INTECH
open science | open minds

OPEN ACCESS
ARTICLE

International Journal of Advanced Robotic Systems

Output-Feedback Adaptive SP-SD-Type Control with an Extended Continuous Adaptation Algorithm for the Global Regulation of Robot Manipulators with Bounded Inputs

Regular Paper

Daniela J. López-Araujo[1], Arturo Zavala-Río[1,*], Víctor Santibáñez[2] and Fernando Reyes[3]

1 Instituto Potosino de Investigación Científica y Tecnológica, Mexico
2 Instituto Tecnológico de la Laguna, Mexico
3 Benemérita Universidad Autonoma de Puebla, Mexico
* Corresponding author E-mail: azavala@ipicyt.edu.mx

Received 30 May 2012; Accepted 11 Oct 2012

DOI: 10.5772/54267

Abstract In this work, an output-feedback adaptive SP-SD-type control scheme for the global position stabilization of robot manipulators with bounded inputs is proposed. Compared with the output-feedback adaptive approaches previously developed in a bounded-input context, the proposed velocity-free feedback controller guarantees the adaptive regulation objective globally (*i.e.* for any initial condition), avoiding discontinuities throughout the scheme, preventing the inputs from reaching their natural saturation bounds and imposing no saturation-avoidance restrictions on the choice of the P and D control gains. Moreover, through its extended structure, the adaptation algorithm may be configured to evolve either in parallel (independently) or interconnected to the velocity estimation (motion dissipation) auxiliary dynamics, giving an additional degree of design flexibility. Furthermore, the proposed scheme is not restricted to the use of a specific saturation function to achieve the required boundedness, but may involve any one within a set of smooth and non-smooth (Lipschitz-continuous) bounded passive functions that include the hyperbolic tangent and the conventional saturation as particular cases. Experimental results on a 3-degree-of-freedom manipulator corroborate the efficiency of the proposed scheme.

Keywords Adaptive Control, Output Feedback, Global Regulation, Saturation, Robot Manipulators

1. Introduction

Since the publication of [1], the Proportional-Derivative with gravity compensation (PDgc) controller [2] has proved to be a useful technique for the regulation of robot manipulators. In its original form, such a control technique achieves global stabilization under ideal conditions, for instance unconstrained input, measurability of all the system (state) variables and exact knowledge of the system parameters. Unfortunately, in actual applications, such underlying assumptions are not generally satisfied, giving rise to unexpected or undesirable effects, such as input saturation and those related to such a nonlinear phenomenon [3], noisy responses and/or deteriorated performance [4], or steady-state errors [5]. However, such inconveniences have not necessarily rendered the PDgc technique useless. Inspired by this control method, researchers have developed alternative (nonlinear or dynamic) PDgc-based approaches that deal with the limitations of the actuator capabilities and/or of the available system data, while keeping the natural energy properties of the original PDgc controller, which are the definition of a unique arbitrarily-located closed-loop equilibrium configuration and motion dissipation. For instance, extensions of the PDgc controller that cope with the input saturation phenomenon have been developed under various analytical frameworks in [6, 7, 8, 9, 10 and 11]. Indeed, assuming the availability of the exact value of all the system parameters and accurate measurements of all the link positions and velocities, a bounded PDgc-based approach was proposed in [6] and [7]. In these works, the P and D terms (at every joint) are each explicitly bounded through specific saturation functions; a continuously differentiable one, or more precisely the hyperbolic tangent function, is used in [6] and the conventional non-smooth one in [7]. In view of their structure, these types of algorithms have been denoted *SP-SD* controllers in [12]. Two alternative schemes that prove to be simpler and/or give rise to improved closed-loop performance were recently proposed in [8]. The first approach includes both the P and D actions (at every joint) within a single saturation function, while in the second one all the terms of the controller (P, D and gravity compensation) are covered by one such function, with the P terms internally embedded within an additional saturation. The exclusive use of a single saturation (at every joint) including all the terms of the controller was further achieved through desired gravity compensation in [13]. Moreover, velocity-free versions of the SP-SD controllers in [7] and [6] (still depending on the exact values of the system parameters) are obtained through the design methodologies developed in [9] and [10]. In [9] global regulation is proven to be achieved when each velocity measurement is replaced by the dirty derivative [14] of the respective position in the SP-SD controller of [7]. A similar replacement in a more general form of the SP-SD controller is proven to achieve global regulation through the design procedure proposed in [10] (where an alternative type of dirty derivative, which involves a saturation function in the auxiliary dynamics that gives rise to the estimated velocity, results from the application of the proposed methodology). Furthermore, an output-feedback dynamic controller with a structure similar to that resulting from the methodology in [10], but which considers a single saturation function (at every joint) where both the position errors and velocity estimation states are involved, was proposed in [11] (where a dissipative linear term on the auxiliary state is added to the saturating velocity error dynamics involved for the dirty derivative calculation). Extensions of this approach to the elastic-joint case were further developed in [15].

Furthermore, SP-SD-type adaptive algorithms that give rise to bounded controllers, while alleviating the system parameter dependence of the gravity compensation term, have been developed in [16, 17, and 18]. In [16] global regulation is aimed for, through a discontinuous scheme that switches among two different control laws, under the consideration of state and output feedback. Both considered control laws keep an SP-SD structure similar to that of [7]; the first one avoids gravity compensation taking high-valued control gains (by means of which the closed-loop trajectories are lead close to the desired position) and the second one considers adaptive gravity compensation terms that are kept bounded by means of discontinuous auxiliary dynamics. Each velocity measurement is replaced by the dirty derivative of the corresponding position in the output-feedback version of the algorithm. Unfortunately, a precise criterion to determine the switching moment (from the first control law to the second one) is not furnished for either of the developed schemes.

In [17] semi-global regulation is proven to be achieved through a state feedback scheme that keeps the same structure as the SP-SD controller of [6] but additionally considers adaptive gravity compensation. The adaptation algorithm is defined in terms of discontinuous auxiliary dynamics, by means of which the parameter estimators are prevented from taking values beyond some pre-specified limit, which consequently keeps the adaptive gravity compensation terms bounded. This approach was further extended in [19] where the control objective is defined in task coordinates and the kinematic parameters, in addition to those involved in the system dynamics, are considered to be uncertain too.

In [18] a controller that keeps the SP-SD structure of [6] is proposed, where each velocity measurement is replaced by the dirty derivative of the corresponding position and an adaptive gravity compensation term with initial-condition-dependent bounds is considered. Based on the proof of the main result, semi-global regulation is claimed to be achieved.

Let us note that, by the way the SP and SD terms are defined in the adaptive schemes mentioned above, the bound of the control signal at every link turns out to be defined in terms of the sum of the P and D control gains (and of an additional term involving the bounds of the parameter estimators). This limits the choice of such gains if the natural actuator bounds (or arbitrary input bounds) are to be avoided. This, in turn, restricts the closed-loop region of attraction in the semi-global stabilization cases. On the other hand, as far as the authors are aware, the semi-global and/or discontinuous approaches developed in [18] and [16] are the only output-feedback bounded adaptive algorithms proposed in the literature. Moreover, a continuous adaptive scheme with continuous auxiliary dynamics, which achieves the global regulation objective, avoiding input saturation and disregarding velocity measurements in the feedback, is still missing in the literature and consequently remains an open problem. These arguments have motivated the present work, which aims to fill in the aforementioned gap.

It is worth adding that recent works have focused on the global regulation problem in the bounded-input context through nonlinear PID-type controllers. This is the case for instance of [20], [21], [22] where state-feedback and output-feedback schemes were presented, and [23] where a controller with the same structure as the state-feedback algorithm presented in [22] was previously proposed. Such PID-type algorithms are not only independent of the exact knowledge of the system parameters, but also disregard the structure of the system dynamics (or of any of its components). However, in a bounded-input context, the design of an output-feedback adaptive scheme that solves the regulation problem globally, avoiding input saturation, and being free of discontinuities, remains an open analytical challenge. Moreover, as will be corroborated in subsequent sections of this work, regulation towards a suitable configuration permits the output-feedback adaptive scheme to provide an estimation (exact under ideal conditions) of the system parameters (involved in the gravity-force vector), which is not the case for other types of controllers.

In this work, an output-feedback adaptive SP-SD-type control scheme for the global regulation of robot manipulators with saturating inputs is proposed. Through its extended structure, the adaptation algorithm may be configured to evolve either in parallel (independently) or interconnected to the velocity estimation (motion dissipation) auxiliary dynamics, giving an additional degree of design flexibility. With respect to the previous output-feedback adaptive approaches developed in a bounded-input context, the proposed velocity-free feedback controller guarantees the adaptive regulation objective globally (*i.e.* for any initial condition), avoiding discontinuities throughout the scheme, preventing the inputs from attaining their natural saturation bounds and imposing no saturation-avoidance restriction on the choice of the P and D control gains. Furthermore, contrarily to the adaptive schemes of the previously cited studies, the approach proposed in this work is not restricted to involving a specific saturation function to achieve the required boundedness, but may involve any one within a set of smooth and non-smooth (Lipschitz-continuous) bounded passive functions that include the hyperbolic tangent and the conventional saturation as particular cases. Experimental results on a 3-degree-of-freedom manipulator corroborate the proposed contribution.

