Output-Feedback PID-type Control with Simple Tuning for the Global Regulation of Robot Manipulators with Input Constraints

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Abstract

An output-feedback PID-type control scheme for the global position stabilization of robot manipulators with bounded inputs is proposed. It guarantees the global regulation objective avoiding input saturation by releasing the feedback not only from the exact knowledge of the system structure and parameter values but also from velocity measurements. With respect to previous approaches of the kind, the proposed scheme remains simple while increasing design/performance-adjustment flexibility. For instance, it does not impose the use of a specific sigmoidal function to achieve the required boundedness but involves a generalized type of saturation functions. More importantly, it is characterized by a very simple control-gain tuning criterion, the simplest hitherto obtained in the considered analytical context. Experimental tests on a 2-degree-of-freedom direct-drive manipulator corroborate the efficiency of the developed scheme.

I. Introduction

Despite the advances hitherto achieved in the design of sophisticated control schemes, implementation of the classical Proportional-Integral-Derivative (PID) controller seems to be a common practice for the regulation of robot manipulators [1]. This is mainly due to the practical certainty on the achievement of the regulation goal experienced through its simple linear structure which avoids involving the system model and exact knowledge of the system parameters. Such benignant characteristics have motivated research work on the stabilization of manipulators through the classical linear PID controller focusing, for instance, on stability [1] [2], robustness [2] [3], stability region estimation [2] [4] and tuning [5] [6]. However, through such a simple linear structure, it has not yet been possible to develop a global proof of the closed-loop stability properties observed in practice. This is why alternative nonlinear versions of the PID controller, mainly oriented to guarantee global regulation, have been proposed for instance in [7] [8]. However, these algorithms implicitly assume that actuators can furnish any required torque value. Unfortunately, this is not possible in practice in view of the saturation nonlinearity that generally relates the controller outputs to the plant inputs in actual feedback systems. Furthermore, disregarding such natural constraints may lead to undesirable system behaviors or degraded closed-loop performances [9]. For this reason, bounded PID-type approaches have been further developed. For instance, semiglobal regulators with different saturating PID-type structures have been proposed in [10] [11]. The closed-loop analysis in these works is developed using the singular perturbation methodology which shows the existence of an appropriate tuning mainly characterized by the requirement of small enough integral action gains and sufficiently high proportional and derivative ones. As far as the authors are aware, the first bounded PID-type control law for global regulation was presented in [12]; the algorithm gives the alternative to include or disregard velocities in the feedback. Nevertheless, the structure of the proposed scheme is quite complex. Other studies have focused on the solution of the global PID position stabilization problem for manipulators with constrained inputs through simpler structures, giving rise to the SP-SI-SD type algorithm developed in [13] *via* passivity theory and later on in [14] through Lyapunov stability analysis. In particular, the work in [14] includes a velocity-free version of the presented controller through the conventional (linear) dirty derivative operator.

The above cited bounded PID-type approaches give a solution to the formulated problem under input and data restrictions. In this direction, special interest have output-feedback schemes, like the velocity-free extensions of the algorithms developed in [12] [14], since they achieve regulation not only without the need for the exact knowledge of the system structure and parameter values but also through the exclusive feedback of the position variables. Nevertheless, some design issues and/or the developed stability analyses have generally conducted to stringent tuning criteria that include a set of conditions that are either not all necessary, or more restrictive than really needed, or whose derivation is not always exhaustive. As might be expected, the resulting complication has been naturally adopted by the previously cited velocity-free algorithms, which generally limits their closed-loop performance improvement ability. This brings to the fore the convenience to count on an output-feedback bounded PID-type global scheme with more design flexibility and less and/or simpler implementation restrictions giving rise to wider performance adjustment possibilities, which constitutes our main motivation. Such a design goal is proven to be achieved in this work through a simple control structure where each one of the P, I and D actions adopts a generalized saturating form through the implication of suitable bounded smooth or non-smooth (Lipschitz-continuous) passive functions that are not *a priori* fixed. More importantly, the proposed output-feedback controller is mainly characterized by its very simple control-gain tuning criterion, and its efficiency is corroborated through experimental tests on a 2-degree-of-freedom (DOF) direct-drive manipulator.

Remark 1: Tuning limitations coming from the developed stability analyses has been characteristic of previous PID-type approaches even in the unconstrained input context [15]. As a matter of fact, simplification of the tuning conditions for PID-type controllers has been a research subject for several years [5] [6] and had never been achieved to be as simple as it is shown in this paper. In this direction, it is important to point out that the sense given in this work to the term *tuning* is the same one given in [5] [6] [15], referring to the inequality conditions on the control parameters —generally in terms of the bound quantifications characterizing Properties 1–4 and independent of the operation region— obtained through the closed-loop analysis to guarantee the pre-specified control objective. This differs from the meaning given in other works where control-gain setting aims at coping with performance requirements or give rise to *acceptable* system behaviors [16] [17], regularly focusing on closed-loop response characteristic aspects such as rising time, overshoot or stabilization time. Such *response-oriented* tuning procedures are frequently based on linear or linearized models and/or linear-model-based methods. Hence, the expected performance is generally ensured at or around the set-point where the tuning procedure is applied. Reference changes would generally require that the tuning method be executed again at the new set-point. Such a tuning method frequently implies several parameter-adjustment stages and, rather than being based on inequality conditions, it regularly involves formulae based on parameter quantifications from input-output tests such as step responses. It is worth further pointing out that the formerly described *control-objective-oriented* parameter tuning may adopt different sights under different problem formulations. For instance, under the consideration of non-vanishing external disturbances and/or model imprecisions, where *practical stabilization* —aiming at the convergence to a (sufficiently small) neighborhood of the desired equilibrium— would generally be a suitable control goal, the expressions on the control gains obtained from the closed-loop analysis may adopt a robustness orientation such as the achievement of a pre-specified tolerance level on the steady-state error, as in [3] (where a desired size of the domain of attraction is simultaneously focused through the control parameter setting).

II. Preliminaries

Let $X \in \mathbb{R}^{m\times n}$ and $y \in \mathbb{R}^n$. Throughout this paper, X_{ij} represents the element of X at its i^{th} row and j^{th} column, and y_i denotes the i^{th} element of y. 0_n stands for the origin of $\mathbb{R}^n$ and I_n represents the $n \times n$ identity matrix. $\|\cdot\|$ denotes the standard Euclidean norm for vectors, *i.e.* $\|y\| = \sqrt{\sum_{i=1}^{n} y_i^2}$, and induced norm for matrices, *i.e.* $\|X\| = \sqrt{\lambda_{\max}\{X^T X\}}$ where $\lambda_{\max}\{X^T X\}$ represents the maximum eigenvalue of $X^T X$. For a continuous scalar function $\psi : \mathbb{R} \to \mathbb{R}$, ψ' denotes its derivative, when differentiable, $D^+\psi$ its upper right-hand (Dini) derivative, *i.e.* $D^+\psi(\varsigma) = \limsup_{h\to 0^+} \frac{\psi(\varsigma+h)-\psi(\varsigma)}{h}$, with $D^+\psi = \psi'$ at points of differentiability [18, Appendix C.2], and ψ^{-1} its inverse, when invertible.

