Extensions to the elastic-joint case, with and without velocity measurements, are presented in [4]. A full-state feedback solution with adaptive gravity compensation is presented in [21]. An approach including adaptive gravity compensation that is additionally free of velocity measurements is proposed in [5]. An adaptive algorithm involving task-space coordinates that additionally deals with uncertainties on the manipulator kinematic model is presented in [6]. A semiglobal full-state feedback approach depending only on *partial* knowledge of the system parameters is developed in [1]. Trajectory tracking has been further treated in [3], [14], [16], [7], all four proposing schemes free of velocity measurements, and the last one presenting an additional full-state algorithm with adaptive gravity compensation. Let us note that the use of special types of saturation functions —a common characteristic of the control laws proposed in all the above mentioned works— has proved to be useful to avoid model (gravity) compensation in the synthesis of PD-type controllers for the global regulation of robot manipulators in an unbounded input context; see for instance [8], [18].

From the above cited works, the schemes proposed in [10], [17] were those developed in the less restrictive analytical setting where the whole system information (*i.e.* the entire set of parameters and states) is considered to be available. Under such a *full-system-information* analytical framework, a fundamental control design methodology for the global regulation of robot manipulators is the one known as *energy shaping plus damping injection* (see for instance [9]), whose foundations are attributed to [19]. Its application under the simplest considerations —disregarding input constraints— gives rise to the PDgc control law (see for instance [9, Rmk. 1]), or

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(under additional tuning constraints) to the PD*d*gc controller. Inspired by Arimoto's *SP-D*-controller-induced quasi-natural potentials [2], such a methodology is applied to the bounded-input case in [10], [17]. As a result, a PDgc-like control law is proposed, where the P and D parts (at every joint) are, each of them, explicitly bounded through specific saturation functions. A continuously differentiable one is used in [10] while the conventional non-smooth one in [17], which constitutes the main difference among both approaches. Through the satisfaction of simple inequalities on the saturation function bounds, the global regulation objective is proved to be achieved avoiding saturation of the control signal in both works. Furthermore, in [1, §3.1], an alternative version of the scheme developed in [17] is proposed, where the gravity compensation term is suppressed and a constant vector, $-\bar{\tau}_I$, is added to the P part within its saturation function. The authors prove that, under simple conditions on the control gains (actually, the typical conditions imposed in the conventional PD*d*gc control scheme to ensure global asymptotical stability of the position error trivial solution; see for instance [11]), there is a globally asymptotically stable equilibrium position error, $\bar{q}^*$, directly defined by $\bar{\tau}_I$ through a non-linear invertible relation $\varphi : \bar{\tau}_I \mapsto \bar{q}^* = \varphi(\bar{\tau}_I)$. One can see, from the analysis in [1, §3.1], that if $-\bar{\tau}_I$ is equal to the gravity force vector evaluated at the desired configuration, then $\bar{q}^* = 0$ (see [1, Eq. (7)]). Thus, under the just mentioned consideration, the algorithm proposed in [1, §3.1] turns out to be an alternative globally stabilizing saturating extension of the PD*d*gc scheme.

Nevertheless, the algorithms developed in [10], [17], and [1, §3.1] are not necessarily the simplest approaches, or those giving rise to the best closed-loop performance, that may be conceived under the availability of the full system information. Consider for instance a PDgc-like or a PD*d*gc-like control law that involves, at every joint, a single saturation function embedding all the terms of the controller, *i.e.* the P, D, and (continuous/desired) gravity terms. This would actually constitute the most direct or natural saturating extension of the PDgc, respectively PD*d*gc, control law to the bounded-input setting. Such (both) approaches result in simpler algorithms in two senses: first, they involve less saturation functions than the schemes proposed in [10], [17], and [1, §3.1]; and second, they do not need to discriminate the terms that shall be bounded since they are simply all of them included within the only saturation involved at every joint.

In this work, an alternative approach for the global regulation of robot manipulators with input saturations, under a full-system-information setting, is proposed. Contrarily to the previous works developed under the same analytical framework, it consists of a special form of the most direct or natural saturating extension of the PD*d*gc (**SPD*d*gc**) control law which involves, at every joint, a single saturation function embedding all the terms of the controller. The closed-loop performance resulting from its implementation is experimentally evaluated and compared to those obtained through the use of the algorithms developed in [10], [17], and [1, §3.1]. As far as the authors are aware, global regulation through a saturating extension of the PD*d*gc scheme where the whole controller expression at every joint is completely embedded within a single saturation function had not been previously proved in the literature. Such an approach turns out to be important not only for its simplicity but also because it is the one that best reproduces what really happens in an actual implementation of a PD*d*gc scheme on a real-life manipulator with real-life actuators.

The work is organized as follows: Section II states the general n-degree-of-freedom serial rigid robot manipulator open-loop dynamics and some of its main properties, as well as considerations, assumptions, definitions, and notations that are involved throughout the study. In Section III, the proposed control scheme is presented. Section IV states the main result, where the stability analysis is developed and the control objective is proved to be achieved. Experimental results are presented in Section V. Finally, conclusions are given in Section VI.

II. Preliminaries

The general n-degree-of-freedom serial rigid robot manipulator dynamics without friction is given by

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{1}$$

where $q,\,\dot{q},\,\ddot{q} \in I\!R^n$ are, respectively, the position (generalized coordinates), velocity, and acceleration vectors, $D(q) \in I\!R^{n\times n}$ is the inertia matrix, and $C(q,\dot{q})\dot{q}$, $g(q)$, $\tau \in I\!R^n$ are, respectively, the vectors of Coriolis and centrifugal, gravity, and external input generalized forces. The terms of such a dynamical model satisfy some well-known properties; see for instance [11, Chap. 4]. Some of them are recalled here.

Property 1: The inertia matrix $D(q)$ is a positive definite symmetric matrix satisfying $\lambda_m I \le D(q) \le \lambda_M I$, $\forall q \in I\!R^n$, where I denotes the $n \times n$ identity matrix, for some positive constants $\lambda_m \le \lambda_M$.

Property 2: The matrix $N(q,\dot{q}) \triangleq \frac{1}{2}\dot{D}(q,\dot{q}) - C(q,\dot{q})$ satisfies $\dot{q}^T N(q,\dot{q})\dot{q} = 0$, $\forall (q,\dot{q}) \in I\!R^n \times I\!R^n$.

