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A generalized global adaptive tracking control scheme for robot manipulators with bounded inputs

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SUMMARY

In this work, a generalized adaptive scheme for the global motion control of robot manipulators with constrained inputs is proposed. It gives rise to various families of bounded adaptive controllers defined through a general class of saturation functions. Compared with adaptive tracking control algorithms previously developed in a bounded-input context, the proposed adaptive approach guarantees the motion control objective for any initial condition, avoiding discontinuities throughout the scheme, preventing the inputs to reach their natural saturation bounds, and permitting innovation on the saturating structure through its generalized form, giving a wide range of possibilities for performance improvement. Experimental results corroborate the efficiency of the proposed scheme. Copyright © 2013 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Tracking control of robot manipulators with bounded inputs has proven to be a challenging task. In addition to the nonautonomous nature of the closed-loop dynamics, the designer has to deal with the analytical complications and practical limitations entailed by the input constraints. For instance, since the desired motion implies a specific form (time variation) of each of the reaction and inherent (generalized) forces involved in the system dynamics, shaped through the external driving devices, only trajectories guaranteeing that the combined effect of such forces remain within the input physical ranges are *tractable*. Any attempt to track a trajectory that does not satisfy such a characteristic would not only fail to achieve the control objective but would also force the actuators to go beyond their natural capabilities giving rise to unexpected or undesirable closed-loop behaviors. Other risks due to the input saturation phenomenon are pointed out for instance in [1, 2, 3, 4].

In order to avoid such input-saturation-induced inconveniences, several bounded tracking control approaches have been proposed in the literature under various analytical frameworks [5, 6, 7, 8, 9, 10, 11]. For instance, assuming that the exact value of the system parameters and accurate

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position measurements are available, an output-feedback scheme has been proposed in [5]. This approach involves a reproduction of the various (reaction and inherent) forces that are present in the system dynamics, calculated involving the current values of the actual position variables but the desired velocity and acceleration time-variations. An adequate desired trajectory $q_d(t)$ keeps such *shaping/compensation* terms suitably bounded and these in turn render $q_d(t)$ a solution of the closed-loop dynamics. In order to achieve the asymptotic tracking objective, correction terms on the position error and on an auxiliary vector state variable that approximates the velocity error —calculated through an auxiliary dynamics— are included in the control expression. Their associated control gains are applied to sigmoidal functions —specifically, the hyperbolic tangent— of the referred (position and approximated velocity) error variables, giving rise to saturating nonlinear P and D type terms. In a frictionless setting, such a control scheme was proven to semi-globally stabilize the closed-loop system. An extended version of the algorithm of [5] was further presented in [6] by additionally including *desired* viscous friction forces. Through such an additional consideration, global tracking is achieved for suitable desired trajectories. An alternative output-feedback tracking approach that keeps a *Computed-Torque*-like structure was developed in [7]. It considers the same form of the gravity, viscous friction, and Coriolis and centrifugal calculated force vectors used of [6], but a special form of inertial (complemented) force vector where saturating nonlinear P and D terms analog to those of [5] (but involving an approximated velocity error vector variable calculated through a different auxiliary dynamics) are included. Semi-global tracking is achieved through such an output-feedback controller of [7]. Further, revisited versions of the controller in [6] have been developed in [8] and [9]. In [8], gains scaling the argument of the hyperbolic tangents are incorporated, while in [9] the hyperbolic tangents are replaced by a more general class of saturating functions. In both works, exponential stability was proven through singular perturbation theory. After the previously mentioned output-feedback schemes, two state-feedback approaches were recently proposed in [10]. While the first of these keeps an SP-SD+ structure, with separated saturating proportional (SP) and saturating derivative (SD) error correction terms, the second one has an SPD+ form where both the P and D error correction terms are included within a single saturation function (at every link). Shaping/compensation terms analog to those of [6] are included, and a generalized type of saturation functions is involved. Both algorithms are proven to achieve global tracking for suitable desired trajectories. Furthermore, an output-feedback extension of the SP-SD+ approach of [10], that may be seen as an improved generalization of the controller of [6], was recently developed in [11]. Keeping analog analytical features of the SP-SD+ approach of [10], but avoiding velocity measurements, such an output-feedback scheme of [11] is proven to achieve global tracking.

Because of the considered shaping/compensation terms, which involve the expressions of the system dynamics, the implementation of the above mentioned saturating schemes becomes difficult when the system parameters are uncertain. In view of such an additional constraint, state-feedback bounded adaptive tracking controllers were alternatively presented in [7] and [12].

The adaptive approach in [7] considers SP and SD type correction terms similarly structured than those defined in [5] and [6] but involving online measurements of both the positions and the velocities. In addition, adaptive *desired compensation* terms of the system dynamics involving parameter estimators are included. The adaptation algorithm is defined in terms of a discontinuous auxiliary dynamics by means of which the parameter estimators are prevented to take values beyond some pre-specified limits, which consequently keeps the adaptive compensation terms bounded. The tracking objective was proven to be achieved for suitable desired trajectories, with a region of attraction that can be enlarged through the control gain values.

In [12], an algorithm similar to that of [7] was presented involving identical SP and SD correction terms but only adaptive *on-line* gravity compensation (no other term of the system dynamics is compensated). A discontinuous adaptation algorithm analog to that involved in [7] is considered. Unfortunately, it is not clear through such an approach how the desired trajectory can be guaranteed to be a solution of the closed-loop system.

It is worth pointing out that the algorithms in [7] and [12] are the only adaptive tracking controllers with predefined bound that the authors are aware of. Other adaptive motion control schemes recently

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proposed in the literature have been developed in an unconstrained input framework [13, 14, 15, 16]. Let us further note that by the way the SP and SD terms are defined in the above mentioned adaptive schemes, the bound of the control signal at every link turns out to be defined in terms of the sum of the P and D control gains (and of an additional term involving the bounds of the parameter estimators). This limits the choice of such gains if the natural actuator bounds (or arbitrary input bounds) are aimed to be avoided. This, in turn, restricts the closed-loop region of attraction. Moreover, as far as the authors are aware, a bounded adaptive motion control scheme guaranteeing the tracking objective globally, preventing input saturation, and avoiding discontinuities throughout the scheme, is still missing in the literature. Even though such achievements have been recently succeeded in a regulation context [17, 18], the development of a tracking controller with analog characteristics remains an open problem requiring a more general and complex formulation within a more elaborated analytical framework. These arguments have motivated the present work which aims at filling in the mentioned gaps.

In this work, we propose a generalized scheme for the global adaptive tracking control of robot manipulators with saturating inputs. It gives rise to various families of bounded adaptive controllers, including adaptive versions of the SP-SD+ and SPD+ algorithms in [10] as well as an adaptive tracking extension of the SPDgc-like algorithm in [19] as particular cases. With respect to the bounded adaptive tracking control algorithms previously developed, the proposed adaptive approach guarantees the motion control objective for any initial condition (globally), avoiding discontinuities throughout the scheme, preventing the inputs to attain their natural saturation bounds, and permitting innovation on the saturating structure through its generalized form, giving a wide range of possibilities for performance improvement. In addition, the approach proposed in this work is not restricted to the use of a specific saturation function to achieve the required boundedness, but can rather involve any one within a set of bounded passive functions that include the hyperbolic tangent as a particular case. The efficiency of the proposed adaptive scheme is corroborated through experimental results.

Let us further add that previous works with adaptive control schemes where the parameter estimates are aimed to remain bounded within pre-specified values generally involve discontinuous adaptation dynamics of the kind of those used in [7] and [12]. This is seen even in recent studies [13, 14, 20, 21]. The discontinuous character of such type of adaptation auxiliary dynamics is not necessarily a disadvantage, but a bounded adaptive scheme that avoids discontinuities constitutes a convenient alternative developed within a simpler analytical context and through simpler and/or more natural ways to cope with the need to bound the parameter estimates. This is achieved through the approach proposed in this work by considering the parameter estimators to be the output variables —and not the auxiliary states— of the adaptation subsystem. The auxiliary (adaptation subsystem) states are in turn released from having to be initiated and evolve within a constrained subset rendering the proposed approach global in an authentic sense. Indeed, all the closed-loop system states, including those involved in the auxiliary adaptation dynamics, can be initiated anywhere. Thus, through its authentic global and continuous characters, the proposed approach overcomes limitations of previous bounded adaptive approaches. Furthermore, as far as the authors are aware, the approach developed in this paper is the first bounded adaptive motion control scheme that achieves the tracking task for any initial condition, avoiding input saturation, and free of discontinuities.

The work is organized as follows: Section 2 presents the general n-degree-of-freedom (n-DOF) serial rigid robot manipulator open-loop dynamics and some of its main properties, as well as considerations, assumptions, notations, and definitions that are involved throughout the study. In Section 3, a generalized approach for the design of global tracking controllers involving exact system parameter values is shown. This proves to furnish a useful structure for the design of the proposed adaptive scheme, which is presented in Section 4. The closed-loop analysis is developed in Section 5, where global adaptive tracking is proved to be achieved avoiding input saturation. Experimental results are presented in Section 6. Finally, conclusions are given in Section 7.

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2. PRELIMINARIES

Let $X \in \mathbb{R}^{m\times n}$ and $y \in \mathbb{R}^n$. Throughout this work, X_{ij} denotes the element of X at its i^{th} row and j^{th} column, X_i represents the i^{th} row of X, and y_i stands for the i^{th} element of y. 0_n represents the origin of $\mathbb{R}^n$, I_n the $n\times n$ identity matrix, and $\mathbb{R}_+$ the set of nonnegative real numbers, *i.e.* $\mathbb{R}_+ = [0,\infty)$. $\|\cdot\|$ denotes the standard Euclidean norm for vectors, *i.e.* $\|y\| = \sqrt{\sum_{i=0}^n y_i^2}$, and induced norm for matrices, *i.e.* $\|X\| = \sqrt{\lambda_{\max}(X^TX)}$, where $\lambda_{\max}(X^TX)$ represents the maximum eigenvalue of X^TX. We denote $\mathcal{B}_r \subset \mathbb{R}^n$ an origin-centered ball of radius $r>0$, *i.e.* $\mathcal{B}_r = \{x\in\mathbb{R}^n : \|x\|\le r\}$. Let $\mathcal{D}$ and $\mathcal{E}$ be subsets (with non-empty interior) of some vector spaces $\mathbb{D}$ and $\mathbb{E}$ respectively. We denote $\mathcal{C}^m(\mathcal{D};\mathcal{E})$ the set of m-times continuously differentiable functions from $\mathcal{D}$ to $\mathcal{E}$ (with differentiability at any point on the boundary of $\mathcal{D}$, when included in the set, meant as the limit from the interior of $\mathcal{D}$). For a dynamic/time variable υ, $\dot\upsilon$ and $\ddot\upsilon$ respectively denote its first- and second-order evolution/change rate. For a continuous scalar function $\phi:\mathbb{R}\to\mathbb{R}$, ϕ' denotes its derivative, when differentiable; $D^+\phi$ its upper right-hand (Dini) derivative, *i.e.* $D^+\phi(\varsigma) = \limsup_{h\to0^+}\frac{\phi(\varsigma+h)-\phi(\varsigma)}{h}$, with $D^+\phi=\phi'$ at points of differentiability [22, App. C.2]; and ϕ^{-1} its inverse, when invertible.