2. Preliminaries

Let $X \in R^{m\times n}$ and $y \in R^n$. Throughout this work, X_{ij} denotes the element of X at its i^{th} row and j^{th} column, X_i represents the i^{th} row of X and y_i stands for the i^{th} element of y. 0_n denotes the origin of R^n and I_n represents the $n\times n$ identity matrix. $\|\cdot\|$ denotes the standard Euclidean norm for vectors, *i.e.*, $\|y\| = \sqrt{\sum_{i=0}^{n} y_i^2}$ and the induced norm for matrices, *i.e.*, $\|X\| = \sqrt{\lambda_{\max}(X^T X)}$, where $\lambda_{\max}(X^T X)$ represents the maximum eigenvalue of $X^T X$. The kernel of X is denoted $\ker(X)$. Consider a continuously differentiable scalar function $\zeta : R \to R$ and a locally Lipschitz-continuous scalar function $\phi : R \to R$, both vanishing at zero, *i.e.*, $\zeta(0) = \phi(0) = 0$. Let ζ' denote the derivative of ζ with respect to its argument and $D^+\phi$ stand for the upper right-hand (Dini) derivative of ϕ [24, App. C.2] and [25, App. I][1]. Thus $\phi(\varsigma) = \int_0^{\varsigma} D^+\phi(r)dr$ and $(\zeta \circ \phi)(\varsigma) = \zeta(\phi(\varsigma)) = \int_0^{\varsigma} \zeta'(\phi(r)) D^+\phi(r)dr$.

Let us consider the general n-degree-of-freedom (n-DOF) serial rigid robot manipulator dynamics with viscous friction [26, 27]:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau \qquad (1)$$

where $q, \dot{q}, \ddot{q} \in R^n$ are, respectively, the position (generalized coordinates), velocity and acceleration vectors.

1 The upper right-hand derivative $D^+\phi$ is defined by $D^+\phi(\varsigma) = \limsup_{h\to 0^+} \frac{\phi(\varsigma+h)-\phi(\varsigma)}{h}$, where $\limsup_{n\to\infty}$ of a sequence of real numbers $\{x_n\}$ is a scalar w satisfying the following two conditions [24, App. C.2]: 1) for every $\varepsilon > 0$, there exists an integer N such that $n > N$ implies $x_n < w + \varepsilon$ and 2) given $\varepsilon > 0$ and $m > 0$, there exists an integer $n > m$ such that $x_n > w - \varepsilon$. In particular, if ϕ is differentiable at ς then $D^+\phi(\varsigma) = \frac{d\phi}{d\varsigma}(\varsigma)$. For a locally Lipschitz-continuous scalar function $\phi(\varsigma)$ that is not differentiable at a countable number of values of ς, say ς_i, $i = 1,2,\ldots$, $D^+\phi(\varsigma)$ is a piecewise continuous function with bounded discontinuities but well defined at ς_i, $i = 1,2,\ldots$

$H(q)\in R^{n\times n}$ is the inertia matrix and $C(q,\dot q)\dot q$, $F\dot q$, $g(q)$, $\tau\in R^n$ are, respectively, the vectors of Coriolis and centrifugal, viscous friction, gravity and external input generalized forces, with $F\in R^{n\times n}$ being a positive definite constant diagonal matrix whose entries $f_i>0$, $i=1,\ldots,n$, are the viscous friction coefficients. Some well-known properties characterizing the terms of such a dynamical model are recalled here (see for instance [2, Chap. 4] and see further [2, Chap. 14] and [28] concerning Property 6 below).

Property 1 *The inertia matrix is a positive definite symmetric bounded matrix i.e.,* $\mu_m I_n \le H(q)\le \mu_M I_n$, $\forall q\in R^n$, *for some positive constants* $\mu_m\le\mu_M$.

Property 2 *The Coriolis matrix satisfies* $\|C(q,\dot q)\|\le k_c\|\dot q\|$, $\forall(q,\dot q)\in R^n\times R^n$, *for some constant* $k_c\ge 0$.

Property 3 *The Coriolis matrix and* $\dot H=\frac{d}{dt}H$ *satisfy* $\dot q^T\left[\frac{1}{2}\dot H(q,\dot q)-C(q,\dot q)\right]\dot q=0$, $\forall(q,\dot q)\in R^n\times R^n$, *and actually* $\dot H(q,\dot q)=C(q,\dot q)+C^T(q,\dot q)$.

Property 4 *The viscous friction coefficient matrix satisfies* $f_m\,||x||^2\le x^TFx\le f_M\,||x||^2$, $\forall x\in R^n$, *where* $0<f_m:=\min_i\{f_i\}\le\max_i\{f_i\}=:f_M$.

Property 5 *The gravity vector* $g(q)$ *is bounded, or equivalently every element of the gravity vector* $g_i(q)$, $i=1,\ldots,n$, *satisfies* $|g_i(q)|\le B_{gi}$, $\forall q\in R^n$ *for some positive constants* B_{gi}, $i=1,\ldots,n$.[2]

Property 6 *The gravity vector can be rewritten as* $g(q,\theta)=G(q)\theta$, *where* $\theta\in R^p$ *is a constant vector whose elements depend exclusively on the system parameters and* $G(q)\in R^{n\times p}$ *(the regression matrix) is a continuous matrix function, whose elements depend exclusively on the configuration variables and do not involve any of the system parameters. Equivalently, the potential energy function of the robot can be rewritten as* $U(q,\theta)=\Upsilon(q)\theta$, *where* $\Upsilon(q)\in R^{1\times p}$ *(the regression vector) is a continuous row vector function whose elements depend exclusively on the configuration variables and do not involve any of the system parameters. Actually,* $G^T(q)=\frac{\partial}{\partial q}\Upsilon^T(q)$, *or equivalently,* $\Upsilon_j(q)=\sum_{i=1}^n\int_{q_i^*}^{q_i}G_{ij}(q_1,\ldots,q_{i-1},r_i,q_{i+1}^*,\ldots,q_n^*)dr_i$, $\forall j\in\{1,\ldots,p\}$, *with* $q^*=(q_1^*,\ldots,q_n^*)^T$ *being the reference configuration where* $U(q^*,\theta)=0$.

Property 7 *Consider the gravity vector* $g(q,\theta)$. *Let* θ_{Mj} *represent an upper bound of* $|\theta_j|$, *such that* $|\theta_j|\le\theta_{Mj}$, $\forall j\in\{1,\ldots,p\}$. *Let* $\theta_M=(\theta_{M1},\ldots,\theta_{Mp})^T$ *and* $\Theta=[-\theta_{M1},\theta_{M1}]\times\cdots\times[-\theta_{Mp},\theta_{Mp}]$. *By Properties 5 and 6, there exist positive constants* $B_{gi}^{\theta_M}\ge B_{gi}$, $i=1,\ldots,n$, *such that* $|g_i(x,y)|=|G_i(x)y|\le B_{gi}^{\theta_M}$, $i=1,\ldots,n$, $\forall x\in R^n$, $\forall y\in\Theta$. *Furthermore, there exist positive constants* $B_{G_{ij}}$, B_{G_i} *and* B_G, *such that* $|G_{ij}(x)|\le B_{G_{ij}}$, $\|G_i(x)\|\le B_{G_i}$ *and* $\|G(x)\|\le B_G$, $\forall x\in R^n$, $i=1,\ldots,n$, $j=1,\ldots,p$.

Let us suppose that the absolute value of each input τ_i (i^{th} element of the input vector τ) is constrained to be smaller than a given saturation bound $T_i>0$, *i.e.* $|\tau_i|\le T_i$, $i=1,\ldots,n$. In other words, letting u_i represent the control signal (controller output) relative to the i^{th} degree of freedom, we have:

$$\tau_i=T_i\,\text{sat}\left(\frac{u_i}{T_i}\right)\qquad(2)$$

$i=1,\ldots,n$, where $\text{sat}(\cdot)$ is the standard saturation function, *i.e.*, $\text{sat}(\varsigma)=\text{sign}(\varsigma)\min\{|\varsigma|,1\}$.

Let us note from (1) and (2) that $T_i\ge B_{gi}$ (see Property 5), $\forall i=1,\ldots,n$, is a necessary condition for the manipulator to be stabilizable at any desired equilibrium configuration $q_d\in R^n$. Thus, the following assumption turns out to be crucial within the analytical setting considered in this work:

Assumption 1 $T_i>B_{gi}$, $\forall i\in 1,\ldots,n$.

The control schemes proposed in this work involve special functions fitting the following definition.

Definition 1 *Given a positive constant* M, *a non-decreasing Lipschitz-continuous function* $\sigma:R\to R$ *is said to be a generalized saturation with bound* M *if*

a) $\varsigma\sigma(\varsigma)>0$ *for all* $\varsigma\ne0$;
b) $|\sigma(\varsigma)|\le M$ *for all* $\varsigma\in R$.

Functions meeting Definition 1 satisfy the following:

Lemma 1 *Let* $\sigma:R\to R$ *be a generalized saturation with bound* M *and* k *be a positive constant. Then*

1. $\lim_{|\varsigma|\to\infty}D^+\sigma(\varsigma)=0$;
2. $\exists\sigma_M'\in(0,\infty)$ *such that* $0\le D^+\sigma(\varsigma)\le\sigma_M'$, $\forall\varsigma\in R$
3. $\frac{\sigma^2(k\varsigma)}{2k\sigma_M'}\le\int_0^\varsigma\sigma(kr)dr\le\frac{k\sigma_M'\varsigma^2}{2}$, $\forall\varsigma\in R$;
4. $\int_0^\varsigma\sigma(kr)dr>0$, $\forall\varsigma\ne0$;
5. $\int_0^\varsigma\sigma(kr)dr\to\infty$ *as* $|\varsigma|\to\infty$;
6. *if* σ *is strictly increasing then, for any constant* $a\in R$, $\bar\sigma(\varsigma)=\sigma(\varsigma+a)-\sigma(a)$ *is a strictly increasing generalized saturation with bound* $\bar M=M+|\sigma(a)|$.

Proof. See Appendix A.

2 Property 5 is not satisfied by all types of robot manipulators but it is, for instance, by those with only revolute joints [2, Sect. 4.3]. This work is addressed to manipulators satisfying Property 5.

