Performance monitoring of heat exchangers
via adaptive observers

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Nomenclature

\( T_{ci}, T_{hi} \) \hspace{1cm} \text{Inlet temperatures in the cold and hot side, respectively, } ^\circ K
\( T_{co}, T_{ho} \) \hspace{1cm} \text{Outlet temperatures in the cold and the hot side, respectively } ^\circ K
\( U \) \hspace{1cm} \text{Heat transfer coefficient, } J/(m^2 \cdot ^\circ K \cdot s)
\( A \) \hspace{1cm} \text{Heat transfer surface area, } m^2
\( c_{pc} \) \hspace{1cm} \text{Specific heat in the cold side, } J/(kg \cdot ^\circ K)
\( c_{ph} \) \hspace{1cm} \text{Specific heat in the hot side, } J/(kg \cdot ^\circ K)
\( \rho_c \) \hspace{1cm} \text{Density of the cold fluid, } kg/m^3
\( \rho_h \) \hspace{1cm} \text{Density of the hot fluid, } kg/m^3
\( V_c \) \hspace{1cm} \text{Volume in the cold side, } m^3
\( V_h \) \hspace{1cm} \text{Volume in the hot side, } m^3
\( v_c \) \hspace{1cm} \text{Flow rate in the cold side, } m^3/s
\( v_h \) \hspace{1cm} \text{Flow rate in the hot side, } m^3/s

Abstract

In this paper, a method for monitoring the performance degradation in a heat exchanger is presented. This method is based on the use of an adaptive observer which estimates the overall heat transfer coefficient \( U \). The monitoring of this parameter can be useful to decide when the heat exchanger needs preventive or corrective maintenance. A simplified mathematical model of the heat exchanger is used to synthesize the adaptive observer. The effectiveness of the proposed method is demonstrated via numerical simulations and through experimental results.

Key words: Performance monitoring, heat exchanger, adaptive observer
1 Introduction

Heat exchangers are present in a wide variety of engineering processes. They are generally used to achieve efficient heat transfer from one fluid to another. Examples of such useful devices are intercoolers, preheaters, boilers and condensers in power plants, just to mention some.

There are several types of heat exchangers:

• recuperative type: the fluids exchange heat on either side of a dividing wall;
• regenerative type: hot and cold fluids occupy the same space containing a matrix of material that works alternatively as a sink or source for heat flow;
• evaporative type, such as cooling towers: a liquid is cooled evaporatively in the same space as coolant.

In this work, the recuperative type of heat exchanger, which is the most common in practice, is considered.

One of the main problems of heat exchangers is the deterioration of the heat transfer surface due to the accumulation of a fouling film. For instance, fouling causes decay in the heat transfer effectiveness. This most often leads to increased energy consumption. In general, fouling is accepted as an unavoidable problem but many efforts are made to try to detect, mitigate and/or correct its occurrence [1–3]. This work is devoted to propose a way to detect performance degradation in a heat exchanger by means of an adaptive observer. Observers, generally referred to as software sensors, are useful to cope with the problems associated to the lack of relevant on-line sensors. They are used to estimate unknown parameters or unmeasured state variables from on-line
and/or off-line measurements, see e.g. [4–7]. Much of the work done in the area of observer design has been based in the application of Kalman filters or extended Kalman filters (EKF) [8,9]. A different category of state estimators has been developed by other authors and implies the use of Luenberger observers or some extensions of them [10,11]. All these observers are used only for state estimation. Nevertheless, it is often the case that some parameter values of the processes are physically unavailable for measurement or they are time-varying. When such is the case, provided that some assumptions are satisfied, it is possible to use adaptive observers for their estimation. An adaptive observer is basically one in which both the parameters and state variables of the system are estimated simultaneously. In the case of linear systems, they have been studied since the 1970’s [12]. Recent works of adaptive observers are often based on variable changes transforming the original system into some canonical form in which the presence of the unknown parameters is simplified to some extent [13,14]. Recently, a simple constant-gain observer has been proposed in [15]. This approach involves two tuning parameters whatever the number of considered differential equations of the model is. Furthermore, the proposed observer is proved not only to be stable but also to yield the estimation error to zero.