Property 3: The gravity vector satisfies:

3.1. $\|g(q)\| \le \gamma$, $\forall q \in I\!R^n$, for some positive constant γ, or equivalently, every element of the gravity vector, *i.e.* $g_i(q)$, $i = 1,\ldots,n$, satisfies $|g_i(q)| \le \gamma_i$, $\forall q \in I\!R^n$, for some positive constants γ_i, $i = 1,\ldots,n$;

3.2. there exists a positive constant κ such that: $\left\|\frac{\partial g}{\partial q}(q)\right\| \le \kappa$, $\forall q \in I\!R^n$, or equivalently, such that $\left|\frac{\partial g_i}{\partial q_j}(q)\right| \le \kappa$, $\forall i,j \in \{1,\ldots,n\}$, $\forall q \in I\!R^n$.

Let us suppose that the absolute value of each input τ_i (i^{th} element of the input vector τ) is constrained to be smaller than a given saturation bound $T_i > 0$, *i.e.* $|\tau_i| \le T_i$, $i = 1,\ldots,n$. In other words, if u_i represents the control signal (controller output) relative to the i^{th} degree of freedom, then

$$\tau_i = T_i \mathrm{sat}\left(\frac{u_i}{T_i}\right) \tag{2}$$

$i = 1,\ldots,n$, where $\mathrm{sat}(\cdot)$ is the standard saturation function, *i.e.* $\mathrm{sat}(\varsigma) = \mathrm{sign}(\varsigma)\min\{|\varsigma|,1\}$. Let us note from (1) that $T_i \ge \gamma_i$ (see Property 3.1), $\forall i \in \{1,\ldots,n\}$, is a necessary condition for the manipulator to be stabilizable at any desired equilibrium configuration $q_d \in I\!R^n$. Thus, the following assumption turns out to be crucial within the analytical setting considered in this work:

Assumption 1: $T_i > \gamma_i$, $\forall i \in \{1, \ldots, n\}$.

Furthermore, the control scheme proposed in this work involves a special type of (saturation) functions fitting the following definition (based on [20, Def. 1]):

Definition 1: Given positive constants L and M, with $L < M$, a function $\sigma : I\!R \to I\!R : \varsigma \mapsto \sigma(\varsigma)$ is said to be a ***strictly increasing linear saturation*** for (L, M) if it is locally Lipschitz, strictly increasing, and satisfies

(a) $\sigma(\varsigma) = \varsigma$ when $|\varsigma| \leq L$

(b) $|\sigma(\varsigma)| < M$ for all $\varsigma \in I\!R$

The following notation will be used throughout the paper: 0_n will generically represent the origin of $I\!R^n$, and $0_{n\times n}$ that of $I\!R^{n\times n}$. The i^{th} element of a vector $x \in I\!R^n$ will be denoted x_i. $\|\cdot\|$ will represent the standard Euclidean norm, *i.e.* $\|x\| = \left[\sum_{i=1}^n x_i^2\right]^{1/2}$.

III. The proposed control scheme

We propose an SPD*dgc* control scheme of the form

$$u = s\big(g(q_d) - K_1\dot{q} - K_2\bar{q}\big) \tag{3}$$

where $\bar{q} = q - q_d$, for any constant (desired equilibrium position) vector $q_d \in I\!R^n$; K_1 and K_2 are positive definite diagonal matrices, $K_1 = \operatorname{diag}\{k_{11}, \ldots, k_{1n}\}$ and $K_2 = \operatorname{diag}\{k_{21}, \ldots, k_{2n}\}$, with $k_{1i} > 0$, $\forall i \in \{1, \ldots, n\}$, and k_{2i}, $i = 1, \ldots, n$, such that

$$k_{2m} \triangleq \min_i \{k_{2i}\} > \kappa \tag{4a}$$

(see Property 3.2); and $s(x) = \big(\sigma_1(x_1)\ ,\ \ldots\ ,\ \sigma_n(x_n)\big)^T$, with $\sigma_i(\cdot)$, $i = 1, \ldots, n$, being **strictly increasing linear saturation functions** for some (L_i, M_i) satisfying

$$\gamma_i < L_i < M_i \leq T_i, \quad i = 1, \ldots, n \tag{4b}$$

IV. Main result

Proposition 1: Under Assumption 1 and the satisfaction of inequalities (4), global asymptotic stabilization of the closed-loop system (1)–(3) towards $(q, \dot{q}) = (q_d, 0_n)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$.

Proof: From (3) and (4b), and the strictly increasing character of $\sigma_i(\cdot)$, $i = 1, \ldots, n$, one sees that $|u_i(t)| = \left|\sigma_i\big(g_i(q_d) - k_{1i}\dot{q}_i(t) - k_{2i}\bar{q}_i(t)\big)\right| < M_i \leq T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$. From this and (2), it follows that $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$. We now focus on the stability analysis. The closed-loop dynamics takes the form

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = s\big(g(q_d) - K_1\dot{q} - K_2\bar{q}\big) \tag{5}$$

Let us define the scalar function

$$V(\bar{q}, \dot{q}) = \frac{1}{2}\dot{q}^T D(q)\dot{q} + \mathcal{U}_{cl}(\bar{q}) \tag{6}$$

with

$$\begin{aligned} \mathcal{U}_{cl}(\bar{q}) &= \int_{0_n}^{\bar{q}} \left[g(r + q_d) - s\big(g(q_d) - K_2 r\big)\right]^T dr \\ &= \sum_{i=1}^n \int_0^{\bar{q}_i} \left[\bar{g}_i(r_i) - \sigma_i\big(g_i(q_d) - k_{2i} r_i\big)\right] dr_i \end{aligned} \tag{7}$$

where[1]

$$\begin{aligned} \bar{g}_1(r_1) &= g_1(r_1 + q_{d1}, q_{d2}, \ldots, q_{dn}) \\ \bar{g}_2(r_2) &= g_2(q_1, r_2 + q_{d2}, q_{d3}, \ldots, q_{dn}) \\ &\vdots \\ \bar{g}_n(r_n) &= g_n(q_1, q_2, \ldots, q_{n-1}, r_n + q_{dn}) \end{aligned}$$