3. The proposed controller

Let $M_a = (M_{a1}, \ldots, M_{ap})^T$ and $\Theta_a = [-M_{a1}, M_{a1}] \times \cdots \times [-M_{ap}, M_{ap}]$, with M_{aj}, $j = 1, \ldots, p$, being positive constants, such that

$$|\theta_j| < M_{aj} \tag{3a}$$

$j = 1, \ldots, p$ and

$$B_{gi}^{M_a} < T_i \tag{3b}$$

$\forall i \in \{1, \ldots, n\}$, where, in accordance to Property 7, $B_{gi}^{M_a}$ are positive constants, such that $|g_i(x,y)| = |G_i(x)y| \leq B_{gi}^{M_a}$, $i = 1, \ldots, n$, $\forall x \in R^n$, $\forall y \in \Theta_a$. Let us note that Assumption 1 ensures the existence of positive values M_{aj}, $\forall j \in \{1, \ldots, p\}$, satisfying inequalities (3). Also notice that inequality (3b) is satisfied if $\sum_{j=1}^{p} B_{G_{ij}} M_{aj} < T_i$, $B_{G_i} \| M_a \| < T_i$, or $B_G \| M_a \| < T_i$, $i = 1, \ldots, n$. Actually, $\sum_{j=1}^{p} B_{G_{ij}} M_{aj}$, $B_{G_i} \| M_a \|$ or $B_G \| M_a \|$ may be taken as the value of $B_{gi}^{M_a}$ as long as inequality (3b) is satisfied.

The proposed output-feedback adaptive control scheme is defined as

$$u(q, \vartheta, \hat{\theta}) = -s_P(K_P \bar{q}) - s_D(K_D \vartheta) + G(q)\hat{\theta} \tag{4}$$

where $\bar{q} = q - q_d$ for any constant (desired equilibrium position) vector $q_d \in R^n$, $G(q)$ is the regression matrix related to the gravity vector, according to Property 6, such that $g(q,\theta) = G(q)\theta$, $K_P \in R^{n \times n}$ and $K_D \in R^{n \times n}$ are positive definite diagonal matrices, *i.e.*, $K_P = \mathrm{diag}[k_{P1}, \ldots, k_{Pn}]$ and $K_D = \mathrm{diag}[k_{D1}, \ldots, k_{Dn}]$ with $k_{Pi} > 0$ and $k_{Di} > 0$ for all $i = 1, \ldots, n$,

$$s_P : R^n \to R^n$$
$$x \mapsto (\sigma_{P1}(x_1), \ldots, \sigma_{Pn}(x_n))^T$$

and

$$s_D : R^n \to R^n$$
$$x \mapsto (\sigma_{D1}(x_1), \ldots, \sigma_{Dn}(x_n))^T$$

with $\sigma_{Pi}(\cdot)$ and $\sigma_{Di}(\cdot)$, $i = 1, \ldots, n$, being generalized saturation functions with bounds M_{Pi} and M_{Di} such that

$$M_{Pi} + M_{Di} < T_i - B_{gi}^{Ma} \tag{5}$$

$i = 1, \ldots, n$,[3] $\vartheta \in R^n$ (the velocity estimator) and $\hat{\theta} \in \Theta_a \subset R^p$ (the parameter estimator) are the output vector variables of (interconnected) auxiliary dynamic subsystems, defined as

$$\dot{q}_c = -AK_D^{-1} s_D\big(K_D(q_c + B\bar{q})\big) \tag{6a}$$

$$\vartheta = q_c + B\bar{q} \tag{6b}$$

(the velocity estimation [or motion dissipation dynamic] algorithm)

$$\dot{\phi}_c = -\varepsilon \Gamma G^T(q)\big[s_P(K_P\bar{q}) + \alpha s_D(K_D\vartheta)\big] \tag{7a}$$

$$\hat{\theta} = s_a(\phi_c - \Gamma \Upsilon^T(q)) \tag{7b}$$

(the parameter estimation [or adaptation] algorithm), where $A \in R^{n \times n}$, $B \in R^{n \times n}$ and $\Gamma \in R^{p \times p}$ are positive definite diagonal matrices *i.e.*, $A = \mathrm{diag}[a_1, \ldots, a_n]$ and $B = \mathrm{diag}[b_1, \ldots, b_n]$, with $a_i > 0$ and $b_i > 0$ for all $i = 1, \ldots, n$ and $\Gamma = \mathrm{diag}[\gamma_1, \ldots, \gamma_p]$ with $\gamma_j > 0$ for all $j = 1, \ldots, p$, q_c and ϕ_c are the (internal) state vectors of the auxiliary dynamics in Eqs. (6a) and (7a) respectively, $\Upsilon(q)$ is the regression vector related to the potential energy function, according to Property 6, *i.e.*, $U(q,\theta) = \Upsilon(q)\theta$,

$$s_a : R^p \to R^p$$
$$x \mapsto (\sigma_{a1}(x_1), \ldots, \sigma_{ap}(x_p))^T$$

with $\sigma_{aj}(\cdot)$, $j = 1, \ldots, p$, being strictly increasing generalized saturation functions with bounds M_{aj} satisfying inequalities (3), α is a constant that may arbitrarily take any real value and ε is a (sufficiently small) positive constant. A block diagram of the proposed output-feedback adaptive control scheme is shown in Fig. 1.

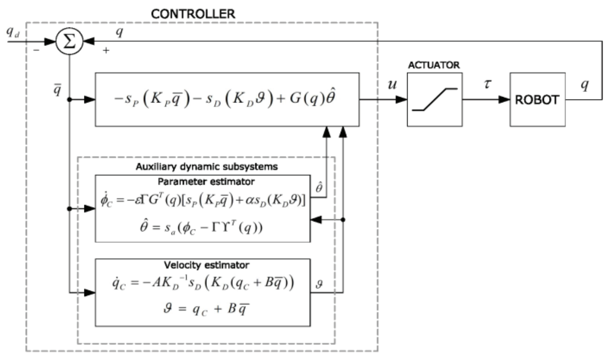

Figure 1. Block diagram of the proposed scheme

Remark 1 Observe that the control scheme in (4), (6) and (7) does not involve the exact values of the elements of θ. It only requires the satisfaction of inequalities (3). In other words, only strict bounds M_{aj} of $|\theta_j|$, $j = 1, \ldots, p$, satisfying inequalities (3b) are involved. Notice further that the velocity vector $\dot{q}$ is not involved in the expressions in Eqs. (4), (6) or (7).

Remark 2 Note that the simplest version of the proposed control scheme arises by taking $\alpha = 0$. However, the α-term extending the adaptation dynamics in (7a) has been included for the sake of generality, since an analogue

3 Observe that the satisfaction of inequalities (3) guarantees positivity of the right-hand side of inequality (5). As will become clear later, inequality (5) constitutes the tuning criterion on M_{Pi} and M_{Di} through which the input variables u_i are prevented from reaching their natural saturation bound T_i along the closed-loop trajectories.

term was considered in a previous approach [18]. Furthermore, the α -term in (7a) has a natural influence in the closed-loop responses which could be used for performance adjustment purposes. This aspect is not explored in this work.

4. Closed-loop analysis

Consider system (1) and (2), taking $u = u(q,\vartheta,\hat{\theta})$ as defined in Eqs. (4), (6) and (7). Define the variable transformation

$$\begin{pmatrix} \bar{q} \\ \vartheta \\ \bar{\phi} \end{pmatrix} : \begin{pmatrix} q \\ q_c \\ \phi_c \end{pmatrix} \mapsto \begin{pmatrix} q - q_d \\ q_c + B(q-q_d) \\ \phi_c - \Gamma Y^T(q) - \phi^* \end{pmatrix} \quad (8)$$

with $\phi^* = (\phi_1^*,\ldots,\phi_p^*)^T$, such that $s_a(\phi^*) = \theta$, or equivalently, $\phi_j^* = \sigma_{aj}^{-1}(\theta_j)$, $j = 1,\ldots,p$.[4] Observe that from the satisfaction of inequalities (3) and (5), we have $| u_i(\bar{q}+q_d,\vartheta,s_a(\bar{\phi}+\phi^*)) | \le M_{Pi} + M_{Di} + B_{gi}^{M_a} < T_i$, $i = 1,\ldots,n$, $\forall(\bar{q},\vartheta,\bar{\phi}) \in R^n \times R^n \times R^p$, whence, in view of (2), one sees that

$$T_i > | u_i(\bar{q}+q_d,\vartheta,s_a(\bar{\phi}+\phi^*)) | = | u_i | = | \tau_i |, \quad i = 1,\ldots,n, \ \forall(\bar{q},\vartheta,\bar{\phi}) \in R^n \times R^n \times R^p \quad (9)$$

Thus, under the consideration of Property 6 and the variable transformation (8), the closed-loop dynamics adopt the (equivalent) form[5]

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} = -s_P(K_P\bar{q}) - s_D(K_D\vartheta) + G(q)\bar{s}_a(\bar{\phi}) \quad (10a)$$

$$\dot{\vartheta} = -AK_D^{-1}s_D(K_D\vartheta) + B\dot{q} \quad (10b)$$

$$\dot{\bar{\phi}} = -\Gamma G^T(q)\left[\varepsilon s_P(K_P\bar{q}) + \alpha\varepsilon s_D(K_D\vartheta) + \dot{q}\right] \quad (10c)$$

where $\bar{s}_a(\bar{\phi}) = s_a(\bar{\phi}+\phi^*) - s_a(\phi^*)$. Note that by point 6 of Lemma 1, the elements of $\bar{s}_a(\bar{\phi})$, *i.e.*, $\bar{\sigma}_{aj}(\bar{\phi}_j) = \sigma_{aj}(\bar{\phi}_j + \phi_j^*) - \sigma_{aj}(\phi_j^*)$, $j = 1,\ldots,p$, turn out to be strictly increasing generalized saturation functions.