Adaptive observers have been applied successfully to a wide variety of dynamical systems, for example, for the estimation of: the sprung mass in automotive suspensions [16], the partial pressure of hydrogen in the anode channel of fuel cells [17], the reaction kinetics in polymerization reactors [4] or bioreactors [7]. This work is devoted to propose a method based on an adaptive observer that can be used to track the overall heat transfer coefficient $U(t)$ of a counter-current heat exchanger. A periodic estimation (this periodicity depending on the use of the heat exchanger) of this coefficient can be useful to deter-
mine when the equipment needs a preventive or corrective maintenance. This is possible if the estimation on $U$ of two different periods, performed under the same operation conditions of the equipment, gives considerably different results. The observer performance is evaluated first via numerical simulations and then using real process data.

This paper is organized as follows. Section 2 presents a simplified model of reasonable accuracy for countercurrent heat exchangers. In Section 3, the problem of estimating the state for a class of nonlinear systems is considered. The observer synthesis for the heat exchanger is based on the mathematical model described in Section 2. Finally, concluding remarks are given in Section 4.

2 Simplified model of a heat exchanger

The recuperative type of heat exchanger may be designed according to one of the following types:

- Parallel-flow (fluids flow in the same direction)
- Counter-flow (fluids flow in the opposite direction)
- Cross-flow (the direction of fluids are perpendicular to each other)

Each of the three types of heat exchangers has advantages and disadvantages. However, among them, the counter-flow heat exchanger design is the most efficient when comparing heat transfer rate per unit surface area. For example, the counter-flow heat exchanger has three significant advantages over the parallel-flow design. First, the thermal stresses throughout the exchanger are minimized thanks to the more uniform temperature difference between the two fluids. Second, the outlet temperature of the cold fluid can approach the
highest temperature of the hot fluid (the inlet temperature). Third, the more uniform temperature difference produces a more uniform rate of heat transfer throughout the heat exchanger.

In this work a counter-flow heat exchanger is considered. The mathematical model presented here, takes into account the following assumptions:

A1 equal inflows and outflows, implying constant volume in both tubes
A2 the heat transfer coefficient is related to the temperatures of the fluids
A3 there is no heat transfer between the external tube and the environment
A4 the thermophysical properties of the fluids are constant
A5 there is no energy storage in the walls
A6 the inlet temperatures are constant.

The system dynamics is obtained through an energy balance rule applied to every element of a lumped model [18]:

\[
\begin{pmatrix}
A_c \\
C_i \\
C_o \\
T_r
\end{pmatrix}
= 
\begin{pmatrix}
\text{Accumulation of energy in the element} \\
\text{Convective flow of energy into the element} \\
\text{Convective flow of energy out of the element} \\
\text{Transfer of heat to (+) / out from (−) the element}
\end{pmatrix}
\pm 
\begin{pmatrix}
\text{Convective flow of energy} \\
\text{Convective flow of energy} \\
\text{Convective flow of energy} \\
\text{Transfer of heat to (+) / out from (−) the element}
\end{pmatrix}
\]

Over a time interval \(\Delta t\), the application of such an energy balance rule considering a single element per fluid (covering the whole tube length; see Fig. 1,
gives rise to [19, Chap. 4]:

\[
\begin{align*}
\rho_c c_p V_c \left[ T_c \right]_{t+\Delta t} - T_c|_t &= (\rho_c c_p \Delta t) v_c T_{ci} - (\rho_c c_p \Delta t) v_c T_{co} + (UA\Delta t) \Delta T \\
\rho_h c_p V_h \left[ T_h \right]_{t+\Delta t} - T_h|_t &= (\rho_h c_p \Delta t) v_h T_{hi} - (\rho_h c_p \Delta t) v_h T_{ho} - (UA\Delta t) \Delta T
\end{align*}
\]

(2)

where \( T_c \) and \( T_h \) respectively represent the cold and hot fluid bulk (average) temperature, and \( \Delta T \) stands for the (mean) temperature difference among the fluids. Since the lumping procedure assumes that every element behaves like a perfectly stirred tank [20], the fluid temperature at each of such elements is generally considered to be uniformly distributed. As a consequence, the outlet temperatures, \( T_{co} \) and \( T_{ho} \), and outlet temperature difference, \( T_{ho} - T_{co} \), may be taken to respectively estimate \( T_c \), \( T_h \), and \( \Delta T \) in (2), i.e., \( T_c = T_{co} \), \( T_h = T_{ho} \), and \( \Delta T = T_{ho} - T_{co} \) [21]. However, these considerations generally result in an oversimplification leading to the derivation of a (2nd order) model that gives rise to dynamic and/or steady-state responses with a noticeable degree of inaccuracy. This was corroborated through numerical simulations in [22], where the consideration of alternative expressions for \( \Delta T \) in (2) were proposed to alleviate the above mentioned deterioration. The results in such work show that the less inaccurate model is the one obtained by approaching \( \Delta T \) in (2) through the logarithmic mean temperature difference (LMTD), typically expressed as (see for instance [23], [24]; recall that a counterflow configuration is being considered):

\[
\Delta T = \Delta T_\ell \triangleq \frac{(T_{ho} - T_{ci}) - (T_{hi} - T_{co})}{\ln \left( \frac{T_{ho} - T_{ci}}{T_{hi} - T_{co}} \right)}
\]