At any $\bar{q} \in I\!R^n$, $\mathcal{U}_{cl}(\bar{q})$ is lower-bounded by $W(\bar{q}) = \sum_{i=1}^n w_i(\bar{q}_i)$ (see the Appendix), where

$$w_i(\bar{q}_i) \triangleq \begin{cases} \frac{k_{li}}{2}\bar{q}_i^2 & \forall |\bar{q}_i| \leq \bar{q}_i^* \\ k_{li}\bar{q}_i^*\left(|\bar{q}_i| - \frac{\bar{q}_i^*}{2}\right) & \forall |\bar{q}_i| > \bar{q}_i^* \end{cases}$$

with $0 < k_{li} \leq k_{2i} - \kappa$ (see inequality (4a)) and $\bar{q}_i^* = \frac{L_i - \gamma_i}{k_{2i}}$. From this and Property 1, we have that

$$V(\bar{q}, \dot{q}) \geq \lambda_m \|\dot{q}\|^2 + W(\bar{q})$$

wherefrom positive definiteness and radial unboundedness of $V(\bar{q}, \dot{q})$ are straightforward. The derivative of V along the system trajectories is given by

$$\begin{aligned} \dot{V}(\bar{q}, \dot{q}) &= \dot{q}^T D(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{D}(q, \dot{q})\dot{q} \\ &\qquad + \dot{q}^T\left[g(q) - s\big(g(q_d) - K_2\bar{q}\big)\right] \\ &= \dot{q}^T\Big[s\big(g(q_d) - K_1\dot{q} - K_2\bar{q}\big) - s\big(g(q_d) - K_2\bar{q}\big)\Big] \\ &= \sum_{i=1}^n \dot{q}_i\Big[\sigma_i\big(g_i(q_d) - k_{2i}\bar{q}_i - k_{1i}\dot{q}_i\big) \\ &\qquad\qquad - \sigma_i\big(g_i(q_d) - k_{2i}\bar{q}_i\big)\Big] \end{aligned}$$

where $D(q)\ddot{q}$ has been replaced by its equivalent expression from the closed-loop dynamics (5), and Property 2 has been used. Notice, from the strictly increasing character of $\sigma_i(\cdot)$, $i = 1, \ldots, n$, that the term

$$\sigma_i\big(g_i(q_d) - k_{2i}\bar{q}_i - k_{1i}\dot{q}_i\big) - \sigma_i\big(g_i(q_d) - k_{2i}\bar{q}_i\big)$$

has the sign of $-\dot{q}_i$. Therefore, $\dot{V}(\bar{q}, \dot{q})$ is (globally) negative semidefinite, with $\dot{V}(\bar{q}, \dot{q}) = 0$ if and only if $\dot{q} = 0_n$. Observe (from (5) and inequalities (4b)) that

$$\begin{aligned} \dot{q}(t) \equiv 0_n &\implies \ddot{q}(t) \equiv 0_n \\ &\implies g\big(\bar{q}(t) + q_d\big) \equiv s\big(g(q_d) - K_2\bar{q}(t)\big) \\ &\implies g\big(\bar{q}(t) + q_d\big) \equiv g(q_d) - K_2\bar{q}(t) \\ &\implies K_2\bar{q}(t) + g\big(\bar{q}(t) + q_d\big) - g(q_d) \equiv 0_n \end{aligned}$$

[1] Since $g(q)$ is the gradient vector of the potential energy function due to gravity, $\mathcal{U}_g(q)$, *i.e.* $g(q) = \nabla_q \mathcal{U}_g(q)$, then: *a)* $\frac{\partial g_i}{\partial q_j} \equiv \frac{\partial g_j}{\partial q_i}$, $\forall i \neq j$, and *b)* the line integral $\int_{q_d}^q g^T(r)dr$ is independent of the path (see for instance [12, p. 107]). Thus, integrating along the axes, we have $\int_{q_d}^q g^T(r)dr = \int_{q_{d1}}^{q_1} g_1(r_1, q_{d2}, \ldots, q_{dn})dr_1 + \int_{q_{d2}}^{q_2} g_2(q_1, r_2, q_{d3}, \ldots, q_{dn})dr_2 + \cdots + \int_{q_{dn}}^{q_n} g_n(q_1, q_2, \ldots, q_{n-1}, r_n)dr_n$. Observe that on every axis, the corresponding coordinate varies (according to the specified integral limits) while the rest of the coordinates remain constant. Further, note that $\frac{\partial}{\partial \bar{q}_j}\Big[g_i(\bar{q} + q_d) - \sigma_i(g_i(q_d) - k_{2i}\bar{q}_i)\Big] = \frac{\partial g_i}{\partial q_j}(\bar{q} + q_d) \equiv \frac{\partial g_j}{\partial q_i}(\bar{q} + q_d) = \frac{\partial}{\partial \bar{q}_i}\Big[g_j(\bar{q} + q_d) - \sigma_j(g_j(q_d) - k_{2j}\bar{q}_j)\Big]$, $\forall i \neq j$. Therefore, a scalar function $\mathcal{U}_{cl}(\bar{q})$ such that $\nabla_{\bar{q}}\mathcal{U}_{cl}(\bar{q}) = g(\bar{q} + q_d) - s\ g(q_d) - K_2\bar{q}$ does exist, and is indeed obtained as expressed by (7) (see for instance [12, p. 107]).