Consider the n-DOF serial rigid robot manipulator dynamics with viscous friction [19]

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau \tag{1}$$

where q, $\dot{q}$, $\ddot{q} \in \mathbb{R}^n$ are, respectively, the position, velocity and acceleration vectors, $H(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, and $C(q,\dot{q})\dot{q}$, $F\dot{q}$, $g(q)$, $\tau \in \mathbb{R}^n$ are respectively the vectors of Coriolis and centrifugal, viscous friction, gravity and external input generalized forces, with $F \in \mathbb{R}^{n\times n}$ being a positive definite constant diagonal matrix whose entries $f_i > 0$, $i = 1,\ldots,n$, are the viscous friction coefficients, and $g(q) = \nabla\mathcal{U}(q)$, with $\mathcal{U}(q)$ being the gravitational potential energy, or equivalently

$$\mathcal{U}(q) = \mathcal{U}(q_0) + \int_{q_0}^{q} g^T(r)dr \tag{2a}$$

with

$$\int_{q_0}^{q} g^T(r)dr = \int_{q_{01}}^{q_1} g_1(r_1, q_{02}, \ldots, q_{0n})dr_1 + \int_{q_{02}}^{q_2} g_2(q_1, r_2, q_{03}, \ldots, q_{0n})dr_2 + \cdots + \int_{q_{0n}}^{q_n} g_n(q_1, \ldots, q_{n-1}, r_n)dr_n \quad \text{(2b)}$$

for any[1] $q, q_0 \in \mathbb{R}^n$. Some well-known properties characterizing the terms of such a dynamical model are recalled here [19, Chap. 4]. Subsequently, we denote $\dot{H}$ the rate of change of H, *i.e.*, $\dot{H} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n} : (q, \dot{q}) \mapsto \left[\frac{\partial H_{ij}}{\partial q}(q)\dot{q}\right]$.

Property 1: $H(q)$ is a continuously differentiable matrix function being positive definite, symmetric, and bounded on $\mathbb{R}^n$, *i.e.* such that $\mu_m I_n \leq H(q) \leq \mu_M I_n$, $\forall q \in \mathbb{R}^n$, for some constants $\mu_M \geq \mu_m > 0$.

Property 2: The Coriolis matrix $C(q, \dot{q})$ satisfies:

2.1. $\|C(q, \dot{q})\| \leq k_C\|\dot{q}\|$, $\forall (q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, for some constant $k_C \geq 0$;
2.2. for all $(q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, $\dot{q}^T \left[\frac{1}{2}\dot{H}(q, \dot{q}) - C(q, \dot{q})\right] \dot{q} = 0$ and actually $\dot{H}(q, \dot{q}) = C(q, \dot{q}) + C^T(q, \dot{q})$.

Property 3: The viscous friction coefficient matrix satisfies $f_m\|\dot{q}\|^2 \leq \dot{q}^T F\dot{q} \leq f_M\|\dot{q}\|^2$, $\forall \dot{q} \in \mathbb{R}^n$, where $0 < f_m \triangleq \min_i\{f_i\} \leq \max_i\{f_i\} \triangleq f_M$.

Property 4: The gravity force term $g(q)$ is a continuously differentiable bounded vector function with bounded Jacobian matrix[2] $\frac{\partial g}{\partial q}$. Equivalently, every element of the gravity force vector, $g_i(q)$, $i = 1, \ldots, n$, satisfies:

4.1. $|g_i(q)| \leq B_{gi}$, $\forall q \in \mathbb{R}^n$, for some positive constant B_{gi};
4.2. $\frac{\partial g_i}{\partial q_j}$, $j = 1, \ldots, n$, exist and are continuous and such that $\left|\frac{\partial g_i}{\partial q_j}(q)\right| \leq \left\|\frac{\partial g}{\partial q}(q)\right\| \leq k_g$, $\forall q \in \mathbb{R}^n$, for some positive constant k_g, and consequently $|g_i(x) - g_i(y)| \leq \|g(x) - g(y)\| \leq k_g\|x - y\|$, $\forall x, y \in \mathbb{R}^n$.

Let us suppose that the absolute value of each input τ_i is constrained to be smaller than a given saturation bound $T_i > 0$, *i.e.*, $|\tau_i| \leq T_i$, $i = 1, \ldots, n$. More precisely, letting u_i represent the control variable (controller output) relative to the i^{th} degree of freedom, we have that

$$\tau_i = T_i \operatorname{sat}(u_i/T_i) \quad (3)$$

where $\operatorname{sat}(\cdot)$ is the standard saturation function, *i.e.* $\operatorname{sat}(\varsigma) = \operatorname{sign}(\varsigma) \min\{|\varsigma|, 1\}$. From Eqs. (1) and (3), one sees that $T_i \geq B_{gi}$, $\forall i \in \{1, \ldots, n\}$, is a necessary condition for the manipulator to be stabilizable at any desired equilibrium configuration $q_d \in \mathbb{R}^n$. Thus, the following assumption turns out to be important within the analytical setting considered here.

Assumption 1: $T_i > \alpha B_{gi}$, $\forall i \in \{1, \ldots, n\}$, for some scalar $\alpha \geq 1$.

The control scheme proposed in this work involves functions fulfilling the following definition [20].

[1]Since $g(q)$ is the gradient of the gravitational potential energy $\mathcal{U}(q)$, a scalar function, then, for any $q, q_0 \in \mathbb{R}^n$, the inverse relation $\mathcal{U}(q) = \mathcal{U}(q_0) + \int_{q_0}^{q} g^T(r)dr$ is independent of the integration path [18, p. 120]. Eq. (2b) considers integration along the axes. This way, on every axis (*i.e.* at every integral in the right-hand side of (2b)), the corresponding coordinate varies (according to the specified integral limits) while the rest of the coordinates remain constant.

[2]Property 4 is satisfied for instance by manipulators having only revolute joints [19, §4.3].

Definition 1: Given a positive constant M, a nondecreasing Lipschitz-continuous function $\sigma : \mathbb{R} \to \mathbb{R}$ is said to be a *generalized saturation* with bound M if

(a) $\varsigma\sigma(\varsigma) > 0$, $\forall\varsigma \neq 0$;

(b) $|\sigma(\varsigma)| \leq M$, $\forall\varsigma \in \mathbb{R}$.

If in addition

(c) $\sigma(\varsigma) = \varsigma$ when $|\varsigma| \leq L$,

for some positive constant $L \leq M$, σ is said to be a *linear saturation* for (L, M).

Functions satisfying Definition 1 have the following properties proven in [20].

Lemma 1: Let $\sigma : \mathbb{R} \to \mathbb{R}$ be a generalized saturation with bound M and let k be a positive constant. Then,

1. $\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) = 0$;
2. $\exists\sigma'_M \in (0, \infty)$ such that $0 \leq D^+\sigma(\varsigma) \leq \sigma'_M$, $\forall\varsigma \in \mathbb{R}$;
3. $\frac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma'_M\varsigma^2}{2}$, $\forall\varsigma \in \mathbb{R}$;
4. $\int_0^\varsigma \sigma(kr)dr > 0$, $\forall\varsigma \neq 0$;
5. $\int_0^\varsigma \sigma(kr)dr \to \infty$ as $|\varsigma| \to \infty$;
6. if σ is strictly increasing then, for any constant $a \in \mathbb{R}$, $\bar{\sigma}(\varsigma) = \sigma(\varsigma + a) - \sigma(a)$ is a strictly increasing generalized saturation function with bound $\bar{M} = M + |\sigma(a)|$.