Let us consider the general n-DOF serial rigid robot manipulator dynamics with viscous friction [23, §2.1], [24, §6.2], [25, §7.2]:

$$H(q)\ddot q + C(q,\dot q)\dot q + F\dot q + g(q) = \tau \tag{1}$$

where $q,\ \dot q,\ \ddot q \in \mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity, and acceleration vectors, $H(q)\in\mathbb{R}^{n\times n}$ is the inertia matrix, and $C(q,\dot q)\dot q$, $F\dot q$, $g(q)$, $\tau\in\mathbb{R}^n$ are, respectively, the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with $F\in\mathbb{R}^{n\times n}$ being a positive definite constant diagonal matrix whose entries $f_i>0$, $i=1,\dots,n$, are the viscous friction coefficients. Some well-known properties characterizing the terms of such a dynamical model are recalled here [23, §2.1], [25, §3.3], [26, Chaps. 4 & 14]. Subsequently, we denote $\dot H$ the change rate of H, *i.e.* $\dot H:\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^{n\times n}:(q,\dot q)\mapsto\left(\frac{\partial H_{ij}}{\partial q}(q)\dot q\right)$.

Property 1
The inertia matrix $H(q)$ is a positive definite symmetric bounded matrix, *i.e.* $\mu_m I_n \le H(q) \le \mu_M I_n$, $\forall q\in\mathbb{R}^n$, for some positive constants $\mu_m\le\mu_M$.

Property 2
The Coriolis matrix $C(q,\dot q)$ satisfies:

2.1. $\dot q^T\left[\frac{1}{2}\dot H(q,\dot q) - C(q,\dot q)\right]\dot q = 0$, $\forall (q,\dot q)\in\mathbb{R}^n\times\mathbb{R}^n$;
2.2. $\dot H(q,\dot q) = C(q,\dot q) + C^T(q,\dot q)$, $\forall (q,\dot q)\in\mathbb{R}^n\times\mathbb{R}^n$;
2.3. $C(w,x+y)z = C(w,x)z + C(w,y)z$, $\forall w,x,y,z\in\mathbb{R}^n$;
2.4. $C(x,y)z = C(x,z)y$, $\forall x,y,z\in\mathbb{R}^n$;
2.5. $\|C(x,y)z\| \le k_C\|y\|\|z\|$, $\forall (x,y,z)\in\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n$, for some constant $k_C\ge0$.

Property 3
The viscous friction coefficient matrix satisfies $f_m\|x\|^2 \le x^TFx \le f_M\|x\|^2$, $\forall x\in\mathbb{R}^n$, where $0 < f_m \triangleq \min_i\{f_i\} \le \max_i\{f_i\} \triangleq f_M$.

Property 4
The gravity vector $g(q)$ is bounded, or equivalently, every element of the gravity vector, $g_i(q)$, $i=1,\dots,n$, satisfies $|g_i(q)|\le B_{gi}$, $\forall q\in\mathbb{R}^n$, for some positive constants B_{gi}, $i=1,\dots,n$.[†]

[†]Property 4 is not satisfied by all types of robot manipulators but it is for instance by those having only revolute joints [26, §4.3]. This work is addressed to manipulators satisfying Property 4.

Property 5
The left-hand side of the robot dynamic model in Eq. (1) can be rewritten as

$$H(q,\theta)\ddot{q} + C(q,\dot{q},\theta)\dot{q} + F(\theta)\dot{q} + g(q,\theta) = Y(q,\dot{q},\ddot{q})\theta$$

where $\theta \in \mathbb{R}^p$ is a constant vector whose elements depend exclusively on the system parameters, and $Y(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times p}$ —the regression matrix— is a continuous matrix function whose elements depend exclusively on the configuration, velocity, and acceleration variables and do not involve any of the system parameters. As a matter of fact, every term of the left-hand side of (1) can be analogously rewritten as $H(q,\theta)\ddot{q} = Y_H(q,\ddot{q})\theta$, $C(q,\dot{q},\theta)\dot{q} = Y_C(q,\dot{q})\theta$, $F(\theta)\dot{q} = Y_F(\dot{q})\theta$, and $g(q,\theta) = Y_g(q)\theta$, and actually $Y(q,\dot{q},\ddot{q}) = Y_H(q,\ddot{q}) + Y_C(q,\dot{q}) + Y_F(\dot{q}) + Y_g(q)$.

Property 6
Consider the manipulator dynamics $Y(q,\dot{q},\ddot{q})\theta = H(q,\theta)\ddot{q} + C(q,\dot{q},\theta)\dot{q} + F(\theta)\dot{q} + g(q,\theta) = \tau$. Let $\theta_{Mj} > 0$ represent an upper bound of $|\theta_j|$, such that $|\theta_j| \le \theta_{Mj}$, $\forall j \in \{1,\dots,p\}$, and let $\theta_M \triangleq \left(\theta_{M1},\dots,\theta_{Mp}\right)^T$ and $\Theta \triangleq [-\theta_{M1},\theta_{M1}] \times \cdots \times [-\theta_{Mp},\theta_{Mp}]$.

a. By Properties 4 and 5, there exist positive constants B^Θ_{gi}, $i = 1,\dots,n$, such that $|g_i(w,z)| = |Y_{gi}(w)z| \le B^\Theta_{gi}$, $i = 1,\dots,n$, $\forall (w,z) \in \mathbb{R}^n \times \Theta$. Furthermore, there exist positive constants $B_{G_{ij}}$, B_{G_i}, and B_G such that $|Y_{gij}(w)| \le B_{G_{ij}}$, $\|Y_{gi}(w)\| \le B_{G_i}$, and $\|Y_g(w)\| \le B_G$, $\forall w \in \mathbb{R}^n$, $i = 1,\dots,n$, $j = 1,\dots,p$.
b. Let $\mathcal{X}$ and $\mathcal{Y}$ be any compact subsets of $\mathbb{R}^n$. By Properties 1, 2.5, 5, and 6a, there exist positive constants B^Θ_{Di}, $i = 1,\dots,n$, such that $|Y_i(w,x,y)z| \le B^\Theta_{Di}$, $i = 1,\dots,n$, $\forall (w,x,y,z) \in \mathbb{R}^n \times \mathcal{X} \times \mathcal{Y} \times \Theta$. Furthermore, there exist positive constants $B_{Y_{ij}}$, B_{Y_i}, and B_Y such that $|Y_{ij}(w,x,y)| \le B_{Y_{ij}}$, $\|Y_i(w,x,y)\| \le B_{Y_i}$, and $\|Y(w,x,y)\| \le B_Y$, $\forall (w,x,y) \in \mathbb{R}^n \times \mathcal{X} \times \mathcal{Y}$, $i = 1,\dots,n$, $j = 1,\dots,p$.

Remark 1
Let us note that under the considerations of Property 6, by Properties 1, 2.5, 3, 5, and 6a, there exist positive constants μ^Θ_M, k^Θ_C, and f^Θ_M, such that $|Y_i(w,x,y)z| \le \mu^\Theta_M\|y\| + k^\Theta_C\|x\|^2 + f^\Theta_M\|x\| + B^\Theta_{gi}$, $i = 1,\dots,n$, $\forall (w,x,y,z) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \Theta$. Observe from this expression that for any $T_i > B^\Theta_{gi}$, there always exist sufficiently small positive values a and b (for instance, such that $\mu^\Theta_M b + k^\Theta_C a^2 + f^\Theta_M a < T_i - B^\Theta_{gi}$) that guarantee $|Y_i(w,x,y)z| < T_i$, $i = 1,\dots,n$, on $\mathbb{R}^n \times \mathcal{B}_a \times \mathcal{B}_b \times \Theta$. $\triangleleft$

Let us suppose that the absolute value of each input τ_i (i^{th} element of the input vector τ) is constrained to be smaller than a given saturation bound $T_i > 0$, *i.e.* $|\tau_i| \le T_i$, $i = 1,\dots,n$. In other words, letting u_i represent the control variable (controller output) relative to the i^{th} degree of freedom, we have that

$$\tau_i = T_i \mathrm{sat}\left(\frac{u_i}{T_i}\right) \tag{2}$$

$i = 1,\dots,n$, where $\mathrm{sat}(\cdot)$ is the standard saturation function, *i.e.* $\mathrm{sat}(\varsigma) = \mathrm{sign}(\varsigma)\min\{|\varsigma|,1\}$.

The control scheme proposed in this work involves special (saturation) functions fitting the following definition.

Definition 1
Given a positive constant M, a nondecreasing Lipschitz-continuous function $\sigma : \mathbb{R} \to \mathbb{R}$ is said to be a ***generalized saturation*** with bound M if

(a) $\varsigma\sigma(\varsigma) > 0$ for all $\varsigma \neq 0$;
(b) $|\sigma(\varsigma)| \le M$ for all $\varsigma \in \mathbb{R}$.

If in addition

(c) $\sigma(\varsigma) = \varsigma$ when $|\varsigma| \le L$,

for some positive constant $L \le M$, σ is said to be a ***linear saturation*** for (L, M) [27].

Any function satisfying Definition 1 has the following properties.

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Lemma 1
Let $\sigma : \mathbb{R} \to \mathbb{R}$ be a generalized saturation function with bound M, and let k be a positive constant. Thus,

1. $\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) = 0$;
2. $D^+\sigma(\varsigma)$ is nonnegative and bounded by a (strictly) positive bound, *i.e.* $\exists \sigma'_M \in (0,\infty)$ such that $0 \leq D^+\sigma(\varsigma) \leq \sigma'_M, \forall \varsigma \in \mathbb{R}$;
3. $|\sigma(k\varsigma + \eta) - \sigma(\eta)| \leq \sigma'_M k|\varsigma|, \forall \varsigma, \eta \in \mathbb{R}$;
4. $|\sigma(k\varsigma)| \leq \sigma'_M k|\varsigma|, \forall \varsigma \in \mathbb{R}$;
5. $\dfrac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \displaystyle\int_0^\varsigma \sigma(kr)dr \leq \dfrac{k\sigma'_M\varsigma^2}{2}, \forall \varsigma \in \mathbb{R}$;
6. $\displaystyle\int_0^\varsigma \sigma(kr)dr > 0, \forall \varsigma \neq 0$;
7. $\displaystyle\int_0^\varsigma \sigma(kr)dr \to \infty$ as $|\varsigma| \to \infty$;
8. if σ is strictly increasing, then
   (a) $\varsigma[\sigma(\varsigma + \eta) - \sigma(\eta)] > 0, \forall \varsigma \neq 0, \forall \eta \in \mathbb{R}$;
   (b) for any constant $a \in \mathbb{R}$, $\bar{\sigma}(\varsigma) = \sigma(\varsigma + a) - \sigma(a)$ is a strictly increasing generalized saturation function with bound $\bar{M} = M + |\sigma(a)|$;
9. if σ is a linear saturation for (L, M) then, for any continuous function $\nu : \mathbb{R} \to \mathbb{R}$ such that $|\nu(\eta)| < L, \forall \eta \in \mathbb{R}$, we have that $\varsigma\big[\sigma\big(\varsigma + \nu(\eta)\big) - \sigma\big(\nu(\eta)\big)\big] > 0, \forall \varsigma \neq 0, \forall \eta \in \mathbb{R}$.