Furthermore, notice that, from Property 3.2 and the satisfaction of (4a), we have that

$$\frac{\partial}{\partial \bar{q}}\left[K_2\bar{q} + g(\bar{q}+q_d) - g(q_d)\right] = K_2 + \frac{\partial g}{\partial q}(\bar{q}+q_d) > 0_{n\times n}$$

$\forall \bar{q} \in I\!R^n$, *i.e.* $K_2 + \frac{\partial g}{\partial q}(\bar{q}+q_d)$ is a positive definite matrix at every $\bar{q}$. Consequently

$$\begin{aligned} K_2\bar{q}(t) + g(\bar{q}(t)+q_d) - g(q_d) \equiv 0_n &\implies \bar{q}(t) \equiv 0_n \\ &\implies q(t) \equiv q_d \end{aligned}$$

Therefore, from La Salle's invariance principle (see for instance [12, Crll. 3.2]), the proposition follows. ■

Remark 1: Let us note that the control input expression in (3) may be rewritten as $u = f_d(\bar{q},\dot{q}) + f_c(\bar{q}) + g(q_d)$, where $f_d(\bar{q},\dot{q}) = s\big(g(q_d) - K_1\dot{q} - K_2\bar{q}\big) - s\big(g(q_d) - K_2\bar{q}\big)$, $f_c(\bar{q}) = s\big(g(q_d) - K_2\bar{q}\big) - s\big(g(q_d)\big)$, and the fact that $s\big(g(q_d)\big) \equiv g(q_d)$ (in view of (4b) and property (a) of Definition 1) is being considered. Further, observe (from the strictly increasing character of $\sigma_i(\cdot)$) that every element of $f_d(\bar{q},\dot{q})$, *i.e.* $\sigma_i\big(g_i(q_d) - k_{1i}\dot{q}_i - k_{2i}\bar{q}_i\big) - \sigma_i\big(g_i(q_d) - k_{2i}\bar{q}_i\big)$, $i = 1,\ldots,n$, has the opposite sign of $\dot{q}_i$, and every element of $f_c(\bar{q})$, *i.e.* $\sigma_i\big(g_i(q_d) - k_{2i}\bar{q}_i\big) - \sigma_i\big(g_i(q_d)\big)$, $i = 1,\ldots,n$, has the opposite sign of $\bar{q}_i$. That is, the proposed controller is equivalent to the sum of three force vectors: a dissipative one opposing to motion (at every joint), $f_d(\bar{q},\dot{q})$, a conservative one opposing to displacement away from the desired equilibrium configuration, $f_c(\bar{q})$, and the (desired) gravity compensation. Actually, the proposed control law may be conceived within the framework of the *energy shaping plus damping injection* methodology. Indeed, u in (3) may be rewritten as the sum of the negative gradients of a *potential function*, $\mathcal{U}_c(\bar{q})$, and a *dissipation function*, $\mathcal{F}_c(\bar{q},\dot{q})$, where $\mathcal{U}_c(\bar{q}) = -\int_{0_n}^{\bar{q}}\left[f_c(r) + g(q_d)\right]^T dr$ (the *controller-induced potential energy*) and $\mathcal{F}_c(\bar{q},\dot{q}) = -\int_{0_n}^{\dot{q}}\left[f_d(\bar{q},r)\right]^T dr$. Observe that $\nabla_{\dot{q}}\mathcal{F}_c(\bar{q},0_n) = -f_d(\bar{q},0_n) = 0_n$, $\forall \bar{q} \in I\!R^n$, and $\dot{q}^T\nabla_{\dot{q}}\mathcal{F}_c(\bar{q},\dot{q}) = -\dot{q}^T f_d(\bar{q},\dot{q}) > 0$, $\forall (\bar{q},\dot{q}) \in I\!R^n \times I\!R^n \setminus \{0_n\}$ (which was demonstrated in the proof of Proposition 1 to show that $\dot{V}(\bar{q},\dot{q})$ is negative semidefinite). Thus, u in (3) may be expressed as $u = -\nabla_{\bar{q}}\mathcal{U}_c(\bar{q}) - \nabla_{\dot{q}}\mathcal{F}_c(\bar{q},\dot{q})$. Moreover, the Lyapunov function in (6) is actually the *closed-loop energy function*, which can be rewritten as $V(\bar{q},\dot{q}) = \mathcal{K}(\bar{q},\dot{q}) + \mathcal{U}_{cl}(\bar{q})$, where $\mathcal{K}(\bar{q},\dot{q}) = \frac{1}{2}\dot{q}^T D(\bar{q}+q_d)\dot{q}$ is the kinetic energy of the system, $\mathcal{U}_{cl}(\bar{q}) = \mathcal{U}_g(\bar{q}) + \mathcal{U}_c(\bar{q})$ is the *closed-loop potential energy*, and $\mathcal{U}_g(\bar{q}) = \int_{0_n}^{\bar{q}} g^T(r+q_d)dr$ is the potential energy due to gravity (taking the origin of the $\bar{q}$-configuration space as the reference point, such that $\mathcal{U}_g(0_n) = 0$). Notice that through the proof of Proposition 1, $\mathcal{U}_{cl}(\bar{q})$ is implicitly proved to be a globally convex function with (unique) minimum at $\bar{q} = 0_n$.

Let us note that the proof of Proposition 1 shows that —through the proposed control law, which keeps the manipulator inputs within their corresponding actuator physical bounds— while the robot is moving, the closed-loop energy decreases. Moreover, the application of LaSalle's invariance principle shows that the manipulator remains motionless exclusively if $q(t) \equiv q_d$. Therefore, we conclude that —for any initial condition different from $(q,\dot{q}) = (q_d, 0_n)$, where the closed-loop energy function has its global (unique) minimum— the closed-loop energy is kept decreasing, forcing the system trajectories to asymptotically approach the desired configuration.

V. EXPERIMENTAL RESULTS

With the end of verifying the effectiveness of the proposed controller, an extensive number of real-time control experiments on a well identified direct-drive manipulator was carried out. The experimental device is a two-axis robot moving on a vertical plane, built at CICESE Research Center [15]. The actuators are direct-drive brushless motors operated in torque mode, so they act as torque sources and accept an analog voltage as a reference of torque signal. The control algorithm is executed at a $2.5\,\mathrm{msec}$ sampling period in a control board (based on a DSP 32-bit floating point microprocessor) mounted on a PC host computer.