Remark 3 Let us note that from Eqs. (10) under stationary conditions, *i.e.*, by considering $\ddot{q} = \dot{q} = \dot{\vartheta} = 0_n$ and $\dot{\bar{\phi}} = 0_p$, q_d proves to be the unique equilibrium position of the closed-loop system (or equivalently, 0_n is the unique equilibrium position error of the closed loop), while the parameter estimation error equilibrium vector $\bar{\phi}_e$ turns out to be defined by the solutions of the equation $G(q_d)\bar{s}_a(\bar{\phi}_e) = 0_n$, and consequently $\bar{s}_a(\bar{\phi}_e) \in \ker(G(q_d))$.

Let

$$\varepsilon_0 = \sqrt{\frac{\mu_m}{\mu_M^2\left(\beta_P + \alpha^2\beta_{D0}\right)}} \quad (11a)$$

$$\varepsilon_1 = \frac{f_m}{\beta_{MP} + \frac{f_M^2}{2} + |\alpha|\left(\beta_{MD} + \frac{f_m}{\varepsilon_3}\right)} \quad (11b)$$

$$\varepsilon_2 = \frac{2\beta_m}{1 + \alpha^2 + \frac{2\beta_m}{\varepsilon_3}|\alpha|} \quad (11c)$$

where

$$\beta_P = \max_i\left\{\sigma'_{PiM}k_{Pi}\right\} \quad , \quad \beta_{D0} = \max_i\left\{\frac{\sigma'_{DiM}k_{Di}}{b_i}\right\}$$

$$\beta_{MP} = k_cB_P + \mu_M\beta_P \quad , \quad \beta_{MD} = k_cB_D + \mu_M\beta_{D1}$$

$$\beta_m = \min_i\left\{\frac{a_i}{b_ik_{Di}}\right\} \quad , \quad \varepsilon_3 = \frac{2\sqrt{f_m\beta_m}}{\beta_{Ma}}$$

$$B_P = \sqrt{\sum_{i=0}^{n} M_{Pi}^2} \quad , \quad B_D = \sqrt{\sum_{i=0}^{n} M_{Di}^2}$$

$$\beta_{D1} = \max_i\left\{\sigma'_{DiM}k_{Di}b_i\right\} \quad , \quad \beta_{Ma} = f_M + \mu_M\beta_{Da}$$

$$\beta_{Da} = \max_i\left\{\sigma'_{DiM}a_i\right\}$$

with σ'_{PiM} and σ'_{DiM} being the positive bounds of $D^+\sigma_{Pi}(\cdot)$ and $D^+\sigma_{Di}(\cdot)$ respectively, in accordance to point 2 of Lemma 1, and μ_m, μ_M, k_c, f_m, and f_M as defined in Properties 1, 2 and 4. We are now ready to state the main analytical result.

Proposition 1 Consider the closed-loop system in Eqs. (10), under the satisfaction of Assumption 1 and inequalities (3) and (5), and the positive constants ε_k, $k = 0,1,2$, defined in Eqs. (11). Then, given any positive definite diagonal matrices K_P, K_D, A, B and Γ, and any $\alpha \in R$ there exists $\varepsilon^* \ge \min\{\varepsilon_0,\varepsilon_1,\varepsilon_2\}$ such that, for any $\varepsilon \in (0,\varepsilon^*)$, the trivial solution $(\bar{q},\vartheta,\bar{\phi})(t) \equiv (0_n,0_n,0_p)$ is stable and, for any initial condition $(\bar{q},\dot{q},\vartheta,\bar{\phi})(0) \in R^n \times R^n \times R^n \times R^p$, $(\bar{q},\vartheta)(t) \to (0_n,0_n)$ *as* $t \to \infty$, and $\bar{s}_a(\bar{\phi}(t)) \to \ker(G(q_d))$ as $t \to \infty$, with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1,\ldots,n$, $\forall t \ge 0$.

4 Notice that their strictly increasing character renders the generalized saturations σ_{aj}, $j = 1,\ldots,p$, (involved in the definition of s_a) invertible.

5 Observe from the variable transformation defined through (8) that $q = \bar{q} + q_d$, and consequently $H(q) = H(\bar{q}+q_d)$, $C(q,\dot{q}) = C(\bar{q}+q_d,\dot{q})$ and $G(q) = G(\bar{q}+q_d)$. However, for the sake of simplicity, $H(q)$, $C(q,\dot{q})$, and $G(q)$ are used throughout the paper.

Proof. From (9) one sees that along the system trajectories $|\tau_i(t)|=|u_i(t)|<T_i, \quad \forall t\geq 0$. This proves that, under the proposed output-feedback adaptive scheme, the input saturation values T_i are never reached. Now, in order to develop the stability/convergence analysis, let us define the scalar function

$$\begin{aligned}V(\bar{q},\dot{q},\vartheta,\bar{\phi}) &= \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})H(q)\dot{q} \\ &+ \alpha\varepsilon s_D^T(K_D\vartheta)H(q)\dot{q} + \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr \\ &+ \int_{0_n}^{\vartheta} s_D^T(K_D r)B^{-1}dr + \int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr\end{aligned}$$

where

$$\begin{aligned}\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr &= \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i \\ \int_{0_n}^{\vartheta} s_D^T(K_D r)B^{-1}dr &= \sum_{i=1}^n \int_0^{\vartheta_i} \sigma_{Di}(k_{Di}r_i)b_i^{-1}dr_i\end{aligned}$$

and

$$\int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr = \sum_{j=1}^{p}\int_0^{\bar{\phi}_j} \bar{s}_{aj}(r_j)\gamma_j^{-1}dr_j$$

with ε satisfying

$$\varepsilon < \varepsilon_M := \min\{\varepsilon_0,\varepsilon_1,\varepsilon_2\} \tag{12}$$

Observe that from Property 1 and point 3 of Lemma 1, we have

$$\begin{aligned}V(\bar{q},\dot{q},\vartheta,\bar{\phi}) &\geq W_0(\bar{q},\dot{q},\vartheta) + (1-\delta_0)\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr \\ &+ (1-\delta_0)\int_{0_n}^{\vartheta} s_D^T(K_D r)B^{-1}dr + \int_{0_p}^{\bar{\phi}} \bar{s}_a^T(r)\Gamma^{-1}dr\end{aligned} \tag{13}$$

where

$$W_0(\bar{q},\dot{q},\vartheta) = \frac{1}{2}\begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{q}\| \\ \|s_D(K_D\vartheta)\|\end{pmatrix}^T Q_0 \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{q}\| \\ \|s_D(K_D\vartheta)\|\end{pmatrix}$$

with

$$Q_0 = \begin{pmatrix}\frac{\delta_0}{\beta_P} & -\varepsilon\mu_M & 0 \\ -\varepsilon\mu_M & \mu_m & -|\alpha|\varepsilon\mu_M \\ 0 & -|\alpha|\varepsilon\mu_M & \frac{\delta_0}{\beta_{D0}}\end{pmatrix}$$

and δ_0 is a positive constant satisfying

$$\frac{\varepsilon^2}{\varepsilon_0^2} < \delta_0 < 1 \tag{14}$$

(see (12)). Furthermore, note that, by (14), $W_0(\bar{q},\dot{q},\vartheta)$ is positive definite (since with $\varepsilon<\varepsilon_M\leq\varepsilon_0$, in accordance with (12), any δ_0 satisfying (14) renders Q_0 positive definite) and observe that $W_0(0_n,\dot{q},0_n)\to\infty$ as $\|\dot{q}\|\to\infty$. From this, inequality (14) and points 4 and 5 of Lemma 1 (through which one sees that the integral terms in the right-hand side of (13) are radially unbounded positive definite functions), $V(\bar{q},\dot{q},\vartheta,\bar{\phi})$ is concluded to be positive definite and radially unbounded. Its upper right-hand derivative along the system trajectories $\dot{V}=D^+V$ [25,App. I] [29,§6.1A] is given by

$$\begin{aligned}&\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) \\ &= \dot{q}^T H(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{H}(q,\dot{q})\dot{q} + \varepsilon s_P^T(K_P\bar{q})H(q)\ddot{q} \\ &\quad + \varepsilon\dot{q}^T\dot{H}(q,\dot{q})s_P(K_P\bar{q}) + \varepsilon\dot{q}^T H(q)s_P'(K_P\bar{q})K_P\dot{q} \\ &\quad + \alpha\varepsilon s_D^T(K_D\vartheta)H(q)\ddot{q} + \alpha\varepsilon\dot{q}^T\dot{H}(q,\dot{q})s_D(K_D\vartheta) \\ &\quad + \alpha\varepsilon\dot{q}^T H(q)s_D'(K_D\vartheta)K_D\dot{q} + s_P^T(K_P\bar{q})\dot{q} \\ &\quad + s_D^T(K_D\vartheta)B^{-1}\dot{\vartheta} + \bar{s}_a^T(\bar{\phi})\Gamma\dot{\bar{\phi}} \\ &= -\dot{q}^T F\dot{q} - \varepsilon s_P^T(K_P\bar{q})F\dot{q} - \varepsilon s_P^T(K_P\bar{q})s_P(K_P\bar{q}) \\ &\quad - (1+\alpha)\varepsilon s_P^T(K_P\bar{q})s_D(K_D\vartheta) \\ &\quad + \varepsilon\dot{q}^T C(q,\dot{q})s_P(K_P\bar{q}) + \varepsilon\dot{q}^T H(q)s_P'(K_P\bar{q})K_P\dot{q} \\ &\quad - \alpha\varepsilon s_D^T(K_P\vartheta)F\dot{q} - \alpha\varepsilon s_D^T(K_P\vartheta)s_D(K_D\vartheta) \\ &\quad + \alpha\varepsilon\dot{q}^T C(q,\dot{q})s_D(K_D\vartheta) \\ &\quad - \alpha\varepsilon\dot{q}^T H(q)s_D'(K_D\vartheta)As_D(K_D\vartheta) \\ &\quad + \alpha\varepsilon\dot{q}^T H(q)s_D'(K_D\vartheta)K_D B\dot{q} \\ &\quad - s_D^T(K_D\vartheta)B^{-1}AK_D^{-1}s_D(K_D\vartheta)\end{aligned}$$

where $H(q)\ddot{q}$, $\dot{\vartheta}$ and $\dot{\bar{\phi}}$ have been replaced by their equivalent expression from the closed-loop manipulator dynamics in Eqs. (10), Property 3 has been used and

$$\begin{aligned}s_P'(K_P\bar{q}) &= \mathrm{diag}[D^+\sigma_{P1}(k_{P1}\bar{q}_1),\ldots,D^+\sigma_{Pn}(k_{Pn}\bar{q}_n)] \\ s_D'(K_D\bar{q}) &= \mathrm{diag}[D^+\sigma_{D1}(k_{D1}\bar{q}_1),\ldots,D^+\sigma_{Dn}(k_{Dn}\bar{q}_n)]\end{aligned}$$