(3)

The use of such a logarithmic expression to approach \( \Delta T \) in (2) has also been considered in [25], where the resulting 2nd order model has been used to
design feedback controllers for the outlet temperature stabilization of coun-
tercurrent heat exchangers. But the model was further refined in such work
through an additional consideration. Namely, the bulk temperatures in the
accumulation terms (left-hand side of (2)) were taken to be the (arithmetic)
average among the inlet and outlet temperatures, i.e. $T_c = (T_{co} + T_{ci})/2$
and $T_h = (T_{ho} + T_{hi})/2$, through which the transient response time is im-
proved. Thus, taking into account all the above mentioned considerations (i.e.
$T_c = (T_{co} + T_{ci})/2$, $T_h = (T_{ho} + T_{hi})/2$, and $\Delta T = \text{LMTD}$) and multiplying
both sides of Eqs. (2) by $2/(\rho_c c_p V_c \Delta t)$, respectively $2/(\rho_h c_p V_h \Delta t)$, we have

$$
\begin{align*}
\frac{[T_{co} + T_{ci}]_{t+\Delta t} - [T_{co} + T_{ci}]_t}{\Delta t} &= \frac{2v_c}{V_c} (T_{ci} - T_{co}) + \frac{2UA}{c_p \rho_c V_c} \Delta T \\
\frac{[T_{ho} + T_{hi}]_{t+\Delta t} - [T_{ho} + T_{hi}]_t}{\Delta t} &= \frac{2v_h}{V_h} (T_{hi} - T_{ho}) - \frac{2UA}{c_p \rho_h V_h} \Delta T
\end{align*}
$$

(3)

Letting $\Delta t \to 0$ and recalling that the inlet temperatures, $T_{ci}$ and $T_{hi}$, are
considered constant (according to Assumption A6), the system dynamics takes
the form

$$
\begin{align*}
\dot{T}_{co} &= \frac{2}{V_c} \left[ v_c (T_{ci} - T_{co}) + \frac{UA}{c_p \rho_c} \Delta T \right] \\
\dot{T}_{ho} &= \frac{2}{V_h} \left[ v_h (T_{hi} - T_{ho}) - \frac{UA}{c_p \rho_h} \Delta T \right]
\end{align*}
$$

(4)

with $\Delta T = \text{LMTD}$. Let us further note that the LMTD expression in (3)
reduces to an indeterminate form when $T_{ho} - T_{ci} = T_{hi} - T_{co}$, which poses a
serious problem to model (4). Such an inconsistency is overcome by taking the
LMTD as

$$
\Delta T = \Delta T_L \triangleq \begin{cases} 
\Delta T_t & \text{if } T_{ho} - T_{ci} \neq T_{hi} - T_{co} \\
\Delta T_0 & \text{if } T_{ho} - T_{ci} = T_{hi} - T_{co} = \Delta T_0
\end{cases}
$$

(5)
as shown in [26], where continuous differentiability as well as other multiple interesting analytical properties of $\Delta T_L$ in (5) are thoroughly demonstrated. Model (4)–(5) has been analytically proved in [27] to keep the main features of the qualitative behavior of heat exchangers. It is worth mentioning that the use of 2nd-order dynamic models using the LMTD to approach $\Delta T$ have been used by several authors as a reliable dynamic representation of heat exchangers. For instance, it was used in [28] for control synthesis, in [29] for stability-limit closed-loop analysis, and in [30] for the development of dynamic simulators. Furthermore, (control or state/disturbance estimator) design based on low-order (plant) models is generally desirable to reduce the resulting algorithm complexity, as pointed out for instance in [31]. Moreover, it is not always possible to install sensors to measure the fluid temperatures at intermediate positions of the exchanger. For this reason, simple models, like (4)–(5), that approach the behavior of the outlet temperatures exclusively in terms of the inlet ones, the flow rates, and the system properties, prove to be useful for certain types of applications that involve state measurements, like feedback control, fault detection, parameter identification, or state/disturbance estimation [25,32].

