On the Stability of AQM Controllers Supporting TCP Flows

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1 Introduction

Since the dynamic fluid-flow model for describing the behavior of the transmission control protocol (TCP) in computer networks was introduced in [10], several control structures have been proposed as active queue management (AQM) to allow the routers to assist TCP management for congestion avoidance. Thus, proportional (P), proportional-integral (PI) and $\mathcal{H}^{\infty}$ AQM controllers have been proposed based on the linearization of the model in [3] and [12]. It was shown there that such controllers improve the performance obtained with standard AQM controllers (e.g. based on Random Early Detection (RED)).

Due to their simplicity, the P and PI controllers proposed in [3] has become a reference for the development of new AQM controllers as they are currently implemented in the Network Simulator [11]. However, such controller designs are based only on sufficient conditions for closed-loop stability of the linearization and, therefore, they do not provide the set of all locally stabilizing P and PI gain values. The knowledge of the set of stabilizing controllers results important for the designer on determining some performance objectives as well as on considering system and controller perturbations.

In the recent paper [7], the complete set of P controllers that locally stabilizes the equilibrium point of a simplified version of the model was obtained. Despite this, to the best of the author's knowledge, there are no specific results for the problem of finding the complete set of PI stabilizing controllers, and one of the aims of this chapter is to focus on it.

In this chapter, we first develop a local stability analysis of a simplified version of the model introduced in [10] for a PI control-based AQM strategy. Necessary and sufficient conditions for stability of the closed-loop linearized system are derived. More explicitly, for a given set of network parameters (round-trip time, number of TCP loads and link capacity), we obtain the complete set of PI controllers that locally stabilizes the equilibrium point. As a subsidiary result, the complete set of robust stabilizing controllers is also obtained. Then, we suggest a delay-dependent controller for a simplified version the model based on the feedback control laws which assign a finite closed-loop spectrum to delay-systems. Stability conditions for a numerically safe implementation of the controller are provided. It is shown that for any given network parameters there is always a delay-dependent controller for which a safe implementation can be obtained.

The chapter is organized as follows: Section 2, introduces the fluid-flow mathematical model. The main results for PI controllers are presented in section 3. Section 4 is devoted to the design of delay-dependent AQM controllers and their practical implementation. We provide numerical examples where appropriate, and conclude in section 5.

2 Fluid-Flow Mathematical Model

We consider the dynamic fluid-flow model introduced in [10] for describing the behavior of n homogeneous TCP-controlled sources and a single congested router

$$\begin{cases} \dot{w}(t) = \frac{1}{\tau(t)} - \frac{1}{2}\frac{w(t)w(t-\tau(t))}{\tau(t-\tau(t))}p(t-\tau(t)), \\ \dot{q}(t) = n(t)\frac{w(t)}{\tau(t)} - c, \end{cases}$$

where $w(t)$ denotes the average of TCP windows size (packets), $q(t)$ is the average queue length (packets), $\tau(t) = \frac{q(t)}{c} + \tau_p$ is the round-trip time (secs) where τ_p represents the propagation delay, c is the link capacity (packets/secs), $n(t)$ is the number of TCP sessions, and $p(\cdot)$ is the probability of a packet marking which represents the AQM control strategy.

For the study of PI AQM controllers we approximate these dynamics by assuming that $n(t) \equiv n$ and $\tau(t) \equiv \tau$ are constants as in [3, 7, 12]. As a result we have the following simplified dynamics:

$$\begin{cases} \dot{w}(t) = \frac{1}{\tau} - \frac{1}{2\tau}w(t)w(t-\tau)p(t-\tau), \\ \dot{q}(t) = \frac{n}{\tau}w(t) - c. \end{cases} \tag{1}$$

For a desired equilibrium queue length q_0, the equilibrium (w_0, q_0, p_0) of (1) is determined by

$$w_0^2 p_0 = 2 \text{ and } w_0 = \frac{\tau c}{n}.$$

In order to get basic results on delay-dependent AQM controller for more general cases, we will consider in section 4 the following dynamic approximation of (1):

$$\begin{array}{l} \dot{w}(t) = \frac{1}{\tau} - \frac{1}{2\tau}w^2(t)p(t-\tau), \\ \dot{q}(t) = \frac{n}{\tau}w(t) - c. \end{array} \tag{2}$$

System (2) approximates the local behavior of (1) about the equilibrium under the assumption $w_0 \gg 1$, see for instance [7] for a mathematical justification.

Our investigation will rely on linearization of the above systems around the equilibrium. Thus, stability will mean local stability near equilibrium, where for simplicity we use stability for the asymptotic stability concept.

3 Proportional-Integral AQM Controller

In order to design a stabilizing PI controller via the linearization of (1) about the equilibrium point, we introduce $\sigma(t) = \int_0^t (q(s) - q_0)\, ds$, and consider the augmented system

$$\begin{cases} \dot{w}(t) = \frac{1}{\tau} - \frac{1}{2\tau} w(t) w(t-\tau) p(t-\tau), \\ \dot{q}(t) = \frac{n}{\tau} w(t) - c, \\ \dot{\sigma}(t) = q(t) - q_0. \end{cases} \tag{3}$$

We now consider a PI controller of the form

$$p(t) = k_p q(t) + \frac{k_p}{I} \sigma(t), \tag{4}$$

where $\frac{k_p}{I} \neq 0$. It can be easily verified that the closed-loop system (3)-(4) has a unique equilibrium point (w_0, q_0, σ_0), where $\sigma_0 = \frac{I}{k_p} (p_0 - k_p q_0)$.