III. The Proposed Control Scheme

The proposed output-feedback controller with generalized SP, SI and SD terms is defined as

$$u(q, \vartheta, \phi) = -s_P(K_P\bar{q}) - s_D(K_D\vartheta) + s_I(K_I\phi) \tag{4}$$

where $\bar{q} = q - q_d$, for any constant desired equilibrium position vector $q_d \in \mathbb{R}^n$; $\phi, \vartheta \in \mathbb{R}^n$ are the output vector variables of the integral-action dynamics, defined as[3]

$$\dot{\phi}_c = -\varepsilon K_P^{-1} s_P(K_P\bar{q}) \tag{5a}$$

$$\phi = -\bar{q} + \phi_c \tag{5b}$$

and the velocity estimation auxiliary subsystem, defined as

$$\dot{\vartheta}_c = -AK_D^{-1} s_D\big(K_D(\vartheta_c + B\bar{q})\big) \tag{6a}$$

$$\vartheta = \vartheta_c + B\bar{q} \tag{6b}$$

(comments related to this subsystem are given in Appendix A), respectively; $K_P = \text{diag}[k_{P1}, \ldots, k_{Pn}]$, $K_D = \text{diag}[k_{D1}, \ldots, k_{Dn}]$, $K_I = \text{diag}[k_{I1}, \ldots, k_{In}]$, $A = \text{diag}[a_1, \ldots, a_n]$ and $B = \text{diag}[b_1, \ldots, b_n]$, with $k_{Di} > 0$, $k_{Ii} > 0$, $a_i > 0$, $b_i > 0$, $\forall i = 1, \ldots, n$, and positive P gains such that

$$k_{Pm} \triangleq \min_i\{k_{Pi}\} > k_g \tag{7}$$

[3] Under time parametrization of the system trajectories, the integral-action dynamics in Eqs. (5) adopts the (equivalent) integral-equation form $\phi(t) = \phi(0) + \bar{q}(0) - \bar{q}(t) - \int_0^t \varepsilon K_P^{-1} s_P\big(K_P\bar{q}(\varsigma)\big)d\varsigma$, for any initial vector values $\phi(0), \bar{q}(0) \in \mathbb{R}^n$.

For any $x \in \mathbb{R}^n$,

$$s_P(x) = \left(\sigma_{P1}(x_1), \ldots, \sigma_{Pn}(x_n)\right)^T$$

$$s_D(x) = \left(\sigma_{D1}(x_1), \ldots, \sigma_{Dn}(x_n)\right)^T$$

$$s_I(x) = \left(\sigma_{I1}(x_1), \ldots, \sigma_{In}(x_n)\right)^T$$

with $\sigma_{Pi}(\cdot)$, $i = 1, \ldots, n$, being *linear saturation functions* for (L_{Pi}, M_{Pi}), $\sigma_{Di}(\cdot)$, $i = 1, \ldots, n$, being *generalized saturation functions* with bounds M_{Di}, and $\sigma_{Ii}(\cdot)$, $i = 1, \ldots, n$, being *strictly increasing generalized saturation functions* with bounds M_{Ii}, such that

$$L_{Pi} > 2B_{gi} \tag{8a}$$

$$M_{Ii} > B_{gi} \tag{8b}$$

$$M_{Pi} + M_{Di} + M_{Ii} < T_i \tag{8c}$$

$i = 1, \ldots, n$; and ε (in Eq. (5a)) is a positive constant satisfying

$$\varepsilon < \varepsilon_M \triangleq \min\{\varepsilon_1, \varepsilon_2, \varepsilon_3\} \tag{9}$$

where

$$\varepsilon_1 \triangleq \sqrt{\frac{\beta_0 \beta_P \mu_m}{\mu_M^2}} \quad , \quad \varepsilon_2 \triangleq 2\beta_0 \beta_m k_{Pm} \quad , \quad \varepsilon_3 \triangleq \frac{f_m}{\beta_M + \frac{f_M^2}{2\beta_0 k_{Pm}}} < \frac{f_m}{\beta_M} \triangleq \varepsilon_4 \tag{10a}$$

with

$$\begin{gathered}\beta_0 \triangleq 1 - \max\left\{\frac{k_g}{k_{Pm}}, \max_i\left\{\frac{2B_{gi}}{L_{Pi}}\right\}\right\} \quad , \quad \beta_P \triangleq \min_i\left\{\frac{k_{Pi}}{\sigma'_{PiM}}\right\} \quad , \quad \beta_m \triangleq \min_i\left\{\frac{a_i}{b_i k_{Di}}\right\} \\ \beta_M \triangleq k_C B_P + \mu_M \sigma'_{PM} \quad , \quad B_P \triangleq \sqrt{\sum_{i=1}^n \left(\frac{M_{Pi}}{k_{Pi}}\right)^2} \quad , \quad \sigma'_{PM} \triangleq \max_i\{\sigma'_{PiM}\}\end{gathered} \tag{10b}$$

(observe that by inequalities (7) and (8a): $0 < \beta_0 < 1$), σ'_{PiM} being the positive bound of $D^+\sigma_{Pi}(\cdot)$, in accordance to item 2 of Lemma 1, and μ_m, μ_M, k_C, f_m, f_M, B_{gi} and k_g as defined through Properties 1–4. A block diagram of the proposed scheme is shown in Fig. 1.

Remark 2: Note that the *input-saturation-avoidance* condition (8c) implies that $M_{Pi} + M_{Ii} < T_i$, while the satisfaction of inequalities (8a)-(8b) implies that $M_{Pi} + M_{Ii} > 3B_{gi}$. Hence, the feasibility of the simultaneous fulfillment of inequalities (8) is ensured by requiring the satisfaction of Assumption 1 with $\alpha = 3$. This is a consequence of the way how the closed-loop analysis is addressed, and its *worst-case* procedure followed at every step giving rise to conditions with certain degree of conservativeness and of a consequent failure tolerance margin. Similar conditions on the control input bounds have been required by other approaches where input constraints have been considered [21]. Previous saturating PID-type schemes that do not explicitly include a similar or analog condition on the control input bounds are not always exhaustive in the search for the whole set of explicit conditions that support the developed closed loop analyses. Moreover, the way how such analyses are addressed

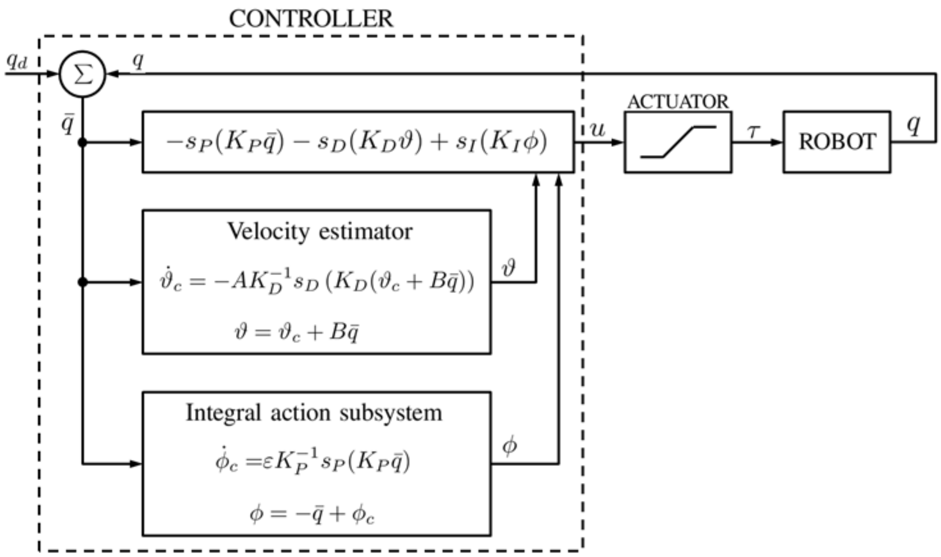

Fig. 1. Block diagram of the proposed output-feedback control scheme

lead to additional constraints on the control gains which complicate the tuning task and restrict the performance adjustment/improvement possibilities. Observe that the control gains in the approach proposed in this work are not tied to the satisfaction of any additional tuning restriction apart from inequality (7) —a standard condition in the literature [15] [19, Chap. 8]— and condition (9) concerning the integral-action-related parameter ε.