Observe that from Properties 1, 2 and 4 and points (b) of Definition 1 and 2 of Lemma 1, we have

$$\begin{aligned}&\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) \\ &\leq -f_m\|\dot{q}\|^2 + \varepsilon f_M\|s_P(K_P\bar{q})\|\,\|\dot{q}\| - \varepsilon\|s_P(K_P\bar{q})\|^2 \\ &\quad + |1+\alpha|\varepsilon\|s_P(K_P\bar{q})\|\,\|s_D(K_D\vartheta)\| + \varepsilon k_C B_P\|\dot{q}\|^2 \\ &\quad + \varepsilon\mu_M\beta_P\|\dot{q}\|^2 + |\alpha|\varepsilon f_M\|\dot{q}\|\,\|s_D(K_D\vartheta)\| \\ &\quad - \varepsilon\alpha\|s_D(K_D\vartheta)\|^2 + |\alpha|\varepsilon k_C B_D\|\dot{q}\|^2 \\ &\quad + |\alpha|\varepsilon\mu_M\beta_{Da}\|\dot{q}\|\,\|s_D(K_D\vartheta)\| + |\alpha|\varepsilon\mu_M\beta_{D1}\|\dot{q}\|^2 \\ &\quad - \beta_m\|s_D(K_D\vartheta)\|^2 \\ &\leq -W_1(\bar{q},\dot{q}) - W_2(\dot{q},\vartheta) - W_3(\bar{q},\vartheta)\end{aligned}$$

where

$$W_1(\bar{q},\dot{q}) = \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}^T Q_1 \begin{pmatrix}\|s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}$$

$$W_2(\dot{q},\vartheta)=\begin{pmatrix} \|\dot{q}\| \\ \|s_D(K_D\vartheta)\| \end{pmatrix}^T Q_2 \begin{pmatrix} \|\dot{q}\| \\ \|s_D(K_D\vartheta)\| \end{pmatrix}$$

$$W_3(\bar{q},\vartheta)=\begin{pmatrix} \|s_P(K_P\bar{q})\| \\ \|s_D(K_D\vartheta)\| \end{pmatrix}^T Q_3 \begin{pmatrix} \|s_P(K_P\bar{q})\| \\ \|s_D(K_D\vartheta)\| \end{pmatrix}$$

with

$$Q_1=\begin{pmatrix} \frac{\varepsilon}{2} & -\frac{\varepsilon f_M}{2} \\ -\frac{\varepsilon f_M}{2} & \delta_1 f_m-\varepsilon\left(\beta_{MP}+|\alpha|\beta_{MD}\right) \end{pmatrix}$$

$$Q_2=\begin{pmatrix} (1-\delta_1)f_m & -\frac{|\alpha|\varepsilon\beta_{Ma}}{2} \\ -\frac{|\alpha|\varepsilon\beta_{Ma}}{2} & (1-\delta_1)\beta_m \end{pmatrix}$$

$$Q_3=\begin{pmatrix} \frac{\varepsilon}{2} & -\frac{|1+\alpha|\varepsilon}{2} \\ -\frac{|1+\alpha|\varepsilon}{2} & \alpha\varepsilon+\delta_1\beta_m \end{pmatrix}$$

where δ_1 is a positive constant satisfying

$$0<\delta_{1m}:=\varepsilon\left[\max\left\{\frac{1}{\varepsilon_1},\frac{1}{\varepsilon_2}\right\}-\frac{|\alpha|}{\bar{\varepsilon}_3}\right] <\delta_1<1-\frac{|\alpha|\varepsilon}{\bar{\varepsilon}_3}=:\delta_{1M}<1 \quad (15)$$

Let us note that the fulfilment of (12) guarantees the existence of values $\delta_1\in(0,1)$ that satisfy (15) (since $\varepsilon<\varepsilon_M\le\min\{\varepsilon_1,\varepsilon_2\}$, in accordance with (12), implies that $\delta_{1m}<\delta_{1M}$), while the satisfaction of (15) renders $W_1(\bar{q},\dot{q})$, $W_2(\dot{q},\vartheta)$ and $W_3(\bar{q},\vartheta)$ positive definite (with respect to their arguments, since, under such a condition, Q_1, Q_2, and Q_3 turn out to be positive definite). Hence, $\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi})\le 0$, $\forall(\bar{q},\dot{q},\vartheta,\bar{\phi})\in R^n\times R^n\times R^n\times R^p$ with $\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi})=0\longleftrightarrow(\bar{q},\dot{q},\vartheta)=(0_n,0_n,0_n)$. Therefore, by Lyapunov's second method[6], the trivial solution $(\bar{q},\vartheta,\bar{\phi})(t)\equiv(0_n,0_n,0_p)$ is concluded to be stable. Now, in view of the radially unbounded character of $V(\bar{q},\dot{q},\vartheta,\bar{\phi})$, the set $\Omega=\{(\bar{q},\dot{q},\vartheta,\bar{\phi})\in R^n\times R^n\times R^n\times R^p : V(\bar{q},\dot{q},\vartheta,\bar{\phi})\le c\}$ is compact for any positive constant c [24, p.128]. Moreover, in view of the semi-negative definite character of $\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi})$, Ω is positively invariant with respect to the closed-loop dynamics [24, p.115]. Furthermore, from previous arguments: $E:=\{(\bar{q},\dot{q},\vartheta,\bar{\phi})\in\Omega: \dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi})=0\}=\{(\bar{q},\dot{q},\vartheta,\bar{\phi})\in\Omega:\bar{q}=\dot{q}=\vartheta=0_n\}$. Further, from Remark 3, the largest invariant set in E, denoted $\mathcal{M}$, is given as $\mathcal{M}=\{(\bar{q},\dot{q},\vartheta,\bar{\phi})\in E:\bar{s}_a(\bar{\phi})\in\ker(G(q_d))\}$. Thus, by the invariance theory [29, Sect. 7.2] (more specifically, by [29, Theorem 7.2.1], see Appendix B), we have $(\bar{q},\dot{q},\vartheta,\bar{\phi})(0)\in\Omega \Rightarrow(\bar{q},\dot{q},\vartheta,\bar{\phi})(t)\to\mathcal{M}$ as $t\to\infty$. Since this holds for any $c>0$ and $V(\bar{q},\dot{q},\vartheta,\bar{\phi})$ is radially unbounded (in view of which Ω may be rendered arbitrarily large), we conclude that for any $(\bar{q},\dot{q},\vartheta,\bar{\phi})(0)\in R^n\times R^n\times R^n\times R^p$, $(\bar{q},\vartheta)(t)\to(0_n,0_n)$ as $t\to\infty$ and $\bar{s}_a(\bar{\phi}(t))\to\ker(G(q_d))$ as $t\to\infty$. Finally, from (12) and its sufficient character, as a condition supporting the proof, the stated stability/convergence result is concluded to hold with $\varepsilon\in(0,\varepsilon^*)$ for some $\varepsilon^*\ge\varepsilon_M$.

Corollary 1 If $G^T(q_d)G(q_d)$ is non-singular then the trivial solution $(\bar{q},\vartheta,\bar{\phi})(t)\equiv(0_n,0_n,0_p)$ is globally asymptotically stable.

Proof. Note on the one hand that non-singularity of $G^T(q_d)G(q_d)$ implies that $\ker(G(q_d))=\{0_p\}$ and on the other hand that $\bar{s}_a(\bar{\phi})=0_p\longleftrightarrow\bar{\phi}=0_p$. Then from Proposition 1 we have that, for any $(\bar{q},\dot{q},\vartheta,\bar{\phi})(0)\in R^n\times R^n\times R^n\times R^p$, $(\bar{q},\vartheta,\bar{\phi})(t)\to(0_n,0_n,0_p)$ as $t\to\infty$, whence the stability of the trivial solution $(\bar{q},\vartheta,\bar{\phi})(t)\equiv(0_n,0_n,0_p)$ is concluded to be globally asymptotical [24, Sect. 4.1], [25, Chap. I, Sect. 2.10-2.11].