The linearization of the closed-loop system (3)-(4) about the equilibrium (w_0, q_0, σ_0) is

$$\dot{\xi}(t) = A\xi(t) + B\xi(t-\tau), \tag{5}$$

where $\xi(t) = \begin{pmatrix} \tilde{w}(t) \\ \tilde{q}(t) \\ \tilde{\sigma}(t) \end{pmatrix}$, $A = \begin{pmatrix} -\frac{n}{\tau^2 c} & 0 & 0 \\ \frac{n}{\tau} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} -\frac{n}{\tau^2 c} & -\frac{\tau c^2}{2n^2} k_p & -\frac{\tau c^2}{2n^2} \frac{k_p}{I} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\tilde{w}(t) = w(t) - w_0$, $\tilde{q}(t) = q(t) - q_0$, and $\tilde{\sigma}(t) = \sigma(t) - \sigma_0$.

Assume for the moment that it is possible to find controller's gains that make (5) stable. Then, it follows that all solutions of (3) starting sufficiently close to (w_0, q_0, σ_0) approach it as t tends to infinity.

It is clear that one cannot investigate the stability of (5) for the delay-free case ($\tau = 0$). This is a particular property of TCP/AQM network systems, where the delay value (round-trip time) cannot be considered zero. Thus, the approach developed in [13], which is based on first determining the set of PI stabilizing controllers for the delay-free system, cannot be directly applied to determine the set of PI stabilizing controllers for (5).

3.1 Stability Analysis

It is well known that (5) is stable if and only if the characteristic function

$$f(s) = s^3 + \frac{n}{\tau^2 c} s^2 + \left[\frac{n}{\tau^2 c} s^2 + \frac{c^2}{2} k_p \left(s + \frac{1}{I} \right) \right] e^{-\tau s}$$

has no zeros with nonnegative real parts [2].

Theorem 1. *System (5) is stable if and only if the controller gains (I, k_p) belong to the stability region $\Phi_{(n,\tau,c)}$, plotted in Fig. 1, whose boundary in the controller gains space (I, k_p) is described by*

$$\partial \Phi_{(n,\tau,c)} = \Bigg\{ (I, k_p) : I = \frac{\omega \cos(\omega\tau) + \frac{n}{\tau^2 c}\sin(\omega\tau)}{\omega\left(\frac{n}{\tau^2 c}\left(1+\cos(\omega\tau)\right) - \omega \sin(\omega\tau)\right)},$$
$$k_p = \frac{2n\omega}{c^2}\left(\omega\cos(\omega\tau) + \frac{n}{\tau^2 c}\sin(\omega\tau)\right), \omega \in (0, \omega^*) \Bigg\},$$

where ω^* is the solution of

$$\tan\left(\frac{\omega\tau}{2}\right) = \frac{n}{\tau^2 c\omega},\ \ \omega \in \left(0, \frac{\pi}{\tau}\right). \tag{6}$$

Proof. First observe that since $\frac{k_p}{I} \neq 0$, $s = 0$ is not a zero of $f(s)$. Suppose that $f(s)$ has a pure imaginary zero $s = j\omega \neq 0$. Then, a direct calculation yields

$$\begin{cases} k_p = \frac{2n}{c^2}\omega\left(\omega\cos(\omega\tau) + \frac{n}{\tau^2 c}\sin(\omega\tau)\right), \\ I = \frac{\omega\cos(\omega\tau) + \frac{n}{\tau^2 c}\sin(\omega\tau)}{\omega\left(\frac{n}{\tau^2 c}(1+\cos(\omega\tau)) - \omega\sin(\omega\tau)\right)}. \end{cases} \tag{7}$$

This parameterization defines a countable number of curves in the parameter space (I, k_p) and each one of them is obtained by varying ω in the following intervals: $(0, \omega_0^*)$, $\left(\omega_k^*, \frac{(2k+1)\pi}{\tau}\right)$ and $\left(\frac{(2k+1)\pi}{\tau}, \omega_{k+1}^*\right)$, $k = 0, 1, 2, \ldots$, where ω_k^* is the solution of

$$\tan\left(\frac{\omega\tau}{2}\right) = \frac{n}{\tau^2 c\omega},\ \ \omega \in \left(\frac{2k\pi}{\tau}, \frac{(2k+1)\pi}{\tau}\right). \tag{8}$$

Since (8) is a transcendental equation we look directly for a numerical solution. This can be found by plotting the two functions $\tan\left(\frac{\omega\tau}{2}\right)$ and $\frac{n}{\tau^2 c\omega}$, see Fig. 2. These curves divide the plane (I, k_p) into a set of connected domains. From the argument principle is easy to show that for all (I, k_p) values inside the open domain $\Phi_{(n,\tau,c)}$, bounded by the curve obtained by varying ω in the interval $(0, \omega_0^*)$ and the coordinate axis $k_p = 0$, the function $f(s)$ has no zeros with strictly positive real part.

Remark 1. When $\tau \to +0$, the stability region $\Phi_{(n,\tau,c)}$ tends to the whole first quadrant of the plane (I, k_p). In other words, for small round-trip time (delay), arbitrarily PI controller's gains locally stabilizes the equilibrium point of (3).

Proof. From parametrization (7) it is not difficult to see that $I(\omega) \to \frac{\tau^2 c}{2n} + \frac{\tau}{2}$ and $k_p(\omega) \to k_p(0) = 0$ when $\omega \to +0$. On the other hand, it holds that $I(\omega) \to +\infty$ and $k_p(\omega) \to k_p(\omega^*) = \frac{2n}{c^2}(\omega^*)^2$ when $\omega \to -\omega^*$. From the above and the fact that $\omega^* \to +\infty$ when $\tau \to +0$, see Fig. 2, the remark follows.

Remark 2. Given nominal network