IV. Closed-Loop Analysis

Consider system (1),(3) taking $u = u(q, \vartheta, \phi)$ as defined through Eqs. (4)–(6). Let us define the variable transformation

$$\begin{pmatrix} \bar{q} \\ \vartheta \\ \bar{\phi} \end{pmatrix} = \begin{pmatrix} q - q_d \\ \vartheta_c + B(q - q_d) \\ -\bar{q} + \phi_c - \phi^* \end{pmatrix} \tag{11}$$

with $\phi^* = (\phi_1^*, \ldots, \phi_n^*)^T$ such that $s_I(K_I\phi^*) = g(q_d)$, or equivalently $\phi_i^* = \sigma_{Ii}^{-1}\big(g_i(q_d)\big)/k_{Ii}$, $i = 1, \ldots, n$ (notice that their strictly increasing character renders the generalized saturation functions σ_{Ii} invertible). Observe that from (8c) and (3), we have for every $i \in \{1, \ldots, n\}$ that

$$T_i > M_{Pi} + M_{Di} + M_{Ii} \geq \big|u_i(\bar{q} + q_d, \vartheta, \bar{\phi} + \phi^*)\big| = |u_i| = |\tau_i| \qquad \forall(\bar{q}, \vartheta, \bar{\phi}) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \tag{12}$$

Thus, under the consideration of the variable transformation (11), the closed-loop dynamics adopts the (equivalent) form

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = -s_P(K_P\bar{q}) - s_D(K_D\vartheta) + \bar{s}_I(\bar{\phi}) + g(q_d) \tag{13a}$$

$$\dot{\vartheta} = -AK_D^{-1}s_D(K_D\vartheta) + B\dot{q} \tag{13b}$$

$$\dot{\bar{\phi}} = -\dot{q} - \varepsilon K_P^{-1}s_P(K_P\bar{q}) \tag{13c}$$

where

$$\bar{s}_I(\bar{\phi}) = s_I(K_I\bar{\phi} + K_I\phi^*) - s_I(K_I\phi^*)$$

Observe that, by item 6 of Lemma 1, the elements of $\bar{s}_I(\bar{\phi})$, *i.e.* $\bar{\sigma}_{Ii}(\bar{\phi}_i) = \sigma_{Ii}(k_{Ii}\bar{\phi}_i + k_{Ii}\phi_i^*) - \sigma_{Ii}(k_{Ii}\phi_i^*)$, $i = 1, \ldots, n$, turn out to be strictly increasing generalized saturation functions.

Proposition 1: Consider the closed-loop system in Eqs. (13), under the satisfaction of Assumption 1 with $\alpha = 3$ and inequalities (8). Thus, for any positive definite diagonal matrices A, B, K_D, K_I and K_P such that inequality (7) is fulfilled, and any ε satisfying inequality (9), global asymptotic stability of the closed-loop trivial solution $(\bar{q}, \vartheta, \bar{\phi})(t) \equiv (0_n, 0_n, 0_n)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$.

Proof: By (12), one sees that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $\forall t \geq 0$. This proves that, under the proposed scheme, the input saturation values, T_i, are never attained. Now, in order to carry out the stability analysis, the following scalar function is defined

$$V(\bar{q}, \dot{q}, \vartheta, \bar{\phi}) = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}H(q)\dot{q} + \mathcal{U}(q) - \mathcal{U}(q_d) - g^T(q_d)\bar{q} + \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr + \int_{0_n}^{\vartheta} s_D^T(K_D r)B^{-1}dr$$

where $\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i$, $\int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr = \sum_{i=1}^n \int_0^{\bar{\phi}_i} \bar{\sigma}_{Ii}(r_i)dr_i$, $\int_{0_n}^{\vartheta} s_D^T(K_D r)B^{-1}dr = \sum_{i=1}^n \int_0^{\vartheta_i} \sigma_{Di}(k_{Di}r_i)b_i^{-1}dr_i$ and recall that $\mathcal{U}$ represents the gravitational potential energy. Note, by recalling Eqs. (2), that the defined scalar function can be rewritten as

$$V(\bar{q}, \dot{q}, \vartheta, \bar{\phi}) = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}H(q)\dot{q} + \gamma_0 \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + \mathcal{U}_{\gamma_0}^c(\bar{q}) + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr + \int_{0_n}^{\vartheta} s_D^T(K_D r)B^{-1}dr$$

where

$$\begin{aligned}\mathcal{U}_{\gamma_0}^c(\bar{q}) &= \int_{0_n}^{\bar{q}} \left[g(r + q_d) - g(q_d) + (1 - \gamma_0)s_P(K_P r)\right]^T dr \\ &= \sum_{i=1}^n \int_0^{\bar{q}_i} \left[\bar{g}_i(r_i) - g_i(q_d) + (1 - \gamma_0)\sigma_{Pi}(k_{Pi}r_i)\right] dr_i\end{aligned}$$

with

$$\begin{aligned}\bar{g}_1(r_1) &= g_1(r_1 + q_{d1}, q_{d2}, \ldots, q_{dn}) \\ \bar{g}_2(r_2) &= g_2(q_1, r_2 + q_{d2}, q_{d3}, \ldots, q_{dn}) \\ &\vdots \\ \bar{g}_n(r_n) &= g_n(q_1, q_2, \ldots, q_{n-1}, r_n + q_{dn})\end{aligned}$$

and γ_0 is a constant satisfying

$$\beta_0 \frac{\varepsilon^2}{\varepsilon_1^2} < \gamma_0 < \beta_0 \tag{14}$$

(observe, from inequality (9) and the definition of β_0, that $0 < \beta_0\varepsilon^2/\varepsilon_1^2 < \beta_0 < 1$). Under this consideration, $\mathcal{U}^c_{\gamma_0}(\bar{q})$ turns out to be lower-bounded by

$$W_{10}(\bar{q}) = \sum_{i=1}^{n} w_i^{10}(\bar{q}_i) \tag{15a}$$

where

$$w_i^{10}(\bar{q}_i) \triangleq \begin{cases} \frac{k_{li}}{2}\bar{q}_i^2 & \text{if } |\bar{q}_i| \leq \bar{q}_i^* \\ k_{li}\bar{q}_i^*\left(|\bar{q}_i| - \frac{\bar{q}_i^*}{2}\right) & \text{if } |\bar{q}_i| > \bar{q}_i^* \end{cases} \tag{15b}$$

with $0 < k_{li} \leq (1-\gamma_0)k_{Pi} - k_g$ and $\bar{q}_i^* = [L_{Pi} - 2B_{gi}/(1-\gamma_0)]/k_{Pi}$ (note that by inequality (14) and the definition of β_0: $0 < (1-\gamma_0)k_{Pi} - k_g$ and $\bar{q}_i^* > 0$); this is proven in [22]. From this, Property 1 and item 3 of Lemma 1, we have that

$$V(\bar{q},\dot{q},\vartheta,\bar{\phi}) \geq W_{11}(\bar{q},\dot{q}) + W_{10}(\bar{q}) + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr + \int_{0_n}^{\vartheta} s_D^T(K_D r)B^{-1}dr \tag{16}$$

where

$$W_{11}(\bar{q},\dot{q}) = \frac{1}{2}\begin{pmatrix} \|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\| \end{pmatrix}^T Q_{11} \begin{pmatrix} \|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\| \end{pmatrix}$$

with

$$Q_{11} = \begin{pmatrix} \gamma_0\beta_P & -\varepsilon\mu_M \\ -\varepsilon\mu_M & \mu_m \end{pmatrix} = \begin{pmatrix} \gamma_0\beta_P & -\frac{\varepsilon}{\varepsilon_1}\sqrt{\beta_0\beta_P\mu_m} \\ -\frac{\varepsilon}{\varepsilon_1}\sqrt{\beta_0\beta_P\mu_m} & \mu_m \end{pmatrix}$$