The entries of the dynamics of this two degree-of-freedom direct-drive robotic arm are given by [15]

$$D(q) = \begin{bmatrix} 2.351 + 0.168\cos q_2 & 0.102 + 0.084\cos q_2 \\ 0.102 + 0.084\cos q_2 & 0.102 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -0.084\dot{q}_2\sin q_2 & -0.084(\dot{q}_1+\dot{q}_2)\sin q_2 \\ 0.084\dot{q}_1\sin q_2 & 0 \end{bmatrix}$$

$$g(q) = 9.81\begin{bmatrix} 3.921\sin q_1 + 0.186\sin(q_1+q_2) \\ 0.186\sin(q_1+q_2) \end{bmatrix}$$

Under such parametric values, Properties 3.1 and 3.2 are satisfied with $\gamma_1 = 40.289\,\mathrm{Nm}$, $\gamma_2 = 1.827\,\mathrm{Nm}$, and $\kappa = 1.406\,\mathrm{Nm/deg}$. According to their manufacturer, the direct-drive motors are able to supply torques within the following bounds: $|\tau_1| \le T_1 = 150\,\mathrm{Nm}$, $|\tau_2| \le T_2 = 15\,\mathrm{Nm}$. It is worth mentioning that, although present in the robot joints, we have decided to consider friction as unmodelled dynamics.

Experiments were run considering the proposed algorithm, (3), and those developed in [10], [17], and [1, §3.1]. That in [10] may be expressed as

$$u = g(q) - K_1 s_1(\dot{q}) - K_2 s_2(\bar{q}) \tag{8}$$

where $s_j(x) = \big(\tanh(\lambda_{j1}x_1)\ ,\ \ldots\ ,\ \tanh(\lambda_{jn}x_n)\big)^T$, $j = 1,2$, for positive constants λ_{ji} and control gains such that $k_{1i} + k_{2i} \le T_i - \gamma_i$; that in [17] by

$$u = g(q) - s_3(K_1\dot{q}) - s_4(K_2\bar{q}) \tag{9}$$

where $s_j(x) = \left(\delta_{j1}\mathrm{sat}\left(\frac{x_1}{\delta_{j1}}\right)\ ,\ \ldots\ ,\ \delta_{jn}\mathrm{sat}\left(\frac{x_n}{\delta_{jn}}\right)\right)^T$, $j = 3,4$, for positive constants δ_{ji} such that $\delta_{3i} + \delta_{4i} < T_i - \gamma_i$; and that in [1, §3.1] (with $-\bar{\tau}_I = g(q_d)$) by

$$u = -s_5(K_1\dot{q}) - s_6\big(K_2\bar{q} - g(q_d)\big) \tag{10}$$

where $s_j(x) = \left(\delta_{j1}\mathrm{sat}\left(\frac{x_1}{\delta_{j1}}\right)\ ,\ \ldots\ ,\ \delta_{jn}\mathrm{sat}\left(\frac{x_n}{\delta_{jn}}\right)\right)^T$, $j = 5,6$, for positive constants δ_{ji} such that $\gamma_i < \delta_{6i} < T_i$ and $\delta_{5i} + \delta_{6i} < T_i$.

In order to achieve a fair experimental comparison, the same desired positions were used for all controllers; namely $q_{d1} = 45°$ and $q_{d2} = 90°$. Initial conditions in all cases were $q_1(0) = q_2(0) = \dot{q}_1(0) = \dot{q}_2(0) = 0$. Special attention was paid in all

TABLE I
CONTROL PARAMETER SELECTED VALUES

controller	**control gains**
(3)	$k_{11} = 2.5$, $k_{12} = 0.5$ [Nm sec/deg] $k_{21} = 35$, $k_{22} = 4$ [Nm/deg]
(8)	$\lambda_{11} = 0.016$, $\lambda_{12} = 0.066$ [sec/deg] $\lambda_{21} = 0.7$, $\lambda_{22} = 0.666$ [1/deg]
(9)	$k_{11} = 0.8$, $k_{12} = 0.04$ [Nm sec/deg] $k_{21} = 35$, $k_{22} = 4$ [Nm/deg]
(10)	$k_{11} = 0.8$, $k_{12} = 0.04$ [Nm sec/deg] $k_{21} = 35$, $k_{22} = 4$ [Nm/deg]
controller	**additional parameters**
(3)	$L_1 = 99$, $M_1 = 100$, $L_2 = 9.5$, $M_2 = 10$ [Nm]
(8)	$k_{11} = 50$, $k_{12} = 6$, $k_{21} = 50$, $k_{22} = 6$ [Nm]
(9)	$\delta_{31} = 50$, $\delta_{32} = 6$, $\delta_{41} = 50$, $\delta_{42} = 6$ [Nm]
(10)	$\delta_{51} = 50$, $\delta_{52} = 6$, $\delta_{61} = 70$, $\delta_{62} = 8$ [Nm]

cases to avoid torque saturation, that is, we took into account the actuator torque limits provided by the manufacturer. The saturation functions in (3) were defined as

$$\sigma_i(\varsigma) = \begin{cases} -L_i + (M_i - L_i)\tanh\left(\frac{\varsigma + L_i}{M_i - L_i}\right) & \text{if } \varsigma < -L_i \\ \varsigma & \text{if } |\varsigma| \leq L_i \\ L_i + (M_i - L_i)\tanh\left(\frac{\varsigma - L_i}{M_i - L_i}\right) & \text{if } \varsigma > L_i \end{cases}$$

The control parameters were adjusted as indicated in Table I.[2] It can be easily verified that for $i = 1, 2$: $T_i > \gamma_i + k_{1i} + k_{2i}$ in (8); $T_i > \gamma_i + \delta_{3i} + \delta_{4i}$ in (9); $T_i > \delta_{5i} + \delta_{6i}$, $T_i > \delta_{6i} > \gamma_i$, and $k_{2m} > \kappa$ in (10); and $T_i > M_i$ and $k_{2m} > \kappa$ in (3).

This is how the control gains in Table I were tuned: following a trial-and-error procedure, numerous simulations were run using a PD*d*gc control law, under the consideration of unbounded inputs, with various control gain combinations. A control gain combination, $k_{ij}^{(\mathrm{PD}d\mathrm{gc})}$, leading to considerably fast and free-of-overshoot responses was kept. Then, experiments with controller (3), using the control gain combination found in the above mentioned simulation case, *i.e.* $k_{ij}^{(3)} = k_{ij}^{(\mathrm{PD}d\mathrm{gc})}$, were performed. Next, experiments with controller (8), adjusting λ_{2j} such that $k_{2j}^{(8)}\lambda_{2j} = k_{2j}^{(3)}$, were carried out; the parameters λ_{1j} were adjusted so as to have as quick responses as possible with negligible overshoots. Next, experiments with controller (9) were performed keeping the same proportional (P) gains adopted by controller (3); the derivative (D) gains were adjusted as in the case of (8), *i.e.* so as to have as fast responses as possible with negligible overshoots. Finally, experiments with controller (10) were performed keeping the same P and D gains used in the scheme (9).