Proof
Points 3 and 4 are a direct consequence of the Lipschitz-continuity of σ and item 2 of the statement (as analogously stated for instance in [22, Lemma 3.3] under continuous differentiability). The rest of the points are proved in [17]. □

The following assumptions are crucial within the analytical setting considered in this work.

Assumption 1
$T_i > B_{gi}, \forall i \in \{1, \ldots, n\}$.

Assumption 2
The desired trajectory $q_d(t)$ (to be tracked) belongs to $\mathcal{Q}_d \triangleq \big\{q_d \in \mathcal{C}^2(\mathbb{R}_+; \mathbb{R}^n) : \|\dot{q}_d(t)\| \leq B_{dv}\,, \|\ddot{q}_d(t)\| \leq B_{da}\,, \forall t \geq 0\big\}$ for some positive constants B_{da} and $B_{dv} < f_m/k_C$ (see Properties 2.5 and 3).

Remark 2
Assumption 1 implies that the actuators and mechanical (power) transmission systems of the robot are capable to hold its physical structure at any point in the configuration space which is reasonably expectable in practice. On the other hand, observe that Assumption 2 does not restrict the location of the target trajectory q_d but rather its first- and second-order change rates. Hence, under Assumption 2, desired trajectories defined anywhere in the configuration space may be tracked as long as they give rise to sufficiently slow motions. Let us further point out that the need to restrict the target trajectories is a direct consequence of the bounded nature of the inputs. Indeed observe, from Eqs. (1) and (2) under the consideration of accurate tracking, *i.e.* $q(t) \equiv q_d(t)$, that only desired trajectories giving rise to left-hand sides of (1) with elements having absolute values lower than the input bounds T_i can be tracked through a control vector u subject to (2). This leads to the need

of additional adjustments on the desired trajectory first- and second-order rate bounds, B_{dv} and B_{da}. Specifications are given in Section 4 in the context of the approach formulated in this work. Furthermore, the closed-loop stability analysis gives rise to the additional restriction on B_{dv} stated through Assumption 2, namely $B_{dv} < f_m/k_C$.[‡] This additional condition adopts coherence through the consideration of the viscous friction terms in the manipulator open-loop dynamics. Certainly, an approach independent of the consideration of friction may be seen as a theoretically stronger result. Nevertheless, it is important to keep in mind that even in such a scenario the desired motion ought to be restricted in order to cope with the input constraints (as previously described). On the other hand, the consideration of viscous friction is meaningful in practice since it is an ever-present phenomenon in mechanical systems [28]. Notwithstanding the dependence on the friction terms, the result developed in this work overcomes limitations of previous bounded adaptive approaches as pointed out in Section 1. ◁

3. GLOBAL TRACKING INVOLVING EXACT SYSTEM PARAMETERS: A GENERALIZED APPROACH

Under the satisfaction of Assumptions 1 and 2, let us consider the following *generalized* expression defining saturating global-tracking controllers for system (1)-(2):

$$u(t,q,\dot{q},\theta) = -s_d(t,\bar{q},\dot{\bar{q}},\theta) - s_P(K_P\bar{q}) + Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\theta \tag{3}$$

where $\bar{q} = q - q_d(t)$ is the position error vector variable. The third term in the right-hand side of (3) is a *hybrid* compensation term (since it involves online position measurements but desired velocities and accelerations), with θ being the system parameter vector and $Y(\cdot,\cdot,\cdot)$ the regression matrix characterizing the system open-loop structure, according to Property 5, *i.e.* such that

$$Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\theta = H(q,\theta)\ddot{q}_d(t) + C\big(q,\dot{q}_d(t),\theta\big)\dot{q}_d(t) + F(\theta)\dot{q}_d(t) + g(q,\theta) \tag{4}$$

The second term in the right-hand side of (3) is a (bounded non-linear) position error correction term where $K_P \in \mathbb{R}^{n\times n}$ is a positive definite diagonal matrix, *i.e.* $K_P = \mathrm{diag}[k_{P1},\ldots,k_{Pn}]$ with $k_{Pi} > 0$ for all $i = 1,\ldots,n$, and, for any $x \in \mathbb{R}^n$, $s_P(x) = \big(\sigma_{P1}(x_1),\ldots,\sigma_{Pn}(x_n)\big)^T$, with $\sigma_{Pi}(\cdot)$, $i = 1,\ldots,n$, being (suitable) **continuously differentiable generalized saturation functions** with bounds M_{Pi}. The first term in the right-hand side of (3) is a *damping* term —whose role is to furnish a velocity-error opposing force through which the corresponding kinetic energy (in the error variable space) be dissipated— where $s_d : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$ is a continuous vector function satisfying

$$s_d(t,x,0_n,z) = 0_n \tag{5}$$

$\forall x \in \mathbb{R}^n$, $\forall z \in \mathbb{R}^p$, $\forall t \geq 0$,

$$\|s_d(t,x,y,z)\| \leq \kappa\|y\| \tag{6}$$

$\forall (t,x,y,z) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, for some positive constant κ, and given $q_d \in \mathcal{Q}_d$ (see Assumption 2) and $z \in \mathbb{R}^p$ such that $\big|Y_i\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)z\big| < T_i$, $i = 1,\ldots,n$, $\forall q \in \mathbb{R}^n$, $\forall t \geq 0$:

$$y^T s_d(t,x,y,z) > 0 \tag{7}$$

$\forall y \neq 0_n$, $\forall x \in \mathbb{R}^n$, $\forall t \geq 0$, and

$$\big|u_i(t,x,y,z)\big| < T_i \tag{8}$$

$i = 1,\ldots,n$, $\forall x \in \mathbb{R}^n$, $\forall y \in \mathbb{R}^n$, $\forall t \geq 0$, for suitable bounds M_{Pi} of $\sigma_{Pi}(\cdot)$.

[‡]Notice that f_m could be small but the quotient f_m/k_C may be several times greater, as is actually the experimental case under consideration in Section 6.

Proposition 1
Consider system (1)-(2) taking $u = u(t, q, \dot{q}, \theta)$ as defined in Eq. (3), under the satisfaction of Assumptions 1 and 2 and the conditions on the vector function s_d stated through the expressions in (5)–(8). Thus, for any positive definite diagonal matrix K_P, global uniform asymptotic stability of the closed loop trivial solution $\bar{q}(t) \equiv 0_n$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$.

Proof
Observe that the satisfaction of (8), under the consideration of (2), shows that $T_i > \big|u_i(t, q, \dot{q}, \theta)\big| = |u_i| = |\tau_i|$, $i = 1, \ldots, n$, $\forall q \in \mathbb{R}^n$, $\forall \dot{q} \in \mathbb{R}^n$, $\forall t \geq 0$. From this expression, one sees that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$. This proves that under the proposed scheme, the input saturation values, T_i, are never reached. Thus, the closed-loop dynamics takes the form

$$H(q)\ddot{\bar{q}} + \big[C(q, \dot{q}) + C\big(q, \dot{q}_d(t)\big)\big]\dot{\bar{q}} + F\dot{\bar{q}} = -s_d(t, \bar{q}, \dot{\bar{q}}, \theta) - s_P(K_P\bar{q}) \tag{9}$$

where[§] Property 2.4 has been used. Let us define the scalar function

$$V_0(t, \bar{q}, \dot{\bar{q}}) = \frac{1}{2}\dot{\bar{q}}^T H(q)\dot{\bar{q}} + \varepsilon s_P^T(K_P\bar{q})H(q)\dot{\bar{q}} + \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr \tag{10}$$

with $\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i$ and ε being a positive constant satisfying

$$\varepsilon < \varepsilon_M \triangleq \min\{\varepsilon_1, \varepsilon_2\} \tag{11}$$

where

$$\varepsilon_1 \triangleq \sqrt{\frac{\mu_m}{\mu_M^2 \beta_P}} \qquad \text{and} \qquad \varepsilon_2 \triangleq \frac{f_m - k_C B_{dv}}{\beta_M + \left(\frac{f_M + \kappa}{2} + k_C B_{dv}\right)^2}$$

(note that the satisfaction of Assumption 2 ensures positivity of ε_2) with

$$\beta_P \triangleq \max_i\{\sigma'_{PiM} k_{Pi}\} \quad , \quad \beta_M \triangleq k_C B_P + \mu_M \beta_P \quad , \quad B_P \triangleq \sqrt{\sum_{i=0}^n M_{Pi}^2}$$

σ'_{PiM} being the positive bound of $\sigma'_{Pi}(\cdot)$ in accordance to item 2 of Lemma 1, κ as defined through (6), and μ_m, μ_M, k_C, f_m, and f_M as defined through Properties 1–3. Observe that from Property 1 and items 4 and 5 of Lemma 1 we have that

$$W_{01}(\bar{q}, \dot{\bar{q}}) \leq V_0(t, \bar{q}, \dot{\bar{q}}) \leq W_{02}(\bar{q}, \dot{q})$$

where

$$W_{01}(\bar{q}, \dot{\bar{q}}) = W_{00}(\bar{q}, \dot{\bar{q}}) + (1 - \alpha)\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr \tag{12}$$

with

$$W_{00}(\bar{q}, \dot{\bar{q}}) = \frac{\mu_m}{2}\|\dot{\bar{q}}\|^2 - \varepsilon\mu_M\|s_P(K_P\bar{q})\|\|\dot{\bar{q}}\| + \frac{\alpha}{2\beta_P}\|s_P(K_P\bar{q})\|^2$$

and α being a positive constant satisfying

$$\frac{\varepsilon^2}{\varepsilon_1^2} < \alpha < 1 \tag{13}$$

[§]Observe that, in the error variable space, $q = \bar{q} + q_d(t)$ and $\dot{q} = \dot{\bar{q}} + \dot{q}_d(t)$, and consequently $H(q) = H\big(\bar{q} + q_d(t)\big)$, $C(q, \cdot) = C\big(\bar{q} + q_d(t), \cdot\big)$, $C(\cdot, \dot{q}) = C\big(\cdot, \dot{\bar{q}} + \dot{q}_d(t)\big)$, $\dot{H}(q, \dot{q}) = \dot{H}\big(\bar{q} + q_d(t), \dot{\bar{q}} + \dot{q}_d(t)\big)$ and $Y(q, \cdot, \cdot) = Y(\bar{q} + q_d(t), \cdot, \cdot)$. However, for the sake of simplicity, $H(q)$, $C(q, \cdot)$, $C(\cdot, \dot{q})$, $\dot{H}(q, \dot{q})$ and $Y(q, \cdot, \cdot)$ will be used instead of $H\big(\bar{q} + q_d(t)\big)$, $C\big(\bar{q} + q_d(t), \cdot\big)$, $C\big(\cdot, \dot{\bar{q}} + \dot{q}_d(t)\big)$, $\dot{H}\big(\bar{q} + q_d(t), \dot{\bar{q}} + \dot{q}_d(t)\big)$ and $Y(\bar{q} + q_d(t), \cdot, \cdot)$.