3 Adaptive observer design

In what follows, $(\cdot)^T$ denotes the transpose of a matrix and $\| \cdot \|$ denotes the Euclidian norm (or the distance function), i.e., given two vectors of equal dimension $\mathbf{x}$ and $\mathbf{y}$,

$$
\| \mathbf{x} - \mathbf{y} \| = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}
$$
Generally speaking, an observer is a dynamical system giving some asymptotic estimate of the actual state of a system, from the knowledge of its inputs and outputs. For example, consider the following general single-input-single-output (SISO) dynamical system:

\[
\begin{aligned}
\dot{x}(t) &= f(x(t), u(t), t) \\
y(t) &= h(x(t), t)
\end{aligned}
\]  

(6)

where \( x(t) \in \mathbb{R}^n \) is the state of the system, \( u \in \mathbb{R} \) and \( y \in \mathbb{R} \) are the input and the output of the system respectively, \( f(\cdot) \in \mathbb{R}^n \) is a smooth function with respect to its arguments and \( h(\cdot) \in \mathbb{R} \) is a real valued function.

A state observer, or simply an observer for system (6) is given by:

\[
\begin{aligned}
\dot{\hat{x}}(t) &= f(\hat{x}(t), u(t), t) + K(\hat{x}(t), t)[\hat{y}(t) - y(t)] \\
y(t) &= h(\hat{x}(t), t)
\end{aligned}
\]  

(7)

The hat (\( \hat{\cdot} \)) represents the estimated value of the variable in question (i.e. \( \hat{x} \) represents the estimated value of \( x \)).

It can be seen that the first equation of the observer (7) is given by two terms. The first term \( f(\hat{x}(t), u(t), t) \) is a 'copy' of system (6) (where the state \( x(t) \) is replaced by its estimate \( \hat{x}(t) \)). The second term \( K(\hat{x}(t), t)[\hat{y}(t) - y(t)] \) is known as the correction term where \( K(\cdot) \) is the gain of the observer which may be constant (e.g. Luenberger observers [10]) or time-varying (e.g. high-gain observers [33]). The factor \( [\hat{y}(t) - y(t)] \) is the process output error.

The objective of observer (7) is to drive the estimation error \( \|\hat{x}(t) - x(t)\| \) to zero at the earliest possible time. This observer assumes constant model
parameters. However, such an assumption may be unrealistic in many practical applications. Parameters, such as the heat transfer coefficient, can change during the operation of thermal processes. In these cases, a more accurate determination of the state can be made by allowing one or more model parameters to vary. The parameter values are then estimated along with the state.

An adaptive observer is a recursive algorithm that is used to estimate the state of a system with unknown parameters or to jointly estimate the state and the unknown parameters of the system. Most designs of such adaptive observers are based on specific forms of the considered systems which can be called for this reason “adaptive observer forms”. In a recent work [15], the author proposed the unifying adaptive observer form (8) which emphasizes properties allowing some asymptotic state estimation in spite of unknown parameters:

\[
\begin{align*}
\dot{y}(t) &= \alpha(y(t), z(t), u(t)) + \beta(y(t), z(t), u(t))\theta(t) \\
\dot{z}(t) &= \gamma(y(t), z(t), u(t))
\end{align*}
\]  

(8)

where \(y(t) \in \mathbb{R}^p\) is the output vector of the system (the measurable states), \(z(t) \in \mathbb{R}^q\) is the vector of the unmeasurable states, \(u(t) \in \mathbb{R}^m\) is the measurable bounded input vector and \(\theta(t) \in \mathbb{R}^r\) is a vector of unknown parameters. \(\alpha(y(t), z(t), u(t))\) and \(\beta(y(t), z(t), u(t))\) are two globally Lipschitz functions with respect to \(z(t)\).

3.1 The proposed adaptive observer

An adaptive observer for a system having the form (8) has been proposed by Besançon [15] as follows:
\[
\begin{aligned}
\dot{\hat{y}}(t) &= \alpha(y(t), \hat{z}(t), u(t)) + \beta(y(t), \hat{z}(t), u(t))\hat{\theta}(t) - k_y (\hat{y}(t) - y(t)) \\
\dot{\hat{z}}(t) &= \gamma(y(t), \hat{z}(t), u(t)) \\
\dot{\hat{\theta}}(t) &= -k_\theta \beta^T(y(t), \hat{z}(t), u(t)) (\hat{y}(t) - y(t))^T
\end{aligned}
\] (9)