By inequality (14), $W_{11}(\bar{q},\dot{q})$ is positive definite (since with $\varepsilon < \varepsilon_M \leq \varepsilon_1$, in accordance to inequality (9), any γ_0 satisfying (14) renders Q_{11} positive definite) and observe that $W_{11}(0_n,\dot{q}) \to \infty$ as $\|\dot{q}\| \to \infty$, while from Eqs. (15) and items 4 and 5 of Lemma 1, it is clear that W_{10} and the integral terms in the right-hand side of (16) are radially unbounded positive definite functions of $\bar{q}$, $\bar{\phi}$ and ϑ, respectively. Thus, $V(\bar{q},\dot{q},\vartheta,\bar{\phi})$ is concluded to be positive definite and radially unbounded. Its upper right-hand derivative along the system trajectories, $\dot{V} = D^+V$ [23, §6.1A], is given by

$$\begin{aligned}\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) = & -\dot{q}^TF\dot{q} - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}F\dot{q} - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}\big[g(q) + s_P(K_P\bar{q}) - g(q_d)\big] \\ & - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}s_D(K_D\vartheta) + \varepsilon\dot{q}^TC(q,\dot{q})K_P^{-1}s_P(K_P\bar{q}) + \varepsilon\dot{q}^Ts_P'(K_P\bar{q})H(q)\dot{q} \\ & - s_D^T(K_D\vartheta)B^{-1}AK_D^{-1}s_D(K_D\vartheta)\end{aligned}$$

where $H(q)\ddot{q}$, $\dot{\bar{\phi}}$ and $\dot{\vartheta}$ have been replaced by their equivalent expressions from the closed-loop dynamics in Eqs. (13), Property 2.2 has been used and $s_P'(K_P\bar{q}) \triangleq \mathrm{diag}\big[D^+\sigma_{P1}(k_{P1}\bar{q}_1),\ldots,D^+\sigma_{Pn}(k_{Pn}\bar{q}_n)\big]$. The resulting expression can be rewritten as

$$\begin{aligned}\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) = & -\dot{q}^TF\dot{q} - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}F\dot{q} - \varepsilon\gamma_1 s_P^T(K_P\bar{q})K_P^{-1}K_PK_P^{-1}s_P(K_P\bar{q}) - \varepsilon\mathcal{W}_{\gamma_1}(\bar{q}) \\ & - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}s_D(K_D\vartheta) + \varepsilon\dot{q}^TC(q,\dot{q})K_P^{-1}s_P(K_P\bar{q}) + \varepsilon\dot{q}^Ts_P'(K_P\bar{q})H(q)\dot{q} \\ & - s_D^T(K_D\vartheta)B^{-1}AK_D^{-1}s_D(K_D\vartheta)\end{aligned}$$

where

$$\mathcal{W}_{\gamma_1}(\bar{q}) = s_P^T(K_P\bar{q})K_P^{-1}\big[(1-\gamma_1)s_P(K_P\bar{q}) + g(q) - g(q_d)\big]$$
$$= \sum_{i=1}^{n}\left[\frac{(1-\gamma_1)}{k_{Pi}}\sigma_{Pi}^2(k_{Pi}\bar{q}_i) + \frac{\sigma_{Pi}(k_{Pi}\bar{q}_i)}{k_{Pi}}\big[g_i(q) - g_i(q_d)\big]\right]$$

and γ_1 is a constant satisfying

$$\beta_0\left[\max\left\{\frac{\varepsilon}{\varepsilon_2}\ ,\ \frac{\varepsilon}{\varepsilon_3}\left(\frac{\varepsilon_4-\varepsilon_3}{\varepsilon_4-\varepsilon}\right)\right\}\right] < \gamma_1 < \beta_0 \tag{17}$$

(from inequality (9) and the definition of β_0, one verifies, after simple developments, that $0 < \beta_0\big[\max\big\{\varepsilon/\varepsilon_2\ ,\ \varepsilon(\varepsilon_4 - \varepsilon_3)/[\varepsilon_3(\varepsilon_4-\varepsilon)]\big\}\big] < \beta_0 < 1$). Under this consideration, $\mathcal{W}_{\gamma_1}(\bar{q})$ turns out to be lower-bounded by

$$W_{20}(\bar{q}) = \sum_{i=1}^{n} w_i^{20}(\bar{q}_i) \tag{18a}$$

where

$$w_i^{20}(\bar{q}_i) = \begin{cases} c_i\bar{q}_i^2 & \text{if } |\bar{q}_i| \leq L_{Pi}/k_{Pi} \\ \frac{d_i}{k_{Pi}}\big(|\sigma_{Pi}(k_{Pi}\bar{q}_i)| - L_{Pi}\big) + c_i\left(\frac{L_{Pi}}{k_{Pi}}\right)^2 & \text{if } |\bar{q}_i| > L_{Pi}/k_{Pi} \end{cases} \tag{18b}$$

with $d_i = (1-\gamma_1)L_{Pi} - 2B_{gi}$, $c_i = \min\left\{h, \frac{d_i k_{Pi}}{L_{Pi}}\right\}$ and $h = (1-\gamma_1)k_{Pm} - k_g$ (notice, from inequality (17) and the definition of β_0, that $d_i > 0$ and $h > 0$, hence $c_i > 0$); this is proven in [22]. From this, Properties 1, 2.1 and 3, and items 2 of Lemma 1 and (b) of Definition 1, we have that

$$\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) \leq -\varepsilon W_{21}(\bar{q},\vartheta) - \varepsilon W_{22}(\bar{q},\dot{q}) - \varepsilon W_{20}(\bar{q})$$

where

$$W_{21}(\bar{q},\vartheta) = \frac{1}{2}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|s_D(K_D\vartheta)\|\end{pmatrix}^T Q_{21}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|s_D(K_D\vartheta)\|\end{pmatrix}$$

$$W_{22}(\bar{q},\dot{q}) = \frac{1}{2}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}^T Q_{22}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}$$

with

$$Q_{21} = \begin{pmatrix}\gamma_1 k_{Pm} & -1 \\ -1 & \frac{2\beta_m}{\varepsilon}\end{pmatrix} = \begin{pmatrix}\gamma_1 k_{Pm} & -1 \\ -1 & \frac{\varepsilon_2}{k_{Pm}\beta_0\varepsilon}\end{pmatrix}$$

$$Q_{22} = \begin{pmatrix}\gamma_1 k_{Pm} & -f_M \\ -f_M & 2\left(\frac{f_m}{\varepsilon} - \beta_M\right)\end{pmatrix} = \begin{pmatrix}\gamma_1 k_{Pm} & -\sqrt{2k_{Pm}\beta_M\beta_0\left(\frac{\varepsilon_4-\varepsilon_3}{\varepsilon_3}\right)} \\ -\sqrt{2k_{Pm}\beta_M\beta_0\left(\frac{\varepsilon_4-\varepsilon_3}{\varepsilon_3}\right)} & 2\beta_M\left(\frac{\varepsilon_4-\varepsilon}{\varepsilon}\right)\end{pmatrix}$$