(see (11)), and

$$W_{02}(\bar{q},\dot{\bar{q}}) = \frac{\mu_M}{2}\|\dot{\bar{q}}\|^2 + \varepsilon\mu_M\beta_P\|\bar{q}\|\|\dot{\bar{q}}\| + \frac{\beta_P}{2}\|\bar{q}\|^2$$

Moreover, $W_{00}(\bar{q},\dot{\bar{q}})$ and $W_{02}(\bar{q},\dot{\bar{q}})$ may be rewritten as

$$W_{00}(\bar{q},\dot{\bar{q}}) = \frac{1}{2}\begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T P_{01}\begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}$$

$$W_{02}(\bar{q},\dot{\bar{q}}) = \frac{1}{2}\begin{pmatrix}\|\bar{q}\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T P_{02}\begin{pmatrix}\|\bar{q}\| \\ \|\dot{\bar{q}}\|\end{pmatrix}$$

with

$$P_{01} = \begin{pmatrix}\frac{\alpha}{\beta_P} & -\varepsilon\mu_M \\ -\varepsilon\mu_M & \mu_m\end{pmatrix} \quad \text{and} \quad P_{02} = \begin{pmatrix}\beta_P & \varepsilon\mu_M\beta_P \\ \varepsilon\mu_M\beta_P & \mu_M\end{pmatrix}$$

Note that, by (11), $W_{00}(\bar{q},\dot{\bar{q}})$ and $W_{02}(\bar{q},\dot{\bar{q}})$ are positive definite (since, with $\varepsilon < \varepsilon_M \leq \varepsilon_1$, any α satisfying (13) renders P_{01} positive definite, while the referred condition on ε renders P_{02} positive definite as well), and observe that $W_{00}(0_n,\dot{\bar{q}}) \to \infty$ as $\|\dot{\bar{q}}\| \to \infty$. From this, inequality (13) (whence $1-\alpha > 0$), and points 6 and 7 of Lemma 1 (through which one sees that the integral term in the right-hand side of (12) is a radially unbounded positive definite function of $\bar{q}$), $V_0(t,\bar{q},\dot{\bar{q}})$ is concluded to be positive definite, radially unbounded, and decrescent. Its derivative along the system trajectories is given by

$$\begin{aligned}
\dot{V}_0(t,\bar{q},\dot{\bar{q}}) = {} & \dot{\bar{q}}^T H(q)\ddot{\bar{q}} + \frac{1}{2}\dot{\bar{q}}^T \dot{H}(q,\dot{q})\dot{\bar{q}} + \varepsilon s_P^T(K_P\bar{q})H(q)\ddot{\bar{q}} + \varepsilon s_P^T(K_P\bar{q})\dot{H}(q,\dot{q})\dot{\bar{q}} \\
& + \varepsilon\dot{\bar{q}}^T K_P s'_P(K_P\bar{q})H(q)\dot{\bar{q}} + s_P^T(K_P\bar{q})\dot{\bar{q}} \\
= {} & \dot{\bar{q}}^T\big[-C(q,\dot{q})\dot{\bar{q}} - C\big(q,\dot{q}_d(t)\big)\dot{\bar{q}} - F\dot{\bar{q}} - s_d(t,\bar{q},\dot{\bar{q}},\theta) - s_P(K_P\bar{q})\big] + \frac{1}{2}\dot{\bar{q}}^T\dot{H}(q,\dot{q})\dot{\bar{q}} \\
& + \varepsilon s_P^T(K_P\bar{q})\big[-C(q,\dot{q})\dot{\bar{q}} - C\big(q,\dot{q}_d(t)\big)\dot{\bar{q}} - F\dot{\bar{q}} - s_d(t,\bar{q},\dot{\bar{q}},\theta) - s_P(K_P\bar{q})\big] \\
& + \varepsilon s_P^T(K_P\bar{q})\dot{H}(q,\dot{q})\dot{\bar{q}} + \varepsilon\dot{\bar{q}}^T K_P s'_P(K_P\bar{q})H(q)\dot{\bar{q}} + s_P^T(K_P\bar{q})\dot{\bar{q}} \\
= {} & -\dot{\bar{q}}^T C\big(q,\dot{q}_d(t)\big)\dot{\bar{q}} - \dot{\bar{q}}^T F\dot{\bar{q}} - \dot{\bar{q}}^T s_d(t,\bar{q},\dot{\bar{q}},\theta) - \varepsilon s_P^T(K_P\bar{q})C\big(q,\dot{q}_d(t)\big)\dot{\bar{q}} \\
& - \varepsilon s_P^T(K_P\bar{q})F\dot{\bar{q}} - \varepsilon s_P^T(K_P\bar{q})s_d(t,\bar{q},\dot{\bar{q}},\theta) - \varepsilon s_P^T(K_P\bar{q})s_P(K_P\bar{q}) \\
& + \varepsilon\dot{\bar{q}}^T\big[C(q,\dot{\bar{q}}) + C\big(q,\dot{q}_d(t)\big)\big]s_P(K_P\bar{q}) + \varepsilon\dot{\bar{q}}^T K_P s'_P(K_P\bar{q})H(q)\dot{\bar{q}}
\end{aligned}$$

where $H(q)\ddot{\bar{q}}$ has been replaced by its equivalent expression from the closed-loop dynamics in (9), Properties 2.1–2.3 have been used, and

$$s'_P(K_P\bar{q}) \triangleq \mathrm{diag}[\sigma'_{P1}(k_{P1}\bar{q}_1),\ldots,\sigma'_{Pn}(k_{Pn}\bar{q}_n)] \tag{14}$$

Observe that from Assumption 2, Properties 1–3, the satisfaction of (6) and (7), items (b) of Definition 1 and 2 of Lemma 1 (recall that for continuously differentiable functions σ_{Pi}, $D^+\sigma_{Pi} = \sigma'_{Pi}$), and the properties of K_P, we have that

$$\dot{V}_0(t,\bar{q},\dot{\bar{q}}) \leq -W_1(\bar{q},\dot{\bar{q}})$$

with

$$\begin{aligned}
W_1(\bar{q},\dot{\bar{q}}) = {} & -k_C B_{dv}\|\dot{\bar{q}}\|^2 + f_m\|\dot{\bar{q}}\|^2 - \varepsilon k_C B_{dv}\|s_P(K_P\bar{q})\|\|\dot{\bar{q}}\| - \varepsilon f_M\|s_P(K_P\bar{q})\|\|\dot{\bar{q}}\| \\
& - \varepsilon\kappa\|s_P(K_P\bar{q})\|\|\dot{\bar{q}}\| + \varepsilon\|s_P(K_P\bar{q})\|^2 - \varepsilon k_C B_P\|\dot{\bar{q}}\|^2 - \varepsilon k_C B_{dv}\|s_P(K_P\bar{q})\|\|\dot{\bar{q}}\| \\
& - \varepsilon\mu_M\beta_P\|\dot{\bar{q}}\|^2 \\
= {} & \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T \begin{pmatrix}\varepsilon & -\varepsilon\left(\frac{f_M+\kappa}{2} + k_C B_{dv}\right) \\ -\varepsilon\left(\frac{f_M+\kappa}{2} + k_C B_{dv}\right) & f_m - k_C B_{dv} - \varepsilon\beta_M\end{pmatrix}\begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}
\end{aligned} \tag{15}$$

Note further that, from the satisfaction of (11), $W_1(\bar{q}, \dot{q})$ is positive definite (since any $\varepsilon < \varepsilon_M \leq \varepsilon_2$ renders the matrix at the right-hand side of (15) positive definite). Thus, by Lyapunov's stability theory (applied to non-autonomous systems, see for instance [22, Theorem 4.9]), the trivial solution $\bar{q}(t) \equiv 0$ is concluded to be globally uniformly asymptotically stable, which completes the proof. □

The generalized formulation presented here gives rise to particular control structures from previous references. Details in this direction are given in Appendix A.

4. THE PROPOSED ADAPTIVE SCHEME

The result of the precedent section cannot be guaranteed as stated in Proposition 1 if the exact knowledge of the system parameters is not available. However, in such a situation, global tracking avoiding input saturation can still be accomplished through auxiliary dynamics in an adaptive control context. This is achieved by means of suitable strict bounds on the elements of θ, as described next.

Let $M_a \triangleq \big(M_{a1}, \ldots, M_{ap}\big)^T$, and $\Theta_a \triangleq [-M_{a1}, M_{a1}] \times \cdots \times [-M_{ap}, M_{ap}]$, with M_{aj}, $j = 1, \ldots, p$, being positive constants such that

$$|\theta_j| < M_{aj} \tag{16a}$$

$\forall j \in \{1, \ldots, p\}$, and

$$B_{gi}^{\Theta_a} < T_i \tag{16b}$$

$\forall i \in \{1, \ldots, n\}$, where, in accordance to Property 6a, $B_{gi}^{\Theta_a}$, $i = 1, \ldots, n$, are positive constants such that $|g_i(w, z)| = |Y_{gi}(w)z| \leq B_{gi}^{\Theta_a}$, $i = 1, \ldots, n$, $\forall (w, z) \in \mathbb{R}^n \times \Theta_a$, and consider (small enough) desired-trajectory-related bound values B_{dv} and B_{da} (in accordance to Assumption 2) such that

$$\big|Y_i\big(q, \dot{q}_d(t), \ddot{q}_d(t)\big)\vartheta\big| \leq B_{Di}^{\Theta_a} < T_i \tag{16c}$$

$i = 1, \ldots, n$, $\forall q \in \mathbb{R}^n$, $\forall \vartheta \in \Theta_a$, $\forall t \geq 0$, where, in accordance to Property 6b, $B_{Di}^{\Theta_a}$, $i = 1, \ldots, n$, are positive constants such that $|Y_i(w, x, y)z| \leq B_{Di}^{\Theta_a}$, $i = 1, \ldots, n$, $\forall (w, x, y, z) \in \mathbb{R}^n \times \mathcal{B}_{B_{dv}} \times \mathcal{B}_{B_{da}} \times \Theta_a$. Let us note that Assumption 1 ensures the existence of such positive values M_{aj}, $j = 1, \ldots, p$, satisfying inequalities (16a) and (16b) while, under Assumption 2, through the fulfillment of (16b), inequalities (16c) can always be satisfied through sufficiently small values of B_{dv} and B_{da} (see Remark 1). Notice further that inequalities (16b) are satisfied if $\sum_{j=1}^{p} B_{G_{ij}} M_{aj} < T_i$, $B_{G_i}\|M_a\| < T_i$, or $B_G\|M_a\| < T_i$, $i = 1, \ldots, n$ (see Property 6a); actually, $\sum_{j=1}^{p} B_{G_{ij}} M_{aj}$, $B_{G_i}\|M_a\|$, or $B_G\|M_a\|$, may be taken as the value of $B_{gi}^{\Theta_a}$ as long as inequality (16b) is fulfilled. Similarly, inequalities (16c) are satisfied if $\sum_{j=1}^{p} B_{Y_{ij}} M_{aj} < T_i$, $B_{Y_i}\|M_a\| < T_i$, or $B_Y\|M_a\| < T_i$, $i = 1, \ldots, n$, where, in accordance to Property 6b and Assumption 2, $B_{Y_{ij}}$, B_{Y_i}, and B_Y are positive constants such that $|Y_{ij}(w, x, y)| \leq B_{Y_{ij}}$, $\|Y_i(w, x, y)\| \leq B_{Y_i}$, and $\|Y(w, x, y)\| \leq B_Y$, respectively, $\forall (w, x, y) \in \mathbb{R}^n \times \mathcal{B}_{B_{dv}} \times \mathcal{B}_{B_{da}}$; in fact, $\sum_{j=1}^{p} B_{Y_{ij}} M_{aj}$, $B_{Y_i}\|M_a\|$, or $B_Y\|M_a\|$, may be taken as the value of $B_{Di}^{\Theta_a}$ as long as inequality (16c) is fulfilled.