such that for any \(\hat{y}(0), \hat{z}(0), y(0), z(0)\) and any measurable bounded \(u(t)\), the estimation errors \(\|\hat{y}(t) - y(t)\|\) and \(\|\hat{z}(t) - z(t)\|\) asymptotically go to zero when \(t\) tends to infinity, while \(\|\hat{\theta}(t) - \theta(t)\|\) remains bounded. Also, if \(\beta^T(y, z, u, t)\) is persistently exciting, and its time derivative is bounded, then \(\|\hat{\theta}(t) - \theta(t)\| \xrightarrow{t \to \infty} 0\). Constants \(k_y > 0\) and \(k_\theta > 0\) are the gains of the observer. Generally, these observer gains are positive and they can have different values. However, it is recommended to take \(k_y < k_\theta\).

If there are no unmeasurable states (which is the case in the application described below), a reduced version of observer (9) is obtained by:

\[
\begin{aligned}
\dot{\hat{y}}(t) &= \alpha(y(t), u(t)) + \beta(y(t), u(t))\hat{\theta}(t) - k_y (\hat{y}(t) - y(t)) \\
\dot{\hat{\theta}}(t) &= -k_\theta \beta^T(y(t), u(t)) (\hat{y}(t) - y(t))^T
\end{aligned}
\] (10)

The goal of this work is to confirm by means of numerical simulations and through experimental results the validity of the proposed simplified adaptive observer (10) in order to design a software sensor for the estimation of the overall heat transfer coefficient in a heat exchanger.

3.2 Application to a heat exchanger

Consider the heat exchanger model given by Eqs. (4). The system parameters \(A, c_{pc}, c_{ph}, \rho_c, \rho_h, V_h,\) and \(V_c\) are known and constant, according to Assumption
A4. The overall heat transfer coefficient may vary in time (if temperatures do), according to Assumption A2. Let \( k_c = A/(c_p\rho cV_c) \) and \( k_h = A/(c_p\rho hV_h) \), then Eqs. (4) can be written in the simplified form:

\[
\begin{align*}
\dot{T}_{co}(t) &= \frac{2v_c(t)}{V_c} (T_{ci} - T_{co}(t)) + 2k_c U(t) \Delta T_L(T_{co}(t), T_{ho}(t)) \\
\dot{T}_{ho}(t) &= \frac{2v_h(t)}{V_h} (T_{hi} - T_{ho}(t)) - 2k_h U(t) \Delta T_L(T_{co}(t), T_{ho}(t))
\end{align*}
\] (11)

We state the following additional assumptions:

A7 the flow rates in the cold and hot side \((v_c(t), v_h(t))\) are measured and they are the inputs of the system \((u_1(t)\) and \(u_2(t)\), respectively),

A8 the hot and cold outlet temperatures \((T_{co}(t)\) and \(T_{ho}(t)\)) are measured and they are the outputs of the system \((y_1(t)\) and \(y_2(t)\), respectively).

These measurements are the only ones needed to make use of the simplified model (4), and they coincide with those available in the ideal case for a single-cell model. It is worth noting that this is the usual case in an industrial environment.

Assumptions A7 and A8 lead to the following matrix representation of the model:

\[
\begin{pmatrix}
\dot{y}_1(t) \\
\dot{y}_2(t)
\end{pmatrix} = \begin{pmatrix}
\frac{2u_1(t)}{V_c} (T_{ci} - y_1(t)) \\
\frac{2u_2(t)}{V_h} (T_{hi} - y_2(t))
\end{pmatrix} + \begin{pmatrix}
2k_c \Delta T_L(y_1(t), y_2(t)) \\
-2k_h \Delta T_L(y_1(t), y_2(t))
\end{pmatrix} U(t)
\] (12)

which has the same form of system (8) without nonmeasurable states \(z(t)\).