Remark 4 Observe that the coefficient ε is involved in the control algorithm through the adaptation subsystem in Eqs. (7). Thus, inequality (12) (obtained as a condition under which the proposed Lyapunov function and its upper right-hand derivative along the system trajectories get the expected analytical properties) may be taken as a tuning criterion on ε, through which the stated stability/convergence result is guaranteed. Furthermore, notice that such a tuning criterion on ε does not require the exact knowledge of the system parameters. Indeed, observe that an estimation of the right-hand side of inequality (12) may be obtained by means of upper and lower bounds of the system parameters and viscous friction coefficients (more precisely, nonzero lower bounds of μ_m and f_m and upper bounds of μ_M, k_c and f_M, see Eqs. (11)). Furthermore, as pointed out in the proof of Proposition 1, the satisfaction of (12) is not necessary, but it is only sufficient for the closed-loop analysis to hold, which permits the consideration of values of ε higher than ε_M, up to certain limit ε^*, without destabilizing the closed loop.

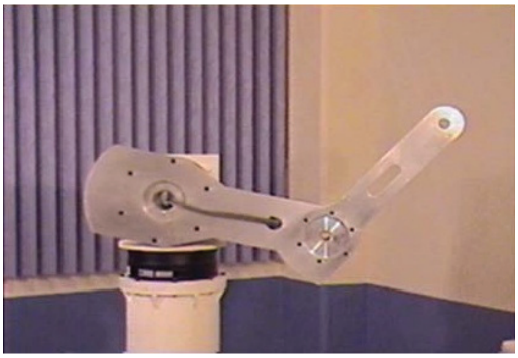

Figure 2. Experimental setup

6 See for instance [25,Chap. II, Sect. 6], where (generalized) statements of Lyapunov's second method are presented under the consideration of locally Lipschitz-continuous Lyapunov functions and their upper right-hand derivative along the system trajectories.

5. Experimental results

In order to experimentally corroborate the efficiency of the proposed scheme, referred to as the SP-SDc-ga controller, real-time control implementations were carried out on a 3-DOF manipulator. The experimental setup, shown in Fig. 2, is a 3-revolute-joint anthropomorphic arm located at the *Benemerita Universidad Autonoma de Puebla*, Mexico. The actuators are direct-drive brushless motors (from Parker Compumotors) operated in torque mode, so they act as a torque source and accept an analogue voltage as a reference of torque signal. Position information is obtained from incremental encoders located on the motors. The setup includes a Pentium host computer and a system of electronic instrumentation, based on the motion control board MFIO3A, manufactured by Precision Microdynamics. The robot software is in open architecture, whose platform is based in C language to run the control algorithm in real time. The control routine registers data generated during the first 2000 samples at a default sample time of $T_s = 2.5$ ms, but T_s can be changed to higher values in accordance to the desired experimental duration. The experiments carried out in the context of this work, whose results are presented below, were run taking $T_s = 0.12$ s. A more detailed technical description of this robot is given in [30].

For the considered experimental manipulator, Properties 5 and 6 are satisfied with

$$G(q) = \begin{pmatrix} 0 & 0 \\ \sin q_2 & \sin(q_2+q_3) \\ 0 & \sin(q_2+q_3) \end{pmatrix}, \quad \theta = \begin{pmatrix} 38.465 \\ 1.825 \end{pmatrix} \text{ [Nm]} \qquad (16)$$

$B_{g1} = 0$, $B_{g2} = 40.29$ Nm, $B_{g3} = 1.825$ Nm, and

$$Y(q) = \begin{pmatrix} 1-\cos q_2 & 1-\cos(q_2+q_3) \end{pmatrix}$$

with $q^* \in \mathcal{U}_0 = \{q \in R^3 : q_2 = q_3 = 0\}$, *i.e.*, such that $U(q^*, \theta) = 0$, $\forall q^* \in \mathcal{U}_0$ (see Property 6). The maximum allowed torques (input saturation bounds) are $T_1 = 50$ Nm, $T_2 = 150$ Nm and $T_3 = 15$ Nm for the first, second and third links respectively. From these data, one easily corroborates that Assumption 1 is fulfilled.

The involved saturation functions were defined as $\sigma_{Pi}(\varsigma) = M_{Pi}\,\mathrm{sat}\left(\varsigma / M_{P_i}\right)$, $\sigma_{Di}(\varsigma) = M_{Di}\,\mathrm{sat}\left(\varsigma / M_{D_i}\right)$, $i = 1,2,3$, and

$$\sigma_{aj}(\varsigma) = \begin{cases} \varsigma & \forall\, |\varsigma| \le L_{aj} \\ \rho_j(\varsigma) & \forall\, |\varsigma| > L_{aj} \end{cases}$$

$j = 1,2$, where

$$\rho_j(\varsigma) = \mathrm{sign}(\varsigma)L_{aj} + (M_{aj} - L_{aj})\tanh\left(\frac{\varsigma - \mathrm{sign}(\varsigma)L_{aj}}{M_{aj} - L_{aj}}\right)$$

with $0 < L_{aj} < M_{aj}$. Let us note that with these saturation functions we have $\sigma'_{PiM} = \sigma'_{DiM} = 1$, $\forall i \in \{1,2,3\}$. The experimental implementations were run fixing the following saturation parameter values (all of them expressed in Nm): $M_{P1} = M_{D1} = 20$, $M_{P2} = M_{D2} = 35$, $M_{P1} = M_{D1} = 4$, $M_{a1} = 70$, $M_{a2} = 5$, and $L_{aj} = 0.9M_{aj}$, $j = 1,2$. These saturation function parameter values were corroborated to satisfy inequalities (3) and (5), taking $B_{gi}^{M_a} = \sum_{j=1}^{2} B_{G_{ij}} M_{aj}$, $i = 1,2,3$, *i.e.* $B_{g1}^{M_a} = 0$, $B_{g2}^{M_a} = 75$ Nm and $B_{g3}^{M_a} = 5$.

For comparison purposes, additional experiments were run implementing the output-feedback adaptive algorithm proposed in [18], referred to as the L00 controller (choice made in terms of the analogue nature of the compared algorithms: output-feedback adaptive developed in a bounded input context; comparison of controllers of a different nature loses coherence), *i.e.*,

$$u = -K_P T_h(\lambda \bar{q}) - K_D T_h(\delta \vartheta) + G_d \hat{\theta} \qquad (17a)$$

where $G_d = G(q_d)$, $T_h(x) = \left(\tanh(x_1), \ldots, \tanh(x_n)\right)^T$, $K_P \in R^n$ and $K_D \in R^n$ are positive definite diagonal matrices, λ and δ are positive constants and $\vartheta \in R^n$ and $\hat{\theta} \in R^p$ are the output variables of (interconnected) auxiliary dynamic subsystems that take the following form:

$$\begin{aligned} \dot{q}_c &= -\bar{\alpha} K(q_c + K\bar{q}) \\ \vartheta &= q_c + K\bar{q} \end{aligned} \qquad (17b)$$

and

$$\begin{aligned} \dot{\phi}_c &= \beta G_d^T \left[\eta T_h(\delta \vartheta) - \mu T_h(\lambda \bar{q})\right] \\ \hat{\theta} &= \phi_c - \beta G_d^T \bar{q} \end{aligned} \qquad (17c)$$

where $K \in R^n$ is a positive definite diagonal matrix and $\bar{\alpha}$, β, η and μ are positive constants. Arguing simplicity of development, the constant matrices involved in this control algorithm are taken in [18] as $K_P = k_P I_n$, $K_D = k_D I_n$ and $K = kI_n$, with k_P, k_D and k being positive constants. However, in order to speed up the closed-loop responses, different P and D control gains were considered at every input control expression. In other words, K_P and K_D in (17a) were taken as $K_P = \mathrm{diag}[k_{P1}, \ldots, k_{Pn}]$ and $K_D = \mathrm{diag}[k_{D1}, \ldots, k_{Dn}]$ with gains k_{Pi} and k_{Di}, $i = 1, \ldots, n$, which each have their own different positive value. Furthermore, observe that through this controller, if input saturation is to be avoided, the control gains must satisfy saturation-avoidance inequalities of the form $k_{Pi} + k_{Di} \le T_i - B_i$, $i = 1,\ldots,n$, for some initial-condition-dependent positive constants B_i, $i = 1,\ldots,n$.

The initial conditions and desired link positions for all the executed experiments were: $q_i(0) = \dot{q}_i(0) = \vartheta_i(0) = 0$, $i = 1,2,3$; $\phi_{c1}(0) = 20$, $\phi_{c2}(0) = 1$ [Nm]; $q_{d1} = q_{d2} = \pi/4$ and $q_{d3} = \pi/2$ [rad]. With this desired configuration, the condition stated by Corollary 1 turns out to be satisfied.[7]

For the proposed controller, with $\alpha = -1$, the selected parameter combination was found through simulation tests, so as to have as good closed-loop responses as possible, mainly in terms of stabilization time (as short as possible) and transient response (avoiding or lowering down overshoot and oscillations as much as possible), and further refining the tuning experimentally. The resulting values were K_P=diag[350 , 400 , 75] Nm/rad, K_D=diag[25 , 50 , 12] Nms/rad, A=diag[15 , 50 , 35] rad/s², B=diag[5 , 10 , 5] s^{-1}, $\Gamma = \mathrm{diag}[5,0.5]$ Nm/rad and $\varepsilon = 1.5$ rad/Nms. As for the L00 controller, a similar tuning procedure was performed disregarding the saturation-avoidance inequalities in view of the considerably poor closed-loop performance observed under their consideration. The resulting values were: K_P=diag[800 , 1300 , 200] Nm, K_D=diag[5,10,10] Nm, $\lambda = 30\ [\mathrm{rad}]^{-1}$, $\delta = 5$ s/rad, $k = 50\ \mathrm{s}^{-1}$, $\bar{\alpha} = 5$, $\beta = 25$ Nm/rad, $\eta = 5$ rad/s and $\mu = 10$ rad/s.