Figures 1 and 2 show the transient of the closed-loop position errors for the shoulder ($\bar{q}_1$) and elbow ($\bar{q}_2$) joints with each of the implemented controllers; those obtained in simulation with the PD*d*gc control law on an unbounded-input setting are included using dashed-lines. Observe that the stabilization objective is achieved. Further, note that among all

[2]Notice that, in the control algorithm expressed by (8), k_{2i} and k_{1i}, $i = 1\ldots, n$, are actually the bounds on the (nonlinearly modulated) P and D parts of the control equation (at every link), respectively, while the values λ_{1i} and λ_{2i}, $i = 1, \ldots, n$, are gains directly acting on the position and velocity error variables, respectively. For this reason, the latter parameters are presented in Table I as control gains while the former ones are treated as additional (bounding) parameters.

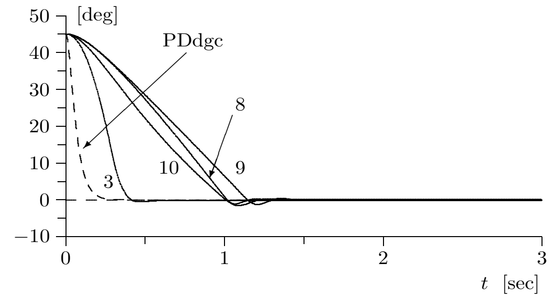

Fig. 1. Graphs of $-\bar{q}_1(t)$ (for each controller)

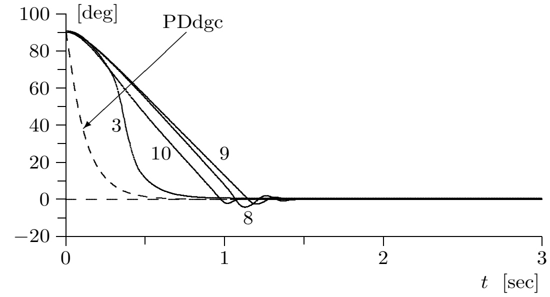

Fig. 2. Graphs of $-\bar{q}_2(t)$ (for each controller)

the closed-loop responses obtained through the experimental set-up, those corresponding to controller (3) are the closest to the ones obtained in simulation with the PD*d*gc control law. Figures 3 and 4 show the applied control inputs, τ_1 and τ_2 respectively. It can be seen that all the experimentally tested controllers clearly satisfy the torque restrictions given by the manufacturer.

Observe that, among all the control schemes that were experimentally tested, the control signals corresponding to (3) are those that make the most of the restricted range of the control variables (according to the saturation function bounds that were defined). This explains why the system responses obtained with (3) are those that best approach the ones gotten with an unbounded PD*d*gc control signal, and is actually a natural feature that distinguishes (3) from the rest of the

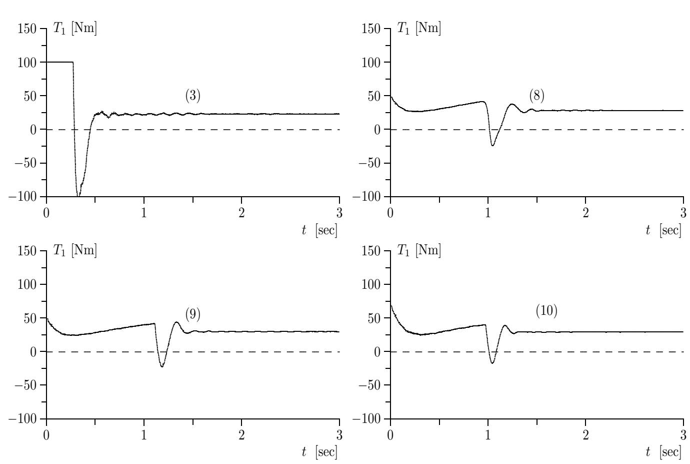

Fig. 3. Applied torques τ_1 (for each controller)

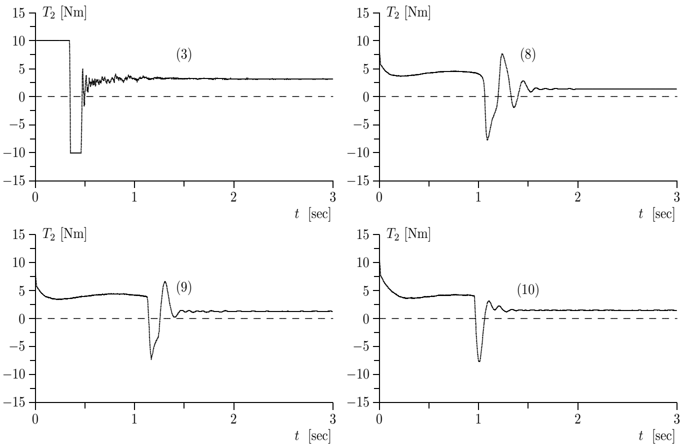

Fig. 4. Applied torques τ_2 (for each controller)