Then, an adaptive observer of the form (10) for system (12) is given by Eqs. (13) and (14):
\[
\begin{pmatrix}
\dot{\hat{y}}_1(t) \\
\dot{\hat{y}}_2(t)
\end{pmatrix}
= 
\begin{pmatrix}
\frac{2u_1(t)}{V_c}(T_{ci} - y_1(t)) \\
\frac{2u_2(t)}{V_h}(T_{hi} - y_2(t))
\end{pmatrix}
+ 
\begin{pmatrix}
2k_c\Delta T_L(y_1(t), y_2(t)) \\
-2k_h\Delta T_L(y_1(t), y_2(t))
\end{pmatrix}
\hat{U}(t)
\]
\[
\dot{\hat{U}}(t) = -2k_0 \left[ k_c\Delta T_L(y_1(t), y_2(t)) - k_h\Delta T_L(y_1(t), y_2(t)) \right]
\times 
\begin{pmatrix}
\dot{\hat{y}}_1(t) - y_1(t) \\
\dot{\hat{y}}_2(t) - y_2(t)
\end{pmatrix}
\]

3.3 Numerical simulations

In order to prove the proposed observer, two series of simulations were prepared according to scheme of Fig. 2. They consist in estimating the heat transfer coefficient, \( U(t) \) as a function of inlet and outlet temperatures. In both cases the model used was given by Eq. (4).

The first sequence of simulations deals with the case of a constant heat transfer coefficient and has a twofold purpose: to verify the convergence of the observer and to get used with the parameters involved in the process and their relations. This case represents the regular and expected operation conditions because, in practice, controlled process maintains a constant heat transfer coefficient during a period of several weeks or months.
Further sequences of simulations concern the case of a variable heat transfer coefficient. They predict heat transfer coefficient changes in a range of conditions (Figs. 6 and 7), which are particularly likely to be the case for alterations due to external perturbations, like fouling problems in heat exchangers walls. The study of performance degradation would not justify the analysis of the exemplified forms of perturbations. However, the illustrated situations examine and incorporate realistic conditions of change, considering in fact different scale in time and arbitrary perturbation successions. The goal of these simulations is rather to ensure the effectiveness of the observer for describing a wide set of realistic conditions affecting the heat transfer coefficient of a heat exchanger.

3.3.1 Case 1: \(U(t)\) is constant

In this case, the process model (4) and the observer given by Eqs. (13)-(14) were simulated using the values given in Table 1. These constants were taken from [21]. A value of \(U(t) = 160 \ J/(m^2 \cdot °K \cdot s)\) was considered. The inlet temperature in the cold side \(T_{ci}\) and the inlet temperature in the hot side \(T_{hi}\) were considered constant at 298 °K and 338 °K respectively. The inlet flow rates were \(v_h = 1.9 \times 10^{-4} \ m^3/s\) and \(v_c = 3.15 \times 10^{-4} \ m^3/s\).

This simulation was performed using a Runge-Kutta first order method (Euler) with an integration step equal to 1.5 s. The initial conditions of the process were: \(T_{co}^0 = 306.82 \ °K\), \(T_{ho}^0 = 325.38 \ °K\). The initial conditions of the observer were quite different from the initial conditions of the process: \(\hat{T}_{co}^0 = 273 \ °K\), \(\hat{T}_{ho}^0 = 350 \ °K\) and \(\hat{U}^0 = 140 \ J/(m^2 \cdot °K \cdot s)\). The observer gains were tuned at \(k_\theta = 50\) and \(k_y = 0.5\).
The result of the estimation of states $T_{co}$, $T_{ho}$ is reported in Fig. 3. The time scale is presented in minutes. It can be seen in this Figure that the convergence of the estimates $\hat{T}_{co}$, $\hat{T}_{ho}$ (dashed lines) appears to be very fast. This fact is not surprising because, actually $T_{co}$, $T_{ho}$ are the measured outputs of the process. As was stated in Section 3, the Euclidian norm of the output error $\|\dot{y}(t) - y(t)\|$, i.e. $\sqrt{(\hat{T}_{co} - T_{co})^2 + (\hat{T}_{ho} - T_{ho})^2}$, decays not only asymptotically but also exponentially to zero when $t$ tends to infinity (Fig. 3) while $\|\hat{\theta}(t) - \theta(t)\|$, i.e. $\|\hat{U}(t) - U(t)\|$, remains bounded (Fig. 4).

Fig. 4 shows the estimation of the heat transfer coefficient $U(t)$. Three different values of $k_\theta$ were used: $k_\theta = 100$, $k_\theta = 75$ and $k_\theta = 50$; $k_y = 0.5$ was maintained constant. It can be seen that the convergence time can be as small as desired by setting larger values of $k_\theta$. However, such tuning is to be avoided since the observer may become too sensitive to measurement noise in real time applications. Conversely, smaller values of $k_\theta$ increase the convergence time. In the three cases presented here, the estimation time for $U(t)$ (of about 10 minutes) is larger than the convergence time for the states $T_{co}$, $T_{ho}$ but convergence is guaranteed when $t$ tends to infinity.