Figures 3-5 show the results for both implemented controllers. Observe that the regulation objective was achieved preventing input saturation and avoiding steady-state position errors. Furthermore, note that despite the presence of a small overshoot, through the SP-SD$_c$-g$_a$ algorithm shorter stabilization times took place in both position error and parameter estimation responses. Let us further note that at 240s, where the experimental data registration was stopped, the parameter estimations were still evolving. This is a consequence of the slow evolution of the adaptation subsystem dynamics, due to the relatively small value of ε in the proposed scheme and the analogue coefficients η and μ in the L00 controller. Nevertheless, the slow evolution of the adaptation subsystem dynamics did not have any influence on the position responses, which had been stabilized during the initial seconds of the experiment. The subsequent parameter estimator evolution was expected to reduce the difference among the estimations obtained through each implemented controller.

7 One can verify from $G(q)$ in (16) that, for the considered manipulator, the desired configurations that satisfy the condition stated by Corollary 1 are those such that $q_{d2} \neq m_1\pi$ and $q_{d2} + q_{d3} \neq m_2\pi$, for any $m_1, m_2 = 0, \pm1, \pm2, \ldots$

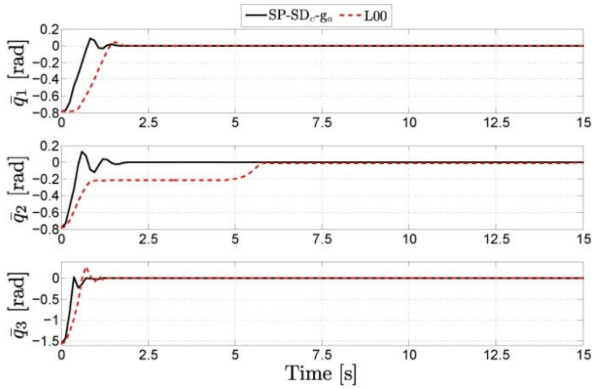

Figure 3. Position errors

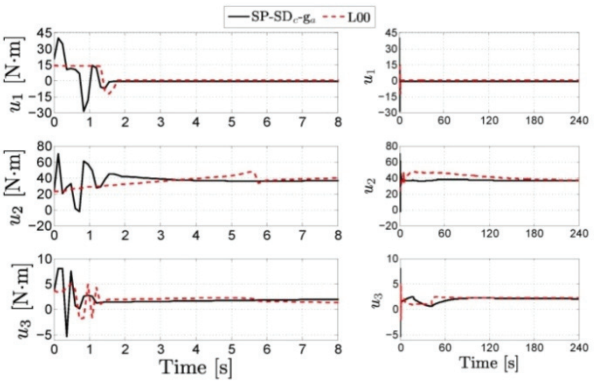

Figure 4. Control signals

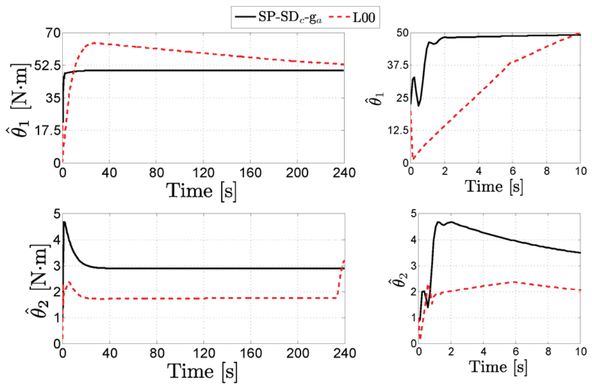

Figure 5. Parameter estimates

6. Conclusions

In this work, an output-feedback adaptive control scheme for the global regulation of robot manipulators with bounded inputs was proposed. With respect to the previous output-feedback adaptive approaches developed in a bounded-input context, the proposed velocity-free feedback controller guarantees the adaptive regulation objective: globally, avoiding discontinuities throughout the scheme, preventing the inputs from reaching their natural saturation limits and imposing no saturation-avoidance restriction on the control gains. Moreover, the developed scheme is not restricted to the use of a specific saturation function to achieve the required boundedness, but may rather involve any one within a set of smooth and non-smooth (Lipschitz-continuous) bounded passive functions that include the hyperbolic tangent and the conventional saturation as particular cases. The efficiency of the proposed scheme was corroborated through experimental tests on a 3-DOF manipulator. Good results were obtained, which were observed to improve those gotten through an algorithm that was previously developed in an analogue analytical context.

A Proof of Lemma 1

1. Since σ is a Lipschitz-continuous function that keeps the sign of its argument (according to point (a) of Definition 1) and is non-decreasing and bounded by M, there exists positive constants $c^- \leq M$ and $c^+ \leq M$ such that

$$\lim_{|\varsigma|\to\infty} \sigma(\varsigma) = \frac{(\text{sign}(\varsigma)-1)c^- + (\text{sign}(\varsigma)+1)c^+}{2} =: \sigma_\infty$$

Hence, we have

$$\begin{aligned}\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) &= \lim_{|\varsigma|\to\infty} \limsup_{h\to 0^+} \frac{\sigma(\varsigma+h)-\varsigma(\varsigma)}{h} \\ &= \limsup_{h\to 0^+} \lim_{|\varsigma|\to\infty} \frac{\sigma(\varsigma+h)-\varsigma(\varsigma)}{h} \\ &= \limsup_{h\to 0^+} \frac{\sigma_\infty - \sigma_\infty}{h} = 0\end{aligned}$$

2. Since σ is a Lipschitz-continuous non-decreasing function, $D^+\sigma(\varsigma)$ exists and is piecewise-continuous on R and $D^+\sigma(\varsigma) \geq 0$, $\forall \varsigma \in R$. Furthermore, because of its piecewise-continuity, $D^+\sigma(\varsigma)$ is bounded on any compact interval on R. Thus, its boundedness holds on R if $\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) < \infty$. Since $\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) = 0$ (according to point 1 of the statement), we conclude boundedness of $D^+\sigma(\varsigma)$ (on R), *i.e.*, there exists a non-negative finite scalar σ_M' such that $D^+\sigma(\varsigma) \leq \sigma_M'$, $\forall \varsigma \in R$. Finally, observe that by virtue of point (a) of Definition 1, there exists $a \in (0,\infty]$ such that $D^+\sigma(\varsigma) > 0$, $\forall \varsigma \in (-a,a)\setminus\{0\}$ whence we conclude that $\sigma_M' > 0$.
3. From Lipschitz-continuity of σ, its satisfaction of point (a) of Definition 1 and the boundedness of $D^+\sigma$ by a positive constant σ_M' (according to point 2 of the statement), it follows that

$$\frac{D^+\sigma(k\varsigma)}{\sigma_M'} |\sigma(k\varsigma)| \leq |\sigma(k\varsigma)| \leq \sigma_M' |k\varsigma|$$

$\forall \varsigma \in R$, whence (considering that σ has the sign of its argument, according to point (a) of Definition 1), we have

$$\int_0^\varsigma \frac{\sigma(kr)}{\sigma_M'} D^+\sigma(kr)dr \leq \int_0^\varsigma \sigma(kr)dr \leq \int_0^\varsigma k\sigma_M' r dr$$

wherefrom we get

$$\frac{\sigma^2(k\varsigma)}{2k\sigma_M'} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma_M'\varsigma^2}{2}$$

$\forall \varsigma \in R$.

4. Strict positivity of $\int_0^\varsigma \sigma(kr)dr$ on $R\setminus\{0\}$ follows from points 3 of the statement and (a) of Definition 1, by noting that $\sigma^2(k\varsigma) > 0$, $\forall \varsigma \neq 0$.
5. By the Lipschitz-continuous and non-decreasing characters of σ and its satisfaction of point (a) of Definition 1, there exist constants $a > 0$, $k_a > 0$, and $c \geq 1$, such that $|\sigma(\varsigma)| \geq k_a |a\,\text{sat}(\varsigma/a)|^c$, whence we get

$$S_a(\varsigma) := \int_0^\varsigma \text{sign}(r) k_a |a\text{sat}(r/a)|^c dr \leq \int_0^\varsigma \sigma(k\varsigma)dr$$

$\forall \varsigma \in R$, with

$$S_a(\varsigma) = \begin{cases} \dfrac{k_a}{c+1}|\varsigma|^{c+1} & \forall |\varsigma| \leq a \\ k_a a^c \left(|\varsigma| - \dfrac{ac}{c+1}\right) & \forall |\varsigma| > a \end{cases}$$

Thus, from these expressions we can observe, on the one hand, that $\lim_{|\varsigma|\to\infty} S_a(\varsigma) \leq \lim_{|\varsigma|\to\infty} \int_0^\varsigma \sigma(kr)dr$ and, on the other, that $S_a(\varsigma) \to \infty$ as $|\varsigma| \to \infty$, wherefrom we conclude that $\int_0^\varsigma \sigma(kr)dr \to \infty$ as $|\varsigma| \to \infty$.

6. Suppose σ is strictly increasing. Let $\psi, \eta, \varsigma \in R$. For any constant $a \in R$, let $\bar{\sigma}(\varsigma) = \sigma(\varsigma + a) - \sigma(a)$.
   - *Lipschitz-continuity.* From the Lipschitz-continuity of σ and point 2 of the statement, we have $|\sigma(\varsigma) - \sigma(\eta)| \leq \sigma_M' |\varsigma - \eta|$, $\forall \varsigma, \eta \in R$. Then

$$\begin{aligned}&|\bar{\sigma}(\varsigma) - \bar{\sigma}(\eta)| \\ &= \left|\left(\sigma(\varsigma+a) - \sigma(a)\right) - \left(\sigma(\eta+a) - \sigma(a)\right)\right| \\ &= |\sigma(\varsigma+a) - \sigma(\eta+a)| \\ &\leq \sigma_M' |(\varsigma+a) - (\eta+a)| \\ &\leq \sigma_M' |\varsigma - \eta|\end{aligned}$$

   $\forall \varsigma, \eta \in R$, which shows that $\bar{\sigma}$ is Lipschitz-continuous.
   - *Strictly increasing monotonicity.* From the strictly increasing monotonicity of σ, we have

$$\begin{aligned}&\bar{\sigma}(\varsigma) > \bar{\sigma}(\eta) \\ &\quad\longleftrightarrow \sigma(\varsigma+a) - \sigma(a) > \sigma(\eta+a) - \sigma(a)\end{aligned}$$

$$\longleftrightarrow \sigma(\varsigma + a) > \sigma(\eta + a)$$
$$\longleftrightarrow \varsigma + a > \eta + a$$
$$\longleftrightarrow \varsigma > \eta$$

which shows that $\bar{\sigma}$ is strictly increasing.