By inequality (17), $W_{21}(\bar{q},\vartheta)$ and $W_{22}(\bar{q},\dot{q})$ are positive definite (since with $\varepsilon < \varepsilon_M \leq \min\{\varepsilon_2,\varepsilon_3\} < \varepsilon_4$, in accordance to inequality (9), any γ_1 satisfying (17) renders Q_{21} and Q_{22} positive definite), while from Eqs. (18), it is clear that W_{20} is a positive definite function of $\bar{q}$. Hence, $\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) \leq 0$ with $\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) = 0 \iff (\bar{q},\dot{q},\vartheta) = (0_n, 0_n, 0_n)$. Further, from the closed-loop dynamics in Eqs. (13), we see that $\bar{q}(t) \equiv \dot{q}(t) \equiv \vartheta(t) \equiv$

Fig. 2. Experimental setup: 2-DOF direct-drive robot manipulator

$0_n \implies \ddot{q}(t) \equiv 0_n \implies \bar{s}_I(\bar{\phi}(t)) \equiv 0_n \implies \bar{\phi}(t) \equiv 0_n$. Therefore, by the invariance theory [23, §7.2], the closed-loop trivial solution $(\bar{q}, \vartheta, \bar{\phi})(t) \equiv (0_n, 0_n, 0_n)$ is concluded to be globally asymptotically stable, which completes the proof. ■

Remark 3: Note that the fulfillment of inequality (9) is not necessary but only sufficient for the closed-loop analysis to hold. As a matter of fact, proving Proposition 1 through inequality (9) is tantamount to show the existence of some $\varepsilon^* \geq \varepsilon_M$ such that, for any $\varepsilon \in (0, \varepsilon^*)$, global stabilization is guaranteed. Hence, the proposed scheme permits successful implementations with values of ε higher than ε_M (up to certain limit, ε^*).

V. Experimental Results

With the aim at corroborating the efficiency of the proposed scheme, several real-time control tests were implemented on a 2-DOF manipulator. The experimental setup, shown in Fig. 2, is a 2-revolute-joint mechanical arm located at the *Instituto Tecnológico de la Laguna*, Mexico, previously used in [20]. The robot actuators are direct-drive brushless servomotors operated in torque mode, *i.e.* they act as torque sources and receive an analog voltage as a torque reference signal. Joint positions are obtained using incremental encoders on the motors. In order to get the encoder data and generate reference voltages, the robot includes a motion control board based on a DSP 32-bit floating point microprocessor. The control algorithm is executed at a 2.5 millisecond sampling period on a PC-host computer.

For the experimental manipulator, the various terms characterizing the system dynamics in (1) are given by

$$H(q) = \begin{bmatrix} 2.351 + 0.168\cos q_2 & 0.102 + 0.084\cos q_2 \\ 0.102 + 0.084\cos q_2 & 0.102 \end{bmatrix} \tag{19a}$$

$$C(q,\dot{q}) = \begin{bmatrix} -0.084\dot{q}_2 \sin q_2 & -0.084(\dot{q}_1 + \dot{q}_2)\sin q_2 \\ 0.084\dot{q}_1 \sin q_2 & 0 \end{bmatrix} \tag{19b}$$

$$g(q) = \begin{bmatrix} 38.465\sin q_1 + 1.825\sin(q_1 + q_2) \\ 1.825\sin(q_1 + q_2) \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} 2.288 & 0 \\ 0 & 0.175 \end{bmatrix} \tag{19c}$$

In particular, Properties 1–4 are satisfied with $\mu_m = 0.088$ kg·m^2, $\mu_M = 2.533$ kg·m^2, $k_C = 0.1455$ kg·m^2, $f_m = 0.175$ kg·m^2/s, $f_M = 2.288$ kg·m^2/s, $B_{g1} = 40.29$ Nm, $B_{g2} = 1.825$ Nm and $k_g = 40.373$ Nm/rad. The maximum allowed torques (input saturation bounds) are $T_1 = 150$ Nm and $T_2 = 15$ Nm for the first and second links respectively. From these data, one easily corroborates that Assumption 1 is fulfilled with $\alpha = 3$.

The saturation functions used for the implementation were defined as $\sigma_{Pi}(\varsigma) = M_{Pi}\text{sat}(\varsigma/M_{Pi})$, $\sigma_{Di}(\varsigma) = M_{Di}\text{sat}(\varsigma/M_{Di})$ and

$$\sigma_{Ii}(\varsigma) = \begin{cases} \varsigma & \text{if } |\varsigma| \leq L_{Ii} \\ \text{sign}(\varsigma)L_{Ii} + (M_{Ii} - L_{Ii})\tanh\left(\frac{\varsigma - \text{sign}(\varsigma)L_{Ii}}{M_{Ii} - L_{Ii}}\right) & \text{if } |\varsigma| > L_{Ii} \end{cases} \tag{20}$$

for $0 < L_{Ii} < M_{Ii}$, $i = 1, 2$.[4] Let us note that with these saturation functions, we have $\sigma'_{PiM} = \sigma'_{IiM} = \sigma'_{DiM} = 1$, $\forall i \in \{1, 2\}$. The saturation parameters were selected in accordance to inequalities (8) as (all of them expressed in Nm): $M_{P1} = 86$, $M_{P2} = 7$, $M_{D1} = 22$, $M_{D2} = 4$, $M_{I1} = 41$, $M_{I2} = 2$, and $L_{Ii} = 0.9M_{Ii}$, $i = 1, 2$.

For comparison purposes, additional experimental tests were implemented using the bounded PID-type scheme presented in [14] (choice made taking into account the analog nature of the compared algorithms: globally stabilizing in a bounded-input context through output feedback, and the recent appearance of [14]), *i.e.*

$$u = -K_P\text{Tanh}(\bar{q}) - K_D\text{Tanh}(\vartheta) - K_I\text{Tanh}(\phi) \tag{21a}$$

$$\begin{aligned} \dot{\vartheta}_c &= -A[\vartheta_c + Bq] \\ \vartheta &= \vartheta_c + Bq \end{aligned} \tag{21b}$$

$$\begin{aligned} \dot{\phi}_c &= \text{Tanh}(\bar{q}) \\ \phi &= \eta^2\bar{q} + \eta\phi_c \end{aligned} \tag{21c}$$

with η being a (sufficiently large) positive constant and $\text{Tanh}(x) = \big(\tanh x_1, \ldots, \tanh x_n\big)^T$ for any $x \in \mathbb{R}^n$.[5] For the sake of simplicity, this algorithm is subsequently referred to as the S10 controller.

In all the experiments, the desired joint positions were fixed at $q_d = \big(q_{d1}, q_{d2}\big)^T = \big(\pi/4, \pi/4\big)^T$ [rad]. The initial conditions were $q(0) = \dot{q}(0) = 0_2$, and, for the proposed scheme, $\phi_c(0)$ was taken so as to have $\phi(0) = 0_2$, while $\phi_c(0) = 0_2$ was taken for the S10 controller in view of the way how it is presented in [14] (recall Footnote 5). The

[4] Note that the achievement of the global regulation task is independent of the specific saturation functions chosen for implementation as long as the design specifications given in Section III be followed. In the particular experimental case presented here, standard (non-smooth) saturations were chosen for the SP and SD actions to show the simplicity permitted by the proposed scheme, while the function in (20) was chosen for the SI actions as an example of a strictly increasing (smooth) saturation that is identical to its argument within a range, permitting the reproduction of the unconstrained input case when the closed-loop trajectories remain *small* enough.