considered controllers (under the same or equivalent tuning). Indeed, observe from Figures 1 and 2 that during an initial time interval, the position error is large enough to generate PD*d*gc values greater than the saturation function bounds in (3) (*i.e.* $|g_i(q_d) - k_{1i}\dot{q}_i - k_{2i}\bar{q}_i| > M_i$ and consequently $|u_i| \approx M_i$). On the contrary, in the rest of the tested controllers, only the saturation functions where the position errors are involved get (during the same initial time interval) their maximum possible value (*i.e.* $|-k_{2i}\tanh(\lambda_{2i}\bar{q}_i)| \approx k_{2i}$ in (8), $\left|-\delta_{4i}\mathrm{sat}\left(\frac{k_{2i}\bar{q}_i}{\delta_{4i}}\right)\right| = \delta_{4i}$ in (9), and $\left|-\delta_{6i}\mathrm{sat}\left(\frac{k_{2i}\bar{q}_i - g_i(q_d)}{\delta_{6i}}\right)\right| = \delta_{6i}$ in (10)), and the other terms (those corresponding to the velocity error and/or the desired gravity compensation terms, *i.e.* $-k_{1i}\tanh(\lambda_{1i}\dot{q}_i) + g_i(q_d)$ in (8), $-\delta_{3i}\mathrm{sat}\left(\frac{k_{1i}\dot{q}_i}{\delta_{3i}}\right) + g_i(q_d)$ in (9), and $-\delta_{5i}\mathrm{sat}\left(\frac{k_{1i}\dot{q}_i}{\delta_{5i}}\right)$ in (10)) do not contribute large enough values to make the most of the control effort permitted by the respective algorithm, as was the case of (3). In this direction, notice that in order for (8), (9), and (10) to produce important control efforts, large enough position and velocity errors are required to take place simultaneously —and that even the desired position q_d may play an important role in this sense— which is not the case of controller (3). Thus, by considering the inclusion of the whole PD*d*gc expression within a single saturation function, as done in (3), unnecessary saturation of its individual terms is avoided and consequently more appropriate control efforts are generated.

Let us finally note, from (3) and Figures 3 and 4, that even though the manipulator inputs are guaranteed to vary within their corresponding actuator physical bounds, they could still get saturated at their corresponding strictly increasing linear saturation function bound M_i. Consequently, one cannot expect to get closed-loop trajectories that are as fast as those that take place with a conventional PD*d*gc controller in an unbounded input context. Nevertheless, provided that all the conditions required by Proposition 1 are satisfied, global stabilization towards $(q, \dot{q}) = (q_d, 0_n)$ is guaranteed.

VI. Conclusions

In this work, a natural saturating extension of the *PD with desired gravity compensation* (PD*d*gc) control law for the global regulation of robot manipulators with bounded inputs was proposed. Through the satisfaction of simple inequalities on the parameters of the *saturation functions* that are involved, the global position stabilization goal was proved to be achieved avoiding saturation of the control signals. Compared to other algorithms previously proposed under the same analytical framework, the scheme developed in this work proves to be simpler. Moreover, control signals producing more appropriate control efforts were experimentally observed. Interesting enough, the proposed approach was proved to be conceivable within the framework of the *energy shaping plus damping injection* methodology.

Appendix

Let $\bar{q}_i^* \triangleq \frac{L_i-\gamma_i}{k_{2i}}$, $\varrho_i(\bar{q}_i) \triangleq \bar{g}_i(\bar{q}_i) - \sigma_i\big(g_i(q_d) - k_{2i}\bar{q}_i\big)$, $\rho_i(\bar{q}_i) \triangleq \bar{g}_i(\bar{q}_i) - g_i(q_d) + k_{2i}\bar{q}_i$, $\mathcal{S}_i^* \triangleq \{\bar{q}_i \in I\!\!R \mid |\bar{q}_i| \le \bar{q}_i^*\}$, $\mathcal{S}_i \triangleq \left\{\bar{q}_i \in I\!\!R \mid \frac{g_i(q_d)-L_i}{k_{2i}} \le \bar{q}_i \le \frac{g_i(q_d)+L_i}{k_{2i}}\right\}$, and

$$w_i(\bar{q}_i) \triangleq \begin{cases} \frac{k_{li}}{2}\bar{q}_i^2 & \forall |\bar{q}_i| \le \bar{q}_i^* \\ k_{li}\bar{q}_i^*\left(|\bar{q}_i| - \frac{\bar{q}_i^*}{2}\right) & \forall |\bar{q}_i| > \bar{q}_i^* \end{cases}$$

where k_{li} is a constant satisfying $0 < k_{li} \le k_{2i} - \kappa$.

Claim 1: $\rho_i(\bar{q}_i)$ is a strictly increasing function satisfying $|\rho_i(\bar{q}_i)| \ge k_{li}|\bar{q}_i|$, $\forall \bar{q}_i \in I\!\!R$.

Proof: Observe that $\rho_i(0) = 0 = [k_{li}\bar{q}_i]_{\bar{q}_i=0}$, and notice, from Property 3.2, that $\frac{d\rho_i}{d\bar{q}_i}(\bar{q}_i) = k_{2i} + \frac{d\bar{g}_i}{d\bar{q}_i}(\bar{q}_i) \ge k_{2i} - \kappa \ge \frac{d[k_{li}\bar{q}_i]}{d\bar{q}_i} = k_{li} > 0$, $\forall \bar{q}_i \in I\!\!R$, wherefrom the claim follows. ■

Claim 2: $|\varrho_i(\bar{q}_i)| \ge \left|k_{li}\bar{q}_i^*\mathrm{sat}\left(\frac{\bar{q}_i}{\bar{q}_i^*}\right)\right|$, $\forall \bar{q}_i \in I\!\!R$.

Proof: First, note from item (a) of Definition 1 that $\sigma_i\big(g_i(q_d) - k_{2i}\bar{q}_i\big) \equiv g_i(q_d) - k_{2i}\bar{q}_i$, $\forall \bar{q}_i \in \mathcal{S}_i$. Furthermore, observe that $\mathcal{S}_i^* \subset \mathcal{S}_i$. Consequently, $|\varrho_i(\bar{q}_i)| = |\rho_i(\bar{q}_i)| \ge k_{li}|\bar{q}_i| = \left|k_{li}\bar{q}_i^*\mathrm{sat}\left(\frac{\bar{q}_i}{\bar{q}_i^*}\right)\right|$, $\forall \bar{q}_i \in \mathcal{S}_i^*$ (where Claim 1 has been considered). On the other hand, from the strictly increasing character of $\rho_i(\bar{q}_i)$ (according to Claim 1), notice that $|\varrho_i(\bar{q}_i)| = |\rho_i(\bar{q}_i)| \ge k_{li}\bar{q}_i^* = \left|k_{li}\bar{q}_i^*\mathrm{sat}\left(\frac{\bar{q}_i}{\bar{q}_i^*}\right)\right|$, $\forall |\bar{q}_i| \in \mathcal{S}_i \setminus \mathcal{S}_i^*$. Finally, observe, from the definition of $\varrho(\bar{q}_i)$, that on $I\!\!R \setminus \mathcal{S}_i$, we have that $|\varrho_i(\bar{q}_i)| \ge L_i - \gamma_i = k_{2i}\bar{q}_i^* > (k_{2i} - \kappa)\bar{q}_i^* \ge k_{li}\bar{q}_i^* = \left|k_{li}\bar{q}_i^*\mathrm{sat}\left(\frac{\bar{q}_i}{\bar{q}_i^*}\right)\right|$, $\forall \bar{q}_i \in I\!\!R \setminus \mathcal{S}_i$. ■

Claim 3: $\int_0^{\bar{q}_i} \varrho_i(r_i)dr_i \ge w_i(\bar{q}_i)$, $\forall \bar{q}_i \in I\!\!R$.