Based on the generalized algorithm in Eq. (3), the proposed adaptive control scheme is defined as

$$u(t, q, \dot{q}, \hat{\theta}) = -s_d(t, \bar{q}, \dot{\bar{q}}, \hat{\theta}) - s_P(K_P\bar{q}) + Y\big(q, \dot{q}_d(t), \ddot{q}_d(t)\big)\hat{\theta} \tag{17}$$

with $s_P(\cdot)$, K_P, and $s_d(\cdot,\cdot,\cdot,\cdot)$ as previously defined, $Y\big(q, \dot{q}_d(t), \ddot{q}_d(t)\big)$ as described in the precedent section (see Eq. (4)), and $\hat{\theta}$ (the parameter estimator) being the output variable of an auxiliary (adaptation) dynamic subsystem defined as

$$\dot{\phi} = -\Gamma Y^T\big(q, \dot{q}_d(t), \ddot{q}_d(t)\big)\big[\dot{\bar{q}} + \varepsilon s_P(K_P\bar{q})\big] \tag{18a}$$

$$\hat{\theta} = s_a(\phi) \tag{18b}$$

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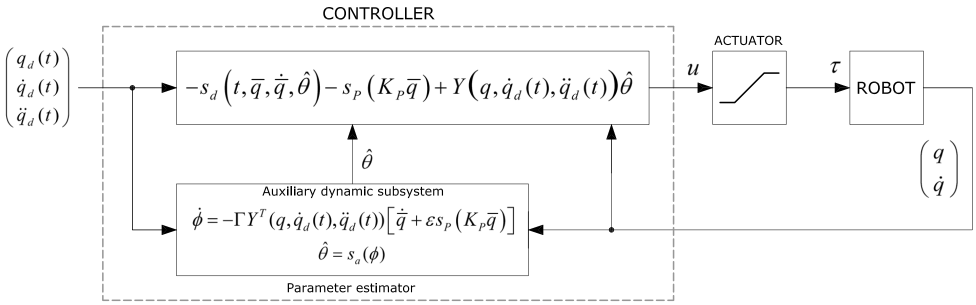

Figure 1. Block diagram of the proposed adaptive control scheme

where ϕ is the (internal) state of the auxiliary dynamics in Eq. (18a); for any $x \in \mathbb{R}^p$, $s_a(x) = \left(\sigma_{a1}(x_1)\ ,\ \ldots\ ,\ \sigma_{ap}(x_p)\right)^T$, $\sigma_{aj}(\cdot)$, $j = 1,\ldots,p$, being **strictly increasing generalized saturation functions** with bounds M_{aj} as defined above, *i.e.* such that inequalities (16) are satisfied; $\Gamma \in \mathbb{R}^{p\times p}$ is a positive definite diagonal constant matrix, *i.e.* $\Gamma = \text{diag}[\gamma_1 \ldots, \gamma_p]$ with $\gamma_j > 0$ for all $j = 1,\ldots,p$; and ε is a positive constant satisfying inequality (11). A block diagram of the proposed adaptive scheme is shown in Fig. 1.

Remark 3
Observe that the control scheme in (17)-(18) does not involve the exact values of the elements of θ. It only requires the satisfaction of inequalities (16). In other words, only strict bounds M_{aj} of $|\theta_j|$, $j = 1,\ldots,p$, (satisfying inequalities (16b)-(16c)) are involved. Notice further that a suitable choice of ε does not require the exact knowledge of the system parameters either. Indeed observe, on the one hand, that an estimation of the right-hand side of inequality (11) may be obtained by means of upper and lower bounds of the system parameters and viscous friction coefficients (more precisely, nonzero lower bounds of μ_m and f_m, and upper bounds of μ_M, k_C, and f_M; see Properties 1, 2.5, and 3). On the other hand, the satisfaction of inequality (11) is not necessary but only sufficient for the closed-loop analysis to hold, as shown in the following section, which permits the consideration of values of ε higher than ε_M (up to certain limit) without destabilizing the closed loop. Note further that, by previous arguments, the satisfaction of the restriction on B_{dv} stated through Assumption 2 does not require the exact knowledge of the system parameters either. ◁

Remark 4
Observe that inequalities (16b) concern exclusively the parameters related to the gravity force vector. For these parameters, it is important to count on suitable bound values (satisfying inequality (16b)) for a proper implementation of the proposed algorithm with analytical certainty of the expected result (under the stated assumptions). For the rest of the system parameters, the restriction on their bounds, stated through inequality (16c), is not as stringent since the fulfillment of such a condition may be accomplished through suitable desired-trajectory-related bound values B_{dv} and B_{da} (recall Remark 1). Thus (under the fulfillment of (16b)), given any $q_d \in \mathcal{Q}_d$, it suffices to adjust its first- and second-order change rates (wherever q_d goes through in the configuration space) to satisfy inequality (16c). For system parameter bounds that do not satisfy (16b), local tracking is still possible for desired trajectories restricted to vary on some configuration space subset $\Omega \subset \mathbb{R}^n$ where $|g_i(x,y)| \leq B_{gi}^{\Omega} < T_i$, $i = 1,\ldots,n$, $\forall(x,y) \in \Omega \times \Theta_a$. Given $q_d \in \mathcal{Q}_d$ such that $q_d(t) \in \Omega$, $\forall t \geq 0$, (under the satisfaction of (16c)) initial conditions sufficiently close to the target values give rise to a closed-loop position trajectory that is kept evolving in Ω —in view of its proximity to $q_d(t)$ implied by the uniform stability property proven in Section 5— thus retrieving the conditions required to ensure a correct functioning of the proposed scheme. ◁

5. CLOSED-LOOP ANALYSIS

Consider system (1)-(2) taking $u = u(t, \bar{q}, \dot{\bar{q}}, \hat{\theta})$ as defined through Eqs. (17)-(18). Observe that (under Assumptions 1 and 2, the satisfaction of inequalities (16), and the consideration of (2)) the fulfilment of (8) shows that

$$T_i > \big|u_i\big(t, q, \dot{q}, s_a(\phi)\big)\big| = |u_i| = |\tau_i| \quad i = 1, \ldots, n \quad \forall (t, q, \dot{q}, \phi) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p \tag{19}$$

Thus, under the consideration of Property 5, the closed-loop system takes the form

$$H(q)\ddot{\bar{q}} + \big[C(q, \dot{q}) + C\big(q, q_d(t)\big)\big]\dot{\bar{q}} + F\dot{\bar{q}} \\ = -s_d\big(t, \bar{q}, \dot{\bar{q}}, s_a(\phi)\big) - s_P(K_P\bar{q}) + Y\big(q, \dot{q}_d(t), \ddot{q}_d(t)\big)\bar{s}_a(\bar{\phi}) \tag{20a}$$

$$\dot{\bar{\phi}} = -\Gamma Y^T\big(q, \dot{q}_d(t), \ddot{q}_d(t)\big)\left[\dot{\bar{q}} + \varepsilon s_P(K_P\bar{q})\right] \tag{20b}$$

where $\bar{\phi} = \phi - \phi^*$ and

$$\bar{s}_a(\bar{\phi}) = s_a(\bar{\phi} + \phi^*) - s_a(\phi^*) \tag{21}$$

with $\phi^* = \big(\phi_1^*, \ldots, \phi_p^*\big)^T$ such that $s_a(\phi^*) = \theta$, or equivalently, $\phi_j^* = \sigma_{aj}^{-1}(\theta_j)$, $j = 1, \ldots, p$.¶ Observe that, by item 8b of Lemma 1, the elements of $\bar{s}_a(\bar{\phi})$ in (21), *i.e.*

$$\bar{\sigma}_{aj}(\bar{\phi}_j) = \sigma_{aj}(\bar{\phi}_j + \phi_j^*) - \sigma_{aj}(\phi_j^*)$$

$j = 1, \ldots, p$, turn out to be strictly increasing generalized saturation functions.

Proposition 2
Consider the closed-loop system in Eqs. (20), under the satisfaction of Assumptions 1 and 2 and the conditions on the vector function s_d stated through the expressions in (5)–(8). Thus, for any positive definite diagonal matrices K_P and Γ, and any ε satisfying (11), the trivial solution $(\bar{q}, \bar{\phi})(t) \equiv (0_n, 0_p)$ is uniformly stable and, for any initial condition $\big(t_0, \bar{q}(t_0), \dot{\bar{q}}(t_0), \bar{\phi}(t_0)\big) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, the closed-loop system solution $(\bar{q}, \bar{\phi})(t)$ is bounded and such that $\bar{q}(t) \to 0_n$ as $t \to \infty$ with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq t_0$.

Proof
By (19), we see that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $\forall t \geq 0$. This proves that, under the proposed adaptive scheme, input saturation is avoided. Now, in order to develop the stability/convergence analysis, let us define the scalar function

$$V_1(t, \bar{q}, \dot{\bar{q}}, \bar{\phi}) = V_0(t, \bar{q}, \dot{\bar{q}}) + \int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr \tag{22}$$

where $\int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr = \sum_{j=1}^{p} \int_0^{\bar{\phi}_j} \bar{\sigma}_{aj}(r_j)\gamma_j^{-1}dr_j$, and $V_0(t, \bar{q}, \dot{\bar{q}})$ is as defined in Eq. (10).‖ Note that, from the analytical properties of $V_0(t, \bar{q}, \dot{\bar{q}})$, shown in the proof of Proposition 1, and items 8b, 6, and 7 of Lemma 1 (through which the integral term in the right-hand side of Eq. (22) is concluded to be a radially unbounded positive definite decrescent function of $\bar{\phi}$), $V_1(t, \bar{q}, \dot{\bar{q}}, \bar{\phi})$ proves to be positive definite, radially unbounded, and decrescent. Its derivative along the system trajectories is

¶Notice that their strictly increasing character renders the generalized saturation functions σ_{aj}, $j = 1, \ldots, p$, (involved in the definition of s_a) invertible.