[5] In place of Eqs. (21c), the work in [14] defines $\phi(t) = \eta^2\bar{q}(t) + \eta\int_0^t \text{Tanh}\big(\bar{q}(\varsigma)\big)d\varsigma$, which imposes the auxiliary variable initial condition $\phi(0) = \eta^2\bar{q}(0)$ (or, equivalently, $\phi_c(0) = 0_n$ in the context of Eqs. (21c)). Instead, Eqs. (21c) —or their (equivalent) time representation $\phi(t) = \phi(0) + \eta^2\big[\bar{q}(t) - \bar{q}(0)\big] + \eta\int_0^t \text{Tanh}\big(\bar{q}(\varsigma)\big)d\varsigma$— keeps the required auxiliary dynamics while permitting any initial condition for ϕ (or, equivalently, for ϕ_c in the context of Eqs. (21c)). This proves to be more appropriate in the global stabilization framework considered in [14] (and what is generally expected from an approach developed within such a framework).

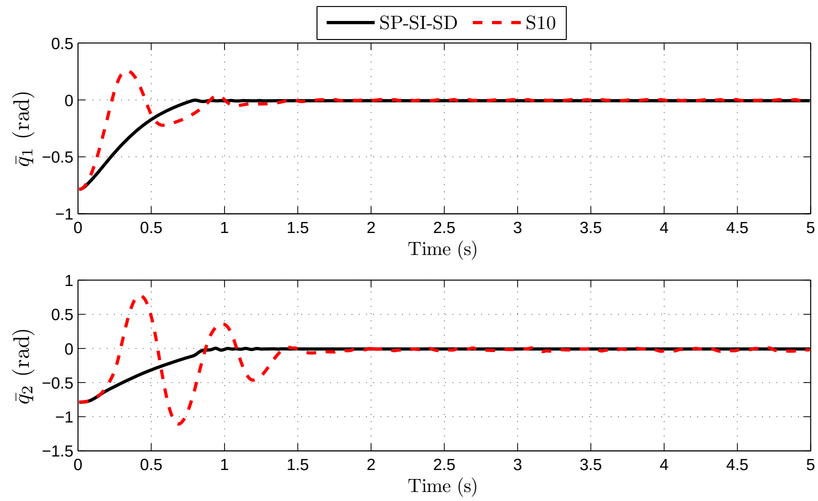

Fig. 3. Experimental results: position errors

control gains for the proposed scheme were selected —under the satisfaction of inequalities (7) and (9)— so as to get fast responses and avoid overshoot. Such a performance requirement was achieved under an additional control-parameter adjustment procedure following the guidelines given in Appendix A. As for the S10 algorithm, the control parameters were tuned so as to get the best possible closed-loop responses while adhering to the saturation-avoidance inequalities and stability conditions (some of which had to be verified numerically) presented in [14]. The resulting tuning values were: $K_P = \text{diag}[9000, 500]$ Nm/rad, $K_D = \text{diag}[10, 4]$ Nms/rad, $K_I = \text{diag}[1000, 300]$ Nm/rad, $A = \text{diag}[60, 40]$ s^{-1}, $B = [70, 20]$ s^{-1} and $\varepsilon = 0.065$ s^{-1} for the proposed scheme, whence one can corroborate that inequalities (7) and (9) are fulfilled, and $K_P = \text{diag}[108, 11.5]$ Nm, $K_D = \text{diag}[0.5, 0.1]$ Nm, $K_I = \text{diag}[40.5, 1.9]$ Nm, $A = \text{diag}[60, 40]$ s^{-1}, $B = [70, 20]$ s^{-1} and $\eta = 170$ s/rad for the S10 controller.

Figs. 3 and 4 show the experimental results. Note that the proposed scheme successfully achieved the regulation objective avoiding overshoot during the transient and preventing input saturation. The S10 controller is also observed to achieve the stabilization goal preventing input saturation but overshoot could not be avoided under the tuning procedure presented in [14]. Note further that the control objective has been achieved despite the imminent measurement noise and unmodelled phenomena, such as the unconsidered friction components (*e.g.* static and dry friction forces). Restricted effect of noise on the closed-loop system responses may be seen as a natural consequence of the output-feedback nature of the proposed approach since only position variables are considered in the control algorithm, avoiding additional noise corruption from speed measurements. On the other hand, it is natural to expect the achievement of the position stabilization objective despite the presence of constant perturbation inputs (of suitable size), or even (suitably bounded) input disturbance terms giving rise to constant values under static conditions. This is in accordance to the nature of the integral action subsystem, which forces $\bar{q} \equiv 0_n$ to be the unique position error equilibrium, while the integral-action term (directly acting on the manipulator dynamics) adopts a suitable steady-state (vector) value to compensate for the static (or constant) perturbation value (in addition to

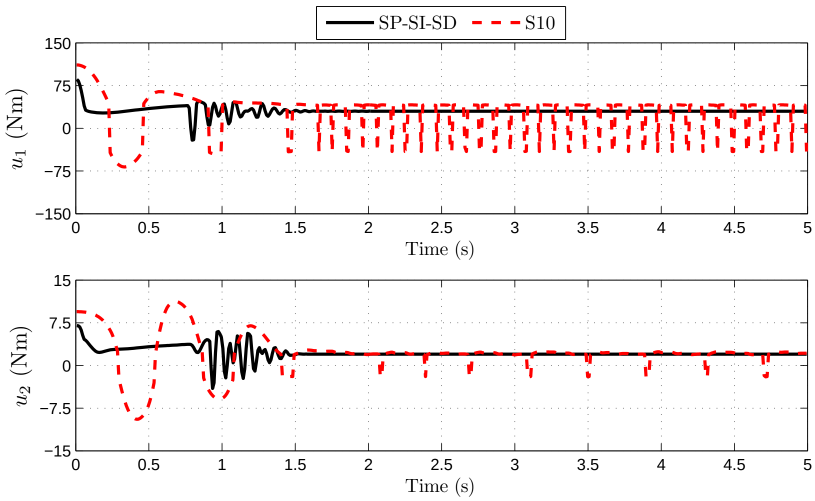

Fig. 4. Experimental results: control signals

the gravity effects). For a clearer appreciation on these capabilities of the PID-type proposed approach, additional simulations were implemented taking model (1),(3) with an additional constant input perturbation term h, *i.e.* $H(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau + h$, $\tau_i = T_i \text{sat}(u_i/T_i)$, in closed loop with the proposed PID-type scheme under the consideration of noisy position measurements $\hat{q} = q + \nu(t)$, *i.e.* with q in Eqs. (4)–(6) replaced by such a $\hat{q}$, with ν being a random (noise) variable with Gaussian distribution such that $|\nu(t)| < \bar{\nu}$, $\forall t$, for some (noise) bound $\bar{\nu} > 0$. The simulations were implemented taking $h = (h_1, h_2)^T$, with $h_i = -0.1\, T_i$, $i = 1, 2$, *i.e.* $h_1 = -15$ Nm and $h_2 = -1.5$ Nm, and $\bar{\nu} = 0.01$ rad. The dynamic properties of the considered experimental manipulator —or equivalently $H(q)$, $C(q,\dot{q})$, $g(q)$ and F in Eqs. (19)— were adopted, together with the control gains and saturation functions and related parameters defined for the above described experimental test (with the proposed control scheme). The results are shown in Figs. 5 and 6 where, just as a reference, the experimental curves (obtained with the proposed control scheme) previously shown were included. One observes that the position stabilization objective is achieved despite the considerable measurement noise level and constant input perturbation that were added, with simulation curves close to the experimental ones. A notorious effect of the noise on the control signals is noticed; this is mainly due to the important proportional gain values that were fixed, which considerably magnifies the corrupted position errors. Notwithstanding, negligible effects of the added noise are observed on the position error responses, which is an important characteristic from a response-oriented performance viewpoint.