Proof: The proof follows directly from Claim 2 by noting that $\varrho_i(0) = 0 = \left[k_{li}\bar{q}_i^*\mathrm{sat}\left(\frac{\bar{q}_i}{\bar{q}_i^*}\right)\right]_{\bar{q}_i=0}$, $\bar{q}_i\varrho_i(\bar{q}_i) > 0$ and $\bar{q}_i\mathrm{sat}\left(\frac{\bar{q}_i}{\bar{q}_i^*}\right) > 0$, $\forall \bar{q}_i \ne 0$, and that $w_i(\bar{q}_i) = \int_0^{\bar{q}_i} k_{li}\bar{q}_i^*\mathrm{sat}\left(\frac{r_i}{\bar{q}_i^*}\right)dr_i$. ■

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References

[1] J. Alvarez-Ramirez, R. Kelly, and I. Cervantes, "Semiglobal stability of saturated linear PID control for robot manipulators," *Automatica*, vol. 39, no.6, pp. 989–995, Jun. 2003.

[2] S. Arimoto, "Fundamental problems of robot control: Part I, Innovations in the realm of robot servo-loops," *Robotica*, vol. 13, no. 1, pp. 19–27, Jan. 1995.

[3] I.V. Burkov, “Stabilization of mechanical systems *via* bounded control and without velocity measurements,” in *Proc. 2nd Russian-Swedish Control Conference*, St. Petersburg, Russia, 1995, pp. 37–41.

[4] I.V. Burkov and L.B. Freidovich, “Stabilization of the position of a Lagrangian system with elastic elements and bounded control with and without measurement of velocities,” *J. Appl. Maths. Mechs.*, vol. 61, no. 3, pp. 433–441, Sep. 1997.

[5] R. Colbaugh, E. Barany, and K. Glass, “Global regulation of uncertain manipulators using bounded controls,” in *Proc. IEEE International Conference on Robotics and Automation*, Albuquerque, New Mexico, 1997, pp. 1148–1155.

[6] W.E. Dixon, “Adaptive regulation of amplitud limited robot manipulators with uncertain kinematics and dynamics,” in *Proc. American Control Conference*, Boston, Massachussets, 2004, pp. 3839–3844.

[7] W.E. Dixon, M.S. de Queiroz, F. Zhang, and D.M. Dawson, “Tracking control of robot manipulators with bounded torque inputs,” *Robotica*, vol. 17, no. 2, pp. 121–129, Mar. 1999.

[8] R. Kelly, “Global positionaing of robot manipulators via PD control plus a class of nonlinear integral actions,” *IEEE Transactions on Robotics and Automation*, vol. 43, no. 7, pp. 934–938, Jul. 1998.

[9] R. Kelly, “Regulation of manipulators in generic task space: en energy shaping plus damping injection approach,” *IEEE Transactions on Robotics and Automation*, vol. 15, no. 2, pp. 381–386, Apr. 1999.

[10] R. Kelly, V. Santibáñez, and H. Berghuis, “Point-to-point robot control under actuator constraints,” *Control Engineering Practice*, vol. 5, no. 11, pp. 1555–1562, Nov. 1997.

[11] R. Kelly, V. Santibáñez, and A. Loría, *Control of Robot Manipulators in Joint Space*. London: Springer, 2005.

[12] H.K. Khalil, *Nonlinear Systems*. Second edition, Upper Saddle River, NJ: Prentice-Hall, 1996.

[13] A. Loría, R. Kelly, R. Ortega, and V. Santibáñez, “On global output feedback regulation of Euler-Lagrange systems with bounded inputs,” *IEEE Trans. on Automatic Control*, vol. 42, no. 8, pp. 1138–1143, Mar. 1997.

[14] A. Loría and H. Nijmeijer, “Bounded output feedback tracking control of fully-actuated Euler-Lagrange systems,” *Systems & Control Letters*, vol. 33, no. 3, pp. 151–161, Mar. 1998.

[15] F. Reyes and R. Kelly, “Experimental evaluation of identification schemes on a direct-drive robot,” *Robotica*, vol. 15, no. 5, pp. 563–571, Sep. 1997.

[16] V. Santibáñez and R. Kelly, “Global asymptotic stability of bounded output feedback tracking control for robot manipulators,” in *Proc. 40th IEEE Conf. on Decision and Ctrl.*, Orlando, FL, 2001, pp. 1378–1379.

[17] V. Santibáñez, R. Kelly, and F. Reyes, “A new set-point controller with bounded torques for robot manipulators,” *IEEE Transactions on Industrial Electronics*, vol. 45, no. 1, pp. 126–133, Feb. 1998.

[18] Y. Su, D. Sun, L. Ren, and J.K. Mills, “Integration of saturated PI synchronous control and PD feedback for control of parallel manipulators,” *IEEE Transactions on Robotics*, vol. 22, no. 1, pp. 202–207, Feb. 2006.

[19] M. Takegaki and S. Arimoto, “A new feedback method for dynamic control of manipulators,” *Journal of Dynamic Systems, Measurement, and Control*, vol. 102, no. 2, pp. 119–125, Jun. 1981.

[20] A.R. Teel, “Global stabilization and restricted tracking for multiple integrators with bounded controls,” *Systems & Control Letters*, vol. 18, no. 3, pp. 165–171, Mar. 1992.

[21] E. Zergeroglu, W. Dixon, A. Behal, and D. Dawson, “Adaptive set-point control of robotic manipulators with amplitud-limited control inputs,” *Robotica*, vol. 18, no. 2, pp. 171–181, Mar. 2000.