Fig. 5 shows the results obtained for different values of $k_y$: $k_y = 0.2$, $k_y = 0.3$ and $k_y = 0.5$; $k_\theta = 50$ was maintained constant. In this case, the convergence time can be as small as desired by setting smaller values of $k_y$. Conversely, greater values of $k_y$ increase the convergence time.

Summarizing, it should be stated that the regular operation analysis case (i.e. $U(t)$ is a constant) has sufficiently verified the convergence of the proposed lumped-parameter-model-based adaptive observer. It should also be remarked that ”sensitivity” of the observer can be reduced (increasing the convergence time) or increased (reducing the convergence time) by varying the tuning parameters $k_\theta$ or $k_y$. 
3.3.2 Case 2: $U(t)$ is time varying

Suspended solids present a major problem in most heat exchangers, in applications such as pellet water coolers as well as catalyst slurry heaters and coolers. If solid residues begin to accumulate on the heat transfer surface, an insulating layer is formed, reducing in this way the heat transfer rate.

Another problem associated with heavy fouling problems is erosion. A typical erosion problem occurs when the local velocity in a heat exchanger becomes excessive and begins to wear away its walls. In many slurries process, a fouling problem can lead to erosion if the local velocity cannot be controlled effectively. Some examples of erosive applications are $TiCl_4$ slurry cooling in titanium dioxide plants and bauxite slurry heating in alumina plants. If the flow is not uniformly distributed or if fouling diverts a large portion of the flow, local velocities may vary significantly within the heat exchanger. This can increase fouling even in low velocity areas and accelerate erosion in higher velocity areas.

For the case of a variable $U(t)$, two types of perturbations are examined: a) a ramp representing a slow degradation of the heat transfer coefficient $U(t)$. This would provide information of a gradual change associated with a performance degradation problem; and b) a step perturbation, representing a sudden change produced by a failure of the process or a malfunction of a sensor, for example.

The simulation was carried out using the same inlet temperatures: $T_{ci} = 298 \, ^oK$ and $T_{hi} = 338 \, ^oK$. The constants in the simulated model were the same given in Table 1. For simplicity, the variation of $U(t)$ occurs in a period of time of about 2.33 hours, from $160 \, J/(m^2 \cdot ^oK \cdot s)$ (for all $t < 40 \, min$) to $120 \, J/(m^2 \cdot ^oK \cdot s)$ (for all $t > 100 \, min$).
Fig. 7 shows the simulation result of the estimation of the heat transfer coefficient $U(t)$. The estimation time is of about 10 min. It can be seen that once the observer converges, it fits well the heat transfer coefficient $U(t)$ in spite of the time-varying nature of this parameter. The initial conditions of the process ($T_{co}^0$, $T_{ho}^0$), the initial conditions of the observer ($\hat{T}_{co}^0$, $\hat{T}_{ho}^0$, $\hat{U}^0$) and the observer gain values ($k_y$, $k_\theta$), were the same used in the precedent simulation.

We can also observe from Fig. 6 the variations of outlet temperatures, $T_{co}$, $T_{ho}$, as a consequence of a change in the heat transfer coefficient, $U(t)$. This is an expected outcome when the heat transfer efficiency is deteriorated.

Finally, we must point out that the performed simulations demonstrate the execution of the adaptive observer when $U(t)$ varies because of degradation (a slow or a gradual change) or because of an operation failure (an abrupt change). The convergence was verified for all the cases.

### 3.4 Experimental results

In order to validate the above theoretical results and to confirm the numerical simulations, several experiments were carried out in a bench-scale pilot plant consisting of a completely instrumented tube heat exchanger (see Fig. 8). The plant operates as a water-cooling process, in this case, the hot water flows through the tube and the cooling water flows in the shell. The adaptive observer was implemented in a personal computer, using the program MATLAB/Simulink® (because the observer implementation is straightforward, it is possible to use different computational programs, for example NI/LabView®). The integration method to solve the observer equations was
the Runge-Kutta first order method (Euler). The discrete sampling period for this experiment was $t_s = 1 \text{ min}$. The constants and physical data used for the internal model of the observer are given in Table 2 (it is worth to note that these physical data were estimated).