VI. Conclusions

An output-feedback PID-type scheme for the global position stabilization of robot manipulators with bounded inputs was proposed. It has proven to relax the complexity of previous approaches of the kind by keeping a simple structure while increasing design/performance-adjustment flexibility. More importantly, it is characterized by its very simple control-gain tuning criterion. This has been the consequence of the designed structure and closed-

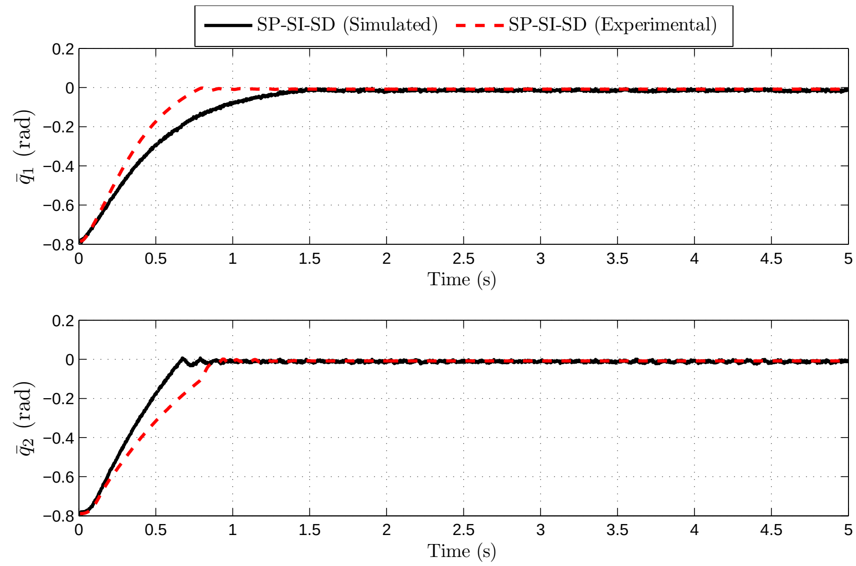

Fig. 5. Simulation results with noise and constant input perturbation: position errors

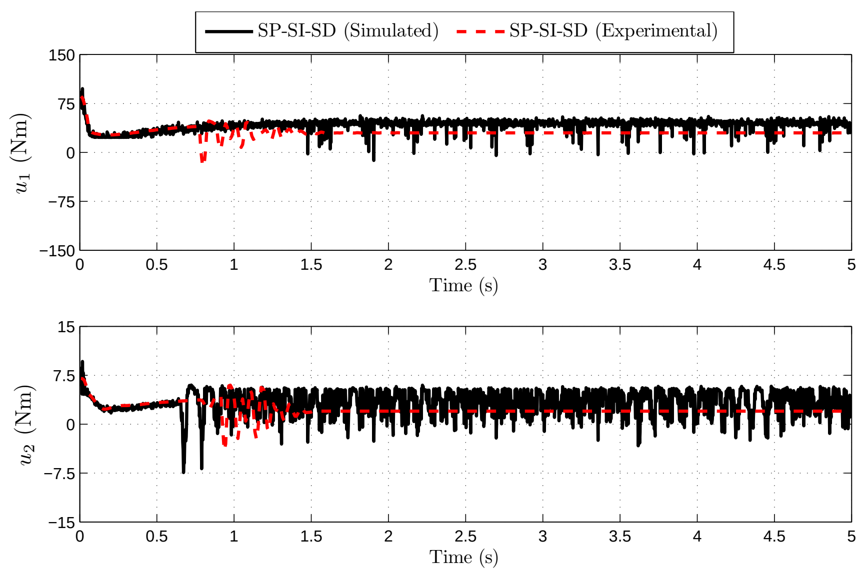

Fig. 6. Simulation results with noise and constant input perturbation: control signals

loop analysis, which have released the control gain selection from unnecessary restrictive conditions while being exhaustive on the search for additional requirements that render feasible the tuning procedure. The efficiency of the proposed algorithm has been corroborated through experimental tests on a 2-DOF direct-drive manipulator.

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Appendix A

The performance-oriented tuning procedure that was used to obtain the experimental results shown Section V is sketched as follows:

1. Set the saturation function parameters (M_{Ii}, M_{Di}, M_{Pi}, L_{Pi}) so as to guarantee the satisfaction of inequalities (8).
2. Set the velocity-estimation-subsystem parameters a_i such that $1/a_i$ be six to ten times the sampling period of the controller, and $b_i \geq a_i$ to speed up the velocity estimation (or motion dissipation) or $b_i < a_i$ to reduce inertial effects (inherent to the velocity estimation dynamics), such as oscillations.
3. Run simulations/experiments with low control gains/coefficients, under the consideration of (7).
4. Increase the integral gains, k_{Ii}, in order to strengthen the elimination of position errors, aiming at reducing stabilization times.
5. Increase the proportional gains, k_{Pi}, in order to reduce the rise time (speed up the closed-loop response).
6. Increase the derivative gains, k_{Di}, in order to reduce inertial effects (particularly added by the integral actions), such as the overshoot.
7. Adjust ε adhering to (9), if possible, or increasing its value as far as the closed-loop stability permits it.
8. Repeat the steps 4–7 until the best possible response is obtained.

Comments

(a) *On the saturation function parameters (concerned in step 1).* Although different values of the saturation function parameters may lead to different effects on the closed-loop performance, their main purpose is to guarantee input-saturation avoidance, through the satisfaction of inequality (8c), simultaneously ensuring the accomplishment of conditions (8a)-(8b) arisen from the closed-loop analysis. We restrict their usage to such a purpose by simply requiring the satisfaction of inequalities (8).

(b) *On the velocity-estimation-subsystem parameters (concerned in step 2).* In its original linear form, where no saturation function is involved, the dirty derivative operator —implemented in the proposed approach through the velocity estimation (nonlinear) subsystem in Eqs. (6)— acts like a set of low-pass filters on the velocity variables, each of them with time constant $1/a_i$ and gain b_i/a_i. Step 2 is stated giving such a sense to the concerned parameters. Even with the (benign) change resulting from the inclusion of the nonlinear function s_D in the velocity-estimation-subsystem dynamics, keeping such an interpretation of the concerned parameters for their adjustment proves to contribute to the achievement of acceptable closed-loop performances.

(c) *On the integral-action-related coefficient ε (concerned in step 7).* In view of Remark 3, arbitrarily small values of ε could be initially tested as long as the stabilization objective be achieved, and further adjustments can then be considered towards the achievement of a performance requirement (lowering down the initial value if the initial test calls into question the suitable accomplishment of the stability requirements or, in the contrary case, as indicated in step 7 aiming at contributing to lower down the stabilization time; if no notorious performance improvement is entailed through its adjustment, it can remain fix during the performance-oriented tuning procedure). Of course, fulfilment of (9) analytically guarantees a correct choice of ε, which could however be conservative. Opting for the satisfaction of (9) leads to the evaluation of expressions (10) at every readjustment of the control parameters. Observe that such expressions (10) are in terms of the control and saturation function parameters as well as the bound values characterizing Properties 1–4, which may in turn be estimated through lower and/or upper bounds on the system parameters.

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