‖The complete expression is given as

$$V_1(t, \bar{q}, \dot{\bar{q}}, \bar{\phi}) = \frac{1}{2}\dot{\bar{q}}^T H(q)\dot{\bar{q}} + \varepsilon s_P^T(K_P\bar{q})H(q)\dot{\bar{q}} + \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + \int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr$$

given by

$$\begin{aligned}
\dot{V}_1(t,\bar{q},\dot{\bar{q}},\bar{\phi}) = {} & \dot{\bar{q}}^T H(q)\ddot{\bar{q}} + \frac{1}{2}\dot{\bar{q}}^T \dot{H}(q,\dot{q})\dot{\bar{q}} + \varepsilon s_P^T(K_P\bar{q})H(q)\ddot{\bar{q}} + \varepsilon\dot{\bar{q}}^T\dot{H}(q,\dot{q})s_P(K_P\bar{q}) \\
& + \varepsilon\dot{\bar{q}}^T H(q)s_P'(K_P\bar{q})K_P\dot{\bar{q}} + s_P^T(K_P\bar{q})\dot{\bar{q}} + \bar{s}_a^T(\bar{\phi})\Gamma^{-1}\dot{\bar{\phi}} \\
= {} & \dot{\bar{q}}^T\Big[-C(q,\dot{q})\dot{\bar{q}} - C\big(q,\dot{q}_d(t)\big)\dot{\bar{q}} - F\dot{\bar{q}} - s_d\big(t,\bar{q},\dot{\bar{q}},s_a(\phi)\big) - s_P(K_P\bar{q}) \\
& + Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\bar{s}_a(\bar{\phi})\Big] + \frac{1}{2}\dot{\bar{q}}^T\dot{H}(q,\dot{q})\dot{\bar{q}} + \varepsilon s_P^T(K_P\bar{q})\Big[-C(q,\dot{q})\dot{\bar{q}} \\
& - C\big(q,\dot{q}_d(t)\big)\dot{\bar{q}} - F\dot{\bar{q}} - s_d\big(t,\bar{q},\dot{\bar{q}},s_a(\phi)\big) - s_P(K_P\bar{q}) + Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\bar{s}_a(\bar{\phi})\Big] \\
& + \varepsilon\dot{\bar{q}}^T\dot{H}(q,\dot{q})s_P(K_P\bar{q}) + \varepsilon\dot{\bar{q}}^T H(q)s_P'(K_P\bar{q})K_P\dot{\bar{q}} + s_P^T(K_P\bar{q})\dot{\bar{q}} \\
& - \bar{s}_a^T(\bar{\phi})Y^T\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\left[\dot{\bar{q}} + \varepsilon s_P(K_P\bar{q})\right] \\
= {} & -\dot{\bar{q}}^T C\big(q,\dot{q}_d(t)\big)\dot{\bar{q}} - \dot{\bar{q}}^T F\dot{\bar{q}} - \dot{\bar{q}}^T s_d\big(t,\bar{q},\dot{\bar{q}},s_a(\phi)\big) - \varepsilon s_P^T(K_P\bar{q})C\big(q,\dot{q}_d(t)\big)\dot{\bar{q}} \\
& - \varepsilon s_P^T(K_P\bar{q})F\dot{\bar{q}} - \varepsilon s_P^T(K_P\bar{q})s_d\big(t,\bar{q},\dot{\bar{q}},s_a(\phi)\big) - \varepsilon s_P^T(K_P\bar{q})s_P(K_P\bar{q}) \\
& + \varepsilon\dot{\bar{q}}^T\left[C(q,\dot{\bar{q}}) + C\big(q,\dot{q}_d(t)\big)\right]s_P(K_P\bar{q}) + \varepsilon\dot{\bar{q}}^T H(q)s_P'(K_P\bar{q})K_P\dot{\bar{q}}
\end{aligned}$$

where $H(q)\ddot{\bar{q}}$ and $\dot{\bar{\phi}}$ have been replaced by their equivalent expression from the closed-loop manipulator dynamics in Eqs. (20), Properties 2.1–2.3 have been used, and $s_P'(K_P\bar{q})$ was defined in (14). Observe that from Assumption 2, Properties 1–3, the satisfaction of (6) and (7), items (b) of Definition 1 and 2 of Lemma 1, and the properties of K_P, we have that

$$\dot{V}_1(t,\bar{q},\dot{\bar{q}},\bar{\phi}) \leq -\dot{\bar{q}}^T s_d\big(t,\bar{q},\dot{\bar{q}},s_a(\phi)\big) - W_1(\bar{q},\dot{\bar{q}}) \leq -W_1(\bar{q},\dot{\bar{q}})$$

where $W_1(\bar{q},\dot{\bar{q}})$ was defined in (15) and shown to be a positive definite function in the proof of Proposition 1. Hence, we have that $\dot{V}_1(t,\bar{q},\dot{\bar{q}},\bar{\phi}) \leq 0$, $\forall (t,\bar{q},\dot{\bar{q}},\bar{\phi}) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, with $\dot{V}_1(t,\bar{q},\dot{\bar{q}},\bar{\phi}) = 0 \iff (\bar{q},\dot{\bar{q}}) = (0_n,0_n)$. Therefore, by Lyapunov stability theory (applied to nonautonomous systems, see for instance [22, Theorem 4.8]), the trivial solution $(\bar{q},\bar{\phi})(t) \equiv (0_n,0_p)$ is concluded to be uniformly stable. Finally, by Theorem 8.4 of [22],** we conclude that for any initial condition $\big(t_0,\bar{q}(t_0),\dot{\bar{q}}(t_0),\bar{\phi}(t_0)\big) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$, the closed-loop system solution $(\bar{q},\bar{\phi})(t)$ is bounded and such that $\bar{q}(t) \to 0_n$ as $t \to \infty$. □

Remark 5

Let $Y_d(t) \triangleq Y\big(q_d(t),\dot{q}_d(t),\ddot{q}_d(t)\big)$. Under the considerations of Proposition 2, if there exist positive constants ν and δ such that

$$\int_t^{t+\delta} Y_d^T(\varsigma)Y_d(\varsigma)d\varsigma \geq \nu I_n \qquad \forall t \geq 0 \tag{23}$$

then the trivial solution $(\bar{q},\bar{\phi})(t) \equiv (0_n,0_p)$ is globally uniformly asymptotically stable. This follows from the application to the closed-loop system in Eqs. (20) of an analysis analog to that developed in [15] in the unconstrained input context, in turn supported by the more general result presented in [29]. When (23) is fulfilled, it is said that $Y_d^T(t)$ is *persistently exciting*. This is a necessary and sufficient condition for *uniform δ-persistent excitation* of $Y_d(t)\bar{s}_a(\bar{\phi})$ with respect to $\bar{s}_a(\bar{\phi})$ [29], in turn a necessary and sufficient condition for the referred global uniform asymptotic stability property

**Theorem 8.4 of [22] states that for state equations with vector field being uniformly bounded at the origin, the existence of a continuously differentiable positive definite decrescent radially unbounded scalar function whose derivative along the system trajectories is upper-bounded by a negative semidefinite continuous function W, guarantees boundedness of all the system trajectories (*i.e.* for any initial condition) as well as their convergence to the state-space subset where W vanishes (the statement in [22, Theorem 8.4] includes also a local version of the result). In the analytical context of the present work, the closed loop internal variables $(\bar{q},\dot{\bar{q}},\bar{\phi})$ are the (natural) system states and the vector field of the consequent state-space representation vanishes at the origin.

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in the context of the closed-loop system in Eqs. (20) [15]. Note however that exact parameter convergence (*i.e.* to the real values) is not necessary to achieve the global tracking objective. Further, the former task implies the need to look for a suitable $q_d(t)$ through which (23) be fulfilled while the latter assumes that a target trajectory is given. Hence, the fulfillment of (23) is in general left only for parameter estimation purposes [15]. ◁

Remark 6
One of the main features of the proposed scheme, compared to previous approaches, is the particular design of the adaptation subsystem. This gives rise to parameter estimations that remain bounded within pre-specified limits without involving discontinuities. This is achieved by detaching the parameter estimator role from the adaptation variable ϕ which is there to counteract tracking errors due to the inherent inaccuracies of the hybrid compensation term (*i.e.* due to the fact that $Y\big(q(t),\dot{q}_d(t),\ddot{q}_d(t)\big)\hat{\theta}(t) \not\equiv Y\big(q(t),\dot{q}_d(t),\ddot{q}_d(t)\big)\theta$; by so acting, the hybrid compensation term in turn approaches the accurate one). The parameter estimator role is transferred to a variable coming out from the adaptation subsystem, namely $\hat{\theta}$, defined in terms of ϕ through a suitable (one-to-one) continuous function s_a stating a (strictly) *passive* relation among them (such that $\bar{\phi}^T\bar{s}_a(\bar{\phi}) > 0$, $\forall\bar{\phi} \neq 0_n$). This permits the inclusion of a suitable *storage* term in the Lyapunov function, that accounts for the *controller-induced-potential* energy due to the parameter estimation error $\bar{s}_a(\bar{\phi})$ (see (21)), namely the last term in the right-hand side of V_1 in Eq. (22). V_1 is thus guaranteed to be a correct Lyapunov function candidate for the closed-loop system. In turn, through the derivative of the *compound* Lyapunov function along the system trajectories, $\dot{V}_1$, the dynamics of ϕ is determined so as to have the required stability/convergence properties (by eliminating though $\dot{\phi}$ the cross terms in $\dot{V}_1$ where $\bar{s}_a(\bar{\phi})$ appears since, in the absence of the exact knowledge of θ, there is no way to get a negative definite $\bar{\phi}$-term) as corroborated through the proof of Proposition 2. Such a continuous dynamics carries out the described corrective role of ϕ. The adequate continuous bounded form of $\hat{\theta}$ is given through s_a by means of suitable generalized saturation functions. The designed algorithm is thus free of discontinuities, permitting the auxiliary adaptive variable ϕ to take initial values and evolve anywhere in $\mathbb{R}^p$ while keeping $\hat{\theta}$ suitably bounded. ◁

Remark 7
Adaptive versions of the SP-SD+, SPD+ and SPDhc+-like controllers described in Appendix A are obtained by considering in the proposed design method the expressions in (31), (33), and (35), respectively, with suitable adjustments on the saturation function parameter conditions. Thus, the adaptive SP-SD+ controller is obtained from (17) by taking

$$s_d(t,\bar{q},\dot{\bar{q}},\hat{\theta}) = s_D(K_D\dot{\bar{q}}) \tag{24}$$

with $s_D(\cdot)$ and K_D as defined in Appendix A and the involved bound values, M_{Pi} and M_{Di}, satisfying

$$M_{Pi} + M_{Di} < T_i - B_{Di}^{\Theta_a} \tag{25}$$

(recall inequality (16c)) $i = 1,\ldots,n$, the adaptive SPD+ scheme is gotten by taking

$$s_d(t,\bar{q},\dot{\bar{q}},\hat{\theta}) = s_P(K_P\bar{q} + K_D\dot{\bar{q}}) - s_P(K_P\bar{q}) \tag{26}$$

with $s_P(\cdot)$ as defined for this case in Appendix A and bound values fulfilling

$$M_{Pi} \leq T_i - B_{Di}^{\Theta_a} \tag{27}$$

$i = 1,\ldots,n$, and the adaptive SPDhc+-like algorithm is obtained by taking

$$\begin{aligned} s_d(t,\bar{q},\dot{\bar{q}},\hat{\theta}) = s_0\big(&Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\hat{\theta} - s_P(K_P\bar{q})\big) \\ &- s_0\big(Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\hat{\theta} - s_P(K_P\bar{q}) - K_D\dot{\bar{q}}\big) \end{aligned} \tag{28}$$

with $s_0(\cdot)$ as defined in Appendix A and the involved saturation function parameters satisfying

$$B_{Di}^{\Theta_a} + M_{Pi} < L_{0i} \leq M_{0i} < T_i \tag{29}$$

$i = 1,\ldots,n$. For these cases, κ in (11) remains as specified in Eqs. (37). ◁

Figure 2. Experimental setup

6. EXPERIMENTAL RESULTS

In order to experimentally corroborate the efficiency of the developed adaptive approach, real-time control implementations were carried out on a 2-DOF direct-drive manipulator. The experimental setup, shown in Fig. 2, is a prototype of the 2-revolute-joint robot arm used in [30, 31], located at the *Instituto Tecnológico de la Laguna*. The actuators are direct-drive brushless motors operated in torque mode, so they act as torque source and accept an analog voltage as a reference of torque signal. The control algorithm is executed at a 2.5 ms sampling period in a control board (based on a DSP 32-bit floating point microprocessor from Texas Instrument) mounted on a PC-host computer. The robot software is in open architecture, whose platform is based in C language to run the control algorithm in real time.

For the considered experimental manipulator, Properties 1–5 are satisfied with[††]

$$Y^T(q,\dot q,\ddot q) = \begin{pmatrix} \ddot q_1 & 0 \\ (2\ddot q_1+\ddot q_2)\cos q_2 - \dot q_2(2\dot q_1+\dot q_2)\sin q_2 & \ddot q_1\cos q_2 + \dot q_1^2 \sin q_2 \\ \ddot q_2 & \ddot q_1+\ddot q_2 \\ \dot q_1 & 0 \\ 0 & \dot q_2 \\ \sin q_1 & 0 \\ \sin(q_1+q_2) & \sin(q_1+q_2) \end{pmatrix}$$

$$\theta^T = \begin{pmatrix} 2.351 & 0.084 & 0.102 & 2.288 & 0.175 & 38.465 & 1.825 \end{pmatrix}$$

$\mu_m = 0.088$ kg m^2, $\mu_M = 2.533$ kg m^2, $k_C = 0.1455$ kg m^2, $f_m = 0.175$ kg m^2/s, $f_M = 2.288$ kg m^2/s, $B_{g1} = 40.29$ Nm, and $B_{g2} = 1.825$ Nm. The maximum allowed torques (input saturation bounds) are $T_1 = 150$ Nm and $T_2 = 15$ Nm for the first and second links respectively. From these data, one easily corroborates that Assumption 1 is fulfilled.

The proposed adaptive scheme in Eqs. (17)-(18) was tested in its SP-SD+, SPD+, and SPDhc+-like forms, under the respective consideration of expressions (24)-(25), (26)-(27), and (28)-(29). The involved saturation functions were defined as

$$\sigma_{Pi}(\varsigma) = \begin{cases} \varsigma & \forall|\varsigma| \le L_{Pi} \\ \mathrm{sign}(\varsigma)L_{Pi} + (M_{Pi}-L_{Pi})\tanh\left(\frac{\varsigma - \mathrm{sign}(\varsigma)L_{Pi}}{M_{Pi}-L_{Pi}}\right) & \forall|\varsigma| > L_{Pi} \end{cases}$$

[††]For the sake of simplicity, the units of the elements of θ, their estimation variables and related bounds and saturation function parameters, the auxiliary states, and the control and adaptation gains are omitted. The angles are expressed and measured in radians.

with $0 < L_{Pi} < M_{Pi}$, $i = 1, 2$, in all the three cases;

$$\sigma_{Di}(\varsigma) = M_{Di}\,\text{sat}(\varsigma/M_{D_i})$$

$i = 1, 2$, in the SP-SD+ case;

$$\sigma_{0i}(\varsigma) = M_{0i}\,\text{sat}(\varsigma/M_{0_i})$$

$i = 1, 2$, in the SPDhc+-like case; and

$$\sigma_{aj}(\varsigma) = \begin{cases} \varsigma & \forall|\varsigma| \leq L_{aj} \\ \text{sign}(\varsigma)L_{aj} + (M_{aj} - L_{aj})\tanh\left(\frac{\varsigma - \text{sign}(\varsigma)L_{aj}}{M_{aj} - L_{aj}}\right) & \forall|\varsigma| > L_{aj} \end{cases}$$

with $0 < L_{aj} < M_{aj}$, $j = 1, \ldots, 7$, in all the three cases. Let us note that with these saturation functions we have $\sigma'_{PiM} = \sigma'_{DiM} = \sigma'_{0iM} = 1$, $\forall i \in \{1, 2\}$, and that in consequence, for the three controllers, inequality (6) is satisfied with $\kappa = \max_i\{k_{Di}\}$ (see Eqs. (37)).

For comparison purposes, additional tests were implemented considering the adaptive controller proposed by [7] —referred to as D$_e$99— (choice made in terms of the analog nature of the compared algorithms: bounded adaptive), *i.e.*

$$u = Y_d(t)\hat{\theta} - K_P T_h(\Lambda_P \bar{q}) - K_D T_h(\Lambda_D r)$$
$$\dot{\hat{\theta}} = P\big(Q(t, r), \hat{\theta}\big)$$

where $Y_d(t) = Y\big(q_d(t), \dot{q}_d(t), \ddot{q}_d(t)\big)$; $T_h(x) = \big(\tanh(x_1), \ldots, \tanh(x_n)\big)^T$; $\Lambda_P = \text{diag}[\lambda_{P1}, \ldots, \lambda_{Pn}]$ and $\Lambda_D = \text{diag}[\lambda_{D1}, \ldots, \lambda_{Dn}]$ with $\lambda_{Pi} = 1$ [rad]$^{-1}$ and $\lambda_{Di} = 1$ s/rad, $\forall i \in \{1, \ldots, n\}$;

$$r = \dot{\bar{q}} + \varepsilon T_h(\bar{q})$$

with ε being a positive constant;

$$Q(t, r) = -\Gamma Y_d^T(t) r$$

K_P, $K_D \in \mathbb{R}^{n\times n}$ and $\Gamma \in \mathbb{R}^{p\times p}$ are positive definite diagonal matrices; the elements of P are defined as

$$P_j(Q, \hat{\theta}) = \begin{cases} Q_j & \text{if } \theta_{jm} < \hat{\theta}_j < \theta_{jM} \text{ or } \big(\hat{\theta}_j \leq \theta_{jm} \text{ and } Q_j \geq 0\big) \text{ or } \big(\hat{\theta}_j \geq \theta_{jM} \text{ and } Q_j \leq 0\big) \\ 0 & \text{if } \big(\hat{\theta}_j \leq \theta_{jm} \text{ and } Q_j < 0\big) \text{ or } \big(\hat{\theta}_j \geq \theta_{jM} \text{ and } Q_j > 0\big) \end{cases}$$

$j = 1, \ldots, p$, with θ_{jm} and θ_{jM} being known lower and upper bounds of θ_j respectively; and the initial auxiliary state values are taken such that $\hat{\theta}_j(0) \in [\theta_{jm}, \theta_{jM}]$, $j = 1, \ldots, p$. The parameter bounds were fixed at $\big(\theta_{1m} \;\; \theta_{2m} \;\; \theta_{3m} \;\; \theta_{4m} \;\; \theta_{5m} \;\; \theta_{6m} \;\; \theta_{7m}\big) = \big(0.588 \;\; 0.021 \;\; 0.025 \;\; 0.572 \;\; 0.044 \;\; 9.616 \;\; 0.456\big)$ (see footnote ††), and $\theta_{jM} = M_{aj}$, $j = 1, \ldots, 7$, (these values are specified below).

At every experimental test, the initial link positions and velocities were taken as $q_i(0) = \dot{q}_i(0) = 0$, $i = 1, 2$. The auxiliary states were initiated at $\phi^T(0) = \big(2.88 \;\; 0.103 \;\; 0.125 \;\; 2.803 \;\; 0.214 \;\; 47.119 \;\; 2.235\big)$ (see footnote ††) in the SP-SD+, SPD+, and SPDhc+-like cases and $\hat{\theta}^T(0) = \big(2.88 \;\; 0.103 \;\; 0.125 \;\; 2.803 \;\; 0.214 \;\; 47.119 \;\; 2.235\big)$ in the case of the D$_e$99 algorithm. The desired trajectory for all the implemented controllers was defined as

$$q_d(t) = \begin{pmatrix} q_{d1}(t) \\ q_{d2}(t) \end{pmatrix} = \begin{pmatrix} \frac{\pi}{2} + \sin(\omega t) \\ \cos(\omega t) \end{pmatrix} \quad \text{[rad]} \tag{30}$$

Let us note that with this desired trajectory, Assumption 2 is satisfied with $B_{dv} = \omega < 1.2027$ rad/s ($\approx f_m/k_C$) and $B_{da} = \omega^2$.

For the adaptive SP-SD+, SPD+, and SPDhc+-like algorithms, a sufficiently small value of ε (satisfying inequality (11)) was taken and the saturation-function parameters as well as ω in (30) were fixed such that inequalities (25), (27), (29), and (16) were satisfied. Within the consequent

Table I. Control parameter and RMS values

prmtr.	SP-SD+	SPDhc+-like	SPD+	D_e99
ε	1.0167×10^{-7}	1.0165×10^{-7}	4.15×10^{-8}	3
K_D	diag$[20, 5]$		diag$[150, 20]$	diag$[10, 3.8]$
K_P	diag$[1500, 300]$			diag$[70, 7.9]$
Γ	diag$[20, 0.5, 0.1, 1.5, 0.1, 10, 0.25]$			
Λ_P				diag$[20, 10]$
Λ_D				diag$[3, 3]$
RMS	0.0138	0.0106	0.0172	0.0314

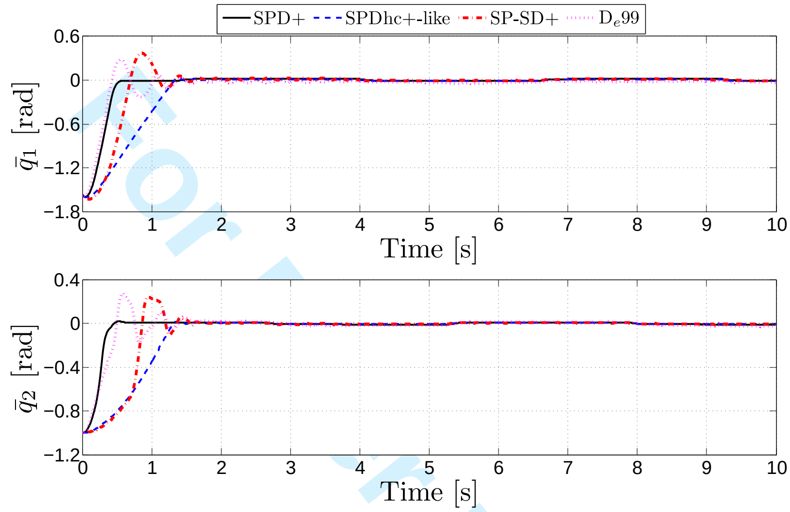

Figure 3. Position errors

limits, the saturation function bounds related to the SP and SD actions and the control and adaptation gains in K_P, K_D and Γ were fixed after several trial-and-error simulation tests so as to have the best possible closed-loop performance —in terms mainly of stabilization time (as short as possible) and transient response (avoiding or lowering down overshoot and oscillations as much as possible)— and then refined experimentally. This was basically done following the guidelines given in Appendix B. As for the D_e99 controller, a similar procedure was followed taking small enough control gains to avoid input saturation (recall that in this approach, the control gains in K_P and K_D respectively bound the P and D terms) but, with the aim to speed up the closed-loop responses, gains λ_{Pi} and λ_{Di}, $i = 1, 2$, (inside the hyperbolic tangent functions involved in the SP and SD actions) greater than unity were fixed. The resulting control parameter values for all the implemented schemes are presented in Table I. As for the saturation function parameters involved in the SP-SD+, SPD+, and SPDhc+-like algorithms, the selected values were (see footnote ††): $M_{P1} = 40$, $M_{D1} = 40$, $M_{P2} = 4$, and $M_{D2} = 4$ in the SP-SD+ case; $M_{P1} = 85$ and $M_{P2} = 8.5$ in the SPD+ case; $M_{01} = 130$, $M_{P1} = 45$, $M_{02} = 13$, and $M_{P2} = 4.5$ in the SPDhc+-like case; and $L_{Pi} = 0.9M_{Pi}$, $i = 1, 2$, $M_a^T = \begin{pmatrix} 2.939 & 0.105 & 0.127 & 2.86 & 0.219 & 48.081 & 2.281 \end{pmatrix}$, and $L_{aj} = 0.9M_{aj}$, $j = 1, \ldots, 7$, in all the three cases. With these values, inequalities (25), (27), (29), (16) and Assumption 2 were corroborated to be satisfied with $\omega = 1.2$ rad/s, taking $B_{gi}^{\Theta_a} = \sum_{j=1}^{7} B_{G_{ij}} M_{aj}$, $i = 1, 2$, *i.e.* $B_{g1}^{\Theta_a} = M_{a6} + M_{a7} = 50.362$ Nm and $B_{g2}^{\Theta_a} = M_{a7} = 2.281$ Nm, and $B_{Di}^{\Theta_a} = \sum_{j=1}^{7} B_{Y_{ij}} M_{aj}$, $i = 1, 2$, *i.e.* $B_{D1}^{\Theta_a} = (M_{a1} + \sqrt{10}M_{a2} + M_{a3})\omega^2 + M_{a4}\omega + M_{a6} + M_{a7} = 58.6872$ Nm and $B_{D2}^{\Theta_a} = (M_{a2} + \sqrt{2}M_{a3})\omega^2 + M_{a5}\omega + M_{a7} = 2.9536$ Nm.

Figures 3 and 4 show the position error evolution and control signals obtained at every experimental test. Note that all the implemented controllers achieved the trajectory tracking

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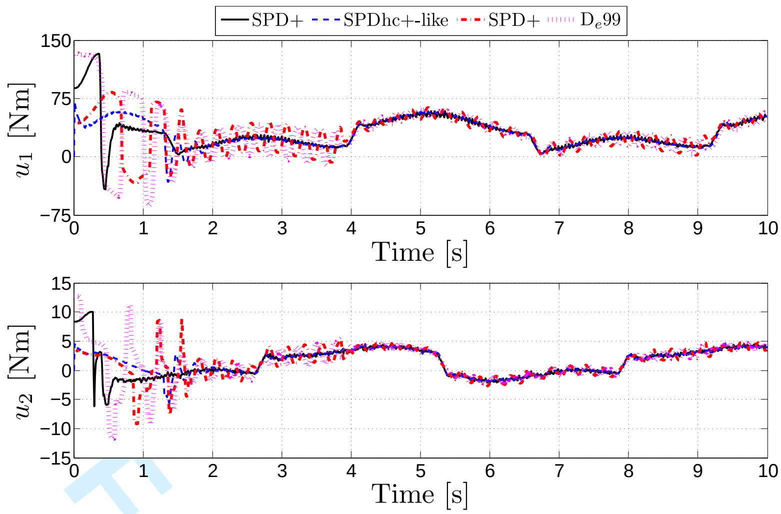

Figure 4. Control signals

objective —avoiding input saturation— in less than 2 seconds, with the SPD+ scheme being the one that gave rise to the fastest responses. This could be achieved preventing overshoot on the position error responses through the SPD+ and SPDhc+-like algorithms, while the SP-SD+ and D_e99 controllers could not avoid it. Let us further note that post-transient effects due to unmodelled phenomena, such as Coulomb friction, were present at all the closed-loop responses. They are observed in the position error graphs as small oscillations. In order to evaluate and compare the performance of the implemented controllers in relation to such a post-transient effect, the root mean square (RMS) of the position errors, *i.e.* $\sqrt{\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\|\bar{q}(t)\|^2 dt}$, was calculated from $t_1 = 2$ s to $t_2 = 10$ s. The values obtained from such a calculation are shown in Table I. Note that under such a criterion, the best performance was obtained through the SPDhc+-like algorithm, while the highest post-transient error was generated by the D_e99 controller. As for the parameter estimators, a considerably slow evolution was observed. This is due to the considerably small value of ε. Moreover, parameter estimations with considerable bias were observed since $q_d(t)$ was not defined so as to fulfill the excitation persistence condition (23). However, accuracy on the parameter estimation is not part of the motion control goal (recall Remark 5). Moreover, neither the slow evolution nor the biased post-transient values of the parameter estimators prevented the trajectory tracking objective to be accomplished —avoiding input saturation— or to be achieved in a considerably short time.

7. CONCLUSIONS

In this work, a generalized adaptive scheme for the global tracking control of robot manipulators with bounded inputs was proposed. Its generalized structure was proven to give rise to adaptive versions/extensions of several PD-type tracking saturating controllers previously developed under the consideration of the exact knowledge of the system parameters. Compared to previous bounded adaptive tracking control algorithms, the proposed adaptive approach guarantees the motion control objective for any initial condition (globally), avoiding discontinuities throughout the scheme, preventing the inputs to reach their natural saturation bounds, permitting the use of any saturation function within a well-specified set to achieve the required boundedness, and permitting innovation on the saturating structure through its generalized form, giving a wide range of possibilities for performance improvement. The efficiency of the proposed adaptive scheme was corroborated through real-time implementations on an actual 2-DOF manipulator. The experimental results showed the coherence of the problem and solution formulations. From a theoretical viewpoint it would still be convenient to relax the result from its dependence on the explicit consideration of the viscous friction forces.

A. SOME PARTICULAR CONTROL STRUCTURES

Let $K_D \in \mathbb{R}^{n\times n}$ be a positive definite diagonal matrix. The control schemes of [10] are retrieved from (3) by respectively defining

$$s_d(t,\bar{q},\dot{\bar{q}},\theta) = s_D(K_D\dot{\bar{q}}) \tag{31}$$

which gives rise to the *SP-SD+* controller

$$u = -s_P(K_P\bar{q}) - s_D(K_D\dot{\bar{q}}) + Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\theta$$

where, for any $x\in\mathbb{R}^n$, $s_D(x) = \big(\sigma_{D1}(x_1),\ldots,\sigma_{Dn}(x_n)\big)^T$, with $\sigma_{Di}(\cdot)$, $i=1,\ldots,n$, being *generalized saturation functions* with bounds M_{Di}, and the involved bound values, M_{Pi} and M_{Di}, satisfying

$$M_{Pi} + M_{Di} < T_i - B_{Di} \tag{32}$$

$i=1,\ldots,n$, with $B_{Di} = \mu_M B_{da} + k_C B_{dv}^2 + f_M B_{dv} + B_{gi}$, and

$$s_d(t,\bar{q},\dot{\bar{q}},\theta) = s_P(K_P\bar{q} + K_D\dot{\bar{q}}) - s_P(K_P\bar{q}) \tag{33}$$

which results in the *SPD+* control law

$$u = -s_P(K_P\bar{q} + K_D\dot{\bar{q}}) + Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\theta$$

with the generalized saturations $\sigma_{Pi}(\cdot)$, $i=1,\ldots,n$, being *strictly increasing*, and bound values fulfilling

$$M_{Pi} < T_i - B_{Di} \tag{34}$$

$i=1,\ldots,n$, both (SP-SD+ and SPD+) cases under the consideration of sufficiently small desired-trajectory-related bound values B_{dv} and B_{da} (see Assumption 2) as stated in [10]. Furthermore, a tracking version of the SPDgc-like controller proposed in [19], that (in addition to SP and D actions) includes the hybrid compensation terms within a single saturation function (at every link), is obtained from (3) by defining

$$s_d(t,\bar{q},\dot{\bar{q}},\theta) = s_0\big(Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\theta - s_P(K_P\bar{q})\big) - s_0\big(Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\theta - s_P(K_P\bar{q}) - K_D\dot{\bar{q}}\big) \tag{35}$$

where, for any $x\in\mathbb{R}^n$, $s_0(x) = \big(\sigma_{01}(x_1),\ldots,\sigma_{0n}(x_n)\big)^T$, with $\sigma_{0i}(\cdot)$, $i=1,\ldots,n$, being *linear saturation functions* for (L_{0i}, M_{0i}), and the involved linear/generalized saturation function parameters satisfying

$$B_{Di} + M_{Pi} < L_{0i} \leq M_{0i} < T_i \tag{36}$$

$i=1,\ldots,n$, with sufficiently small desired-trajectory-related bound values B_{dv} and B_{da} as stated in [10]. Observe from (36) that, by virtue of item (c) of Definition 1, we have that $s_0\big(Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\theta - s_P(K_P\bar{q})\big) \equiv Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\theta - s_P(K_P\bar{q})$, giving rise to an *SPDhc+-like* controller of the form

$$u = s_0\big(Y\big(q,\dot{q}_d(t),\ddot{q}_d(t)\big)\theta - s_P(K_P\bar{q}) - K_D\dot{\bar{q}}\big)$$

One can verify that, in the three cases, the expressions in (5)–(8) are satisfied. In particular, from points 3 and 4 of Lemma 1, one sees that $s_d(t,\bar{q},\dot{\bar{q}},\theta)$ in every one of the above cases in (31), (33), and (35) satisfies inequality (6) with

$$\kappa = \max_i\{\sigma'_{iM}k_{Di}\} \tag{37a}$$

where

$$\sigma'_{iM} = \begin{cases} \sigma'_{DiM} & \text{in the SP-SD+ case} \\ \sigma'_{PiM} & \text{in the SPD+ case} \\ \sigma'_{0iM} & \text{in the SPDhc+-like case} \end{cases} \tag{37b}$$

σ'_{DiM}, σ'_{PiM}, and σ'_{0iM} respectively being the positive bounds of $D^+\sigma_{Di}(\cdot)$, $\sigma'_{Pi}(\cdot)$, and $D^+\sigma_{0i}(\cdot)$, in accordance to item 2 of Lemma 1.

B. A SKETCH OF THE TUNING PROCEDURE

The tuning procedure followed to get the implementation results shown in this work can be sketched as follows:

1. Set the saturation function bounds in accordance to inequalities (25) / (27) / (29) and (16).
2. Run simulations/experiments with low proportional, derivative and adaptation gains.
3. According to the post-transient variation, increase the proportional gains in order to reduce the tracking errors.
4. Increase the derivative gains in order to get a transient response with the shortest possible rise time simultaneously keeping the overshoot as low as possible.
5. Increase ε trying to adhere to (11) if possible or as far as the closed-loop stability permits it.
6. Increase the update gains in Γ in order to improve the parameter estimator responses.
7. Repeat the procedure until the best possible response is obtained.

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