The instrumentation and operation variables are the following: the inlet temperatures were kept constant at $T_{ci} = 301.5 \degree K$ (measured with a SIKA glass thermometer) and $T_{hi} = 343.1 \degree K$ (measured via an Engelhard Pyro-Controle Pt-100 temperature transmitter). The volumetric flowrates were measured via two Platon flowmeters. In the hot side, we had $v_h = 1.33 \times 10^{-5} \text{ m}^3/\text{s}$. In the cold side, $v_c$ was time varying between $6.67 \times 10^{-6} \text{ m}^3/\text{s}$ and $7.5 \times 10^{-6} \text{ m}^3/\text{s}$ as shown in Fig. 9. The outlet temperature in the cold side $T_{co}$ was measured using an Engelhard Pyro-Controle Pt-100 temperature transmitter. The outlet temperature in the hot side $T_{ho}$ was measured using a SIKA glass thermometer.

The measured inlet and outlet temperatures were registered and used off-line in order to: a) estimate the overall heat transfer coefficient, $U(t)$, by means of an adaptive observer, and b) to estimate the outlet temperatures, associated to the inlet conditions. The gains of the observer were tuned at $k_y = 15$ and $k_\theta = 1 \times 10^4$. The final estimated value of $U(t)$ was $1049.4 \text{ J/(m}^2 \cdot \degree K \cdot \text{s})$, this is an acceptable value and corresponds well to the heat exchanger used for this experiment.

The estimated outlet temperatures serve thus to validate the proposed observer. As it can be seen in Figs. 10 and 11, it is evident that the estimates $\hat{T}_{co}$, $\hat{T}_{ho}$ converge towards the measured values $T_{co}$, $T_{ho}$ respectively.

The resulting heat transfer coefficient $U(t)$ estimates are shown in Fig. 12. As outlined before, these estimations agree absolutely with the values obtained by simulation. In particular, it must be noted that: a) in absence of
a directly applied perturbation in operation variables, \( U(t) \) converges to a constant value. As anticipated, for a short experiment, variations of \( U(t) \) are absent. b) For a direct perturbation in operation variables (i.e. temperatures, pressure or flow), the observer follows exactly the same experimental path and after restoring initial conditions, the experimental \( U(t) \) returns also to the stable corresponding value.

4 Conclusions

In this work, an adaptive observer was developed for monitoring the performance degradation in a counter-flow heat exchanger. Based on temperature measurements, the observer estimates the global heat transfer coefficient \( U(t) \). The designed scheme derive from a simplified lumped-parameter model (4), which does not account for the variation of temperature in the axial direction. Hence, the major problem appears to be the lost of spatial information, since it is not possible to relate the heat transfer coefficient to the axial position over the heat exchanger. In contrast, the premier advantage is the fact that both, implementation and calibration are simple. This is because the gains \( k_y \) and \( k_\theta \) are constants and consequently, it is not necessary to solve a dynamical system (this is the case of Kalman filters where a Riccati equation must be solved and numerical instability can easily arise from accumulated errors [34]). In addition, it should be pointed out that the proposed observer needs a limited knowledge of the system behavior (i.e. a simple model provides enough information), and the estimation routine does not require neither the dynamics of \( U(t) \), nor any data assumptions for initializing \( U(t) \) computation. To prove the convergence of the observer, two simulations were carried out.
(Figs. 4 and 5). Results show that the estimated values $T_{co}$, $T_{ho}$ and $U(t)$ compares quite well with the simulated process values $T_{co}$, $T_{ho}$ and $U(t)$, even though no relation exists between the $U(t)$ simulation-model and the observer. The real-time experiments with a bench-scale pilot plant demonstrate good agreement in temperature measurement and simulation, but evidently, $U(t)$ can not be measured directly. Nevertheless, according to Fig. 12, it shows a typical and characteristic behavior: a) As expected, at short times, the estimated $\hat{U}(t)$ remains the same for a set of operation conditions. In practice, any variation of this parameter due to performance degradation can take some weeks or several years to appear. b) To show the sensitivity to changes in process variables, a series of temperature perturbations was carried out. Each time, the corresponding response of $U(t)$ was observed, as well as the corresponding stable value that is attained after a certain period.

The goal of this work is not to provide a global heat transfer coefficient for modeling purposes, but it is to give a reasonably simple, yet versatile methodology by which the value of $U(t)$ can be monitored continuously. Any variation of $U(t)$ with time may be related to the thermal performance degradation of heat exchangers. This information can be used by process engineers to decide whether the heat exchanger needs a maintenance action, in order to restore the heat transfer rates and pressure drops required by the process. Furthermore, this monitoring can readily be used to program preventive and corrective maintenance proceedings, since in many traditional heat exchanger designs, cleaning would appear to be a costly, time consuming and frequently necessary.
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Table 2

Physical data used in the experiments

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