- $\varsigma\bar{\sigma}(\varsigma) > 0$, $\forall \varsigma \neq 0$. Since σ is strictly increasing, we have

$$\bar{\sigma}(\varsigma) = \sigma(\varsigma + a) - \sigma(a) > 0$$
$$\longleftrightarrow \sigma(\varsigma + a) > \sigma(a)$$
$$\longleftrightarrow \varsigma > 0 \quad \forall a \in R$$

and

$$\bar{\sigma}(\varsigma) = \sigma(\varsigma + a) - \sigma(a) < 0$$
$$\longleftrightarrow \sigma(\varsigma + a) < \sigma(a)$$
$$\longleftrightarrow \varsigma < 0 \quad \forall a \in R$$

whence one sees that, for any $a \in R$, $\varsigma\bar{\sigma}(\varsigma) > 0$, $\forall \varsigma \neq 0$.

- $|\bar{\sigma}(\varsigma)| \leq \bar{M} = M + |\sigma(a)|$, $\forall \varsigma \in R$. Since $|\sigma(\varsigma)| \leq M$, $\forall \varsigma \in R$, we have that

$$\begin{aligned} |\bar{\sigma}(\varsigma)| &= |\sigma(\varsigma + a) - \sigma(a)| \\ &\leq |\sigma(\varsigma + a)| + |\sigma(a)| \\ &\leq M + |\sigma(a)| = \bar{M} \end{aligned}$$

$\forall \varsigma \in R$.

Thus, according to Definition 1, $\bar{\sigma}$ is concluded to be a strictly increasing generalized saturation with bound $\bar{M} = M + |\sigma(a)|$.

B Theorem 7.2.1 of [29]

Theorem 7.2.1 in [29] states a version of La Salle's Invariance Principle that considers autonomous systems with continuous dynamics and makes use of continuous scalar functions and their upper-right derivative along the system trajectories (in contrast, for instance, with the statement presented in [24, Theorem 4.4], which is addressed to autonomous state equations with locally Lipschitz-continuous vector fields and makes use of continuously differentiable scalar functions). Consider the autonomous system

$$\dot{x} = f(x) \tag{18}$$

where $f : D \to R^n$ is continuous, $D \subset R^n$ is an open connected set and $f(0_n) = 0_n \in D$. Theorem 7.2.1 in [29] is stated as follows.

Theorem Assume that there exists a continuous function $V : D \to R$, such that $D^+V(x) \leq 0$ for all $x \in D$ and such that, for some constant $c \in R$, the set Ω is a closed and bounded component of the set $\{x \in D : V(x) \leq c\}$. Let M be the largest invariant set with respect to (18) in the set $E = \{x \in D : D^+V(x) = 0\}$. Then every solution $x(t)$ of (18) with $x(t_0) \in \Omega$ approaches the set M as $t \to \infty$.

The use of the upper right-hand derivative of V along the system trajectories, D^+V, in the statement of this theorem, is corroborated in [29, §6.1A].

7. Acknowledgements

The first and third authors were supported by CONACYT, Mexico (3rd author: Project No. 134534). One of the authors, V. Santibáñez, thanks José A. Yarza for his invaluable help during the experimental essays.

8. References

[1] Takegaki M, Arimoto S (1981) A New Feedback Method for Dynamic Control of Manipulators. J. of Dyn. Syst., Meas., and Control 102: 119-125.

[2] Kelly R, Santibáñez V, Loría A (2005) Control of Robot Manipulators in Joint Space. London: Springer.

[3] Krikelis NJ, Barkas SK (1984) Design of Tracking Systems Subject to Actuator Saturation and Integrator Wind-Up. Int. J. of Control 39: 667-682.

[4] Berghuis H, Nijmeijer H (1993) A Passivity Approach to Controller-Observer Design for Robots. IEEE Trans. on Rob. and Automation 9: 740-754.

[5] Tomei P (1991) Adaptive PD Controller for Robot Manipulator. IEEE Trans. Rob. Aut. 7: 565-570.

[6] Kelly R, Santibáñez V, Berghuis H (1997) Point-to-Point Robot Control Under Actuator Constraints. Cont. Engineering Practice 5: 1555-1562.

[7] Santibáñez V, Kelly R, Reyes F (1998) A New Set-Point Controller with Bounded Torques for Robot Manipulators. IEEE Tr. Ind. Elect. 45: 126-133.

[8] Zavala-Río A, Santibáñez V (2006) Simple Extensions of the PD-with-Gravity-Compensation Control Law for Robot Manipulators with Bounded Inputs. IEEE Tr. Cont. Syst. Tech. 14: 958-965.

[9] Santibáñez V, Kelly R (1997) On Global Regulation of Robot Manipulators: Saturated Linear State Feedback and Saturated Linear Output Feedback. European J. of Control 3: 104-113.

[10] Loría A, Kelly R, Ortega R, Santibáñez V (1997) On Global Output Feedback Regulation of Euler-Lagrange Systems with Bounded Inputs. IEEE Trans. on Automatic Control 42: 1138-1143.

[11] Burkov IV (1995) Stabilization of Mechanical Systems via Bounded Control and without Velocity Measurements. In: Proc. 2nd Russian-Swedish Control Conf., St. Petersburg, Russia, pp. 37-41.

[12] Santibáñez V, Kelly R (1996) Global Regulation for Robot Manipulators under SP-SD Feedback. In: Proc. 1996 IEEE Int. Conference on Robotics and Automation, Minneapolis, MN, pp. 927-932.

[13] Zavala-Río A, Santibáñez V (2007) A Natural Saturating Extension of the PD-with-Desired-Gravity-Compensation Control Law for Robot Manipulators with Bounded Inputs. IEEE Transactions on Robotics 23: 386-391.

[14] Ortega R, Loría A, Kelly R, Praly L (1994) On Passivity-Based Output Feedback Global Stabilization of Euler-Lagrange Systems. In: Proc. 33th IEEE Conference on Decision and Control, Lake Buena Vista, FL, pp. 381-386.

[15] Burkov IV, Freidovich LB (1997) Stabilization of the Position of a Lagrangian System with Elastic Elements and Bounded Control with and without Measurement of Velocities. J. Appl. Maths. Mechs. 61: 433-441.

[16] Colbaugh R, Barany E, Glass K (1997) Global Regulation of Uncertain Manipulators Using Bounded Controls. In: Proc. 1997 IEEE Int. Conf. Rob. and Aut., Albuquerque, NM, pp. 1148-1155.

[17] Zergeroglu E, Dixon W, Behal A, Dawson D (2000) Adaptive Set-Point Control of Robotic Manipulators with Amplitude-Limited Control Inputs. Robotica 18: 171-181.

[18] Laib A (2000) Adaptive Output Regulation of Robot Manipulators under Actuator Constraints. IEEE Trans. on Rob. and Automation 16: 29-35.

[19] Dixon WE (2007) Adaptive Regulation of Amplitude Limited Robot Manipulators with Uncertain Kinematics and Dynamics. IEEE Transactions on Automatic Control 52: 488-493.

[20] Yarza A, Santibáñez V, Moreno-Valenzuela J (2011) Global Asymptotic Stability of the Classical PID Controller by Considering Saturation Effects in Industrial Robots. International Journal of Advanced Robotic Systems 8: 34-42.

[21] Santibáñez V, Kelly R, Zavala-Río A, Parada P (2008) A New Saturated Nonlinear PID Global Regulator for Robot Manipulators. In: Proc. 17th IFAC Wld. Cong., Seoul, Korea, pp. 11690-11695.

[22] Su Y, Müller PC, Zheng C (2010) Global Asymptotic Saturated PID Control for Robot Manipulators. IEEE Tr. Cont. Syst. Tech. 18: 1280-1288.

[23] Meza JL, Santibáñez V, Hernández VM (2005) Saturated Nonlinear PID Global Regulator for Robot Manipulators: Passivity-Based Analysis. In: Proc. 16th IFAC World Congress, Prague, Czech Republic.

[24] Khalil HK (2002) Nonlinear Systems, 3rd ed. Upper Saddle River, NJ: Prentice-Hall.

[25] Rouche N, Habets P, Laloy M, (1977) Stability Theory by Lyapunov's Direct Method. New York: Springer-Verlag.

[26] Arimoto S (1996) Control Theory of Non-Linear Mechanical Systems: A Passivity-Based and Circuit-Theoretic Approach Oxford: Oxford University Press.

[27] Sciavicco L, Siciliano B (2000) Modelling and Control of Robot Manipulators, 2nd ed. London: Springer.

[28] Gautier M, Khalil W (1988) On the Identification of the Inertial Parameters of Robots. In: Proc. 27th Conf. Dec. Ctl., Austin, TX, pp. 2264-2269.

[29] Michel AN, Hou L, Liu D (2008) Stability of Dynamical Systems. Boston: Birkhäuser.

[30] Reyes F, Rosado A (2005) Polynomial Family of PD-type Controllers for Robot Manipulators. Control Engineering Practice 13: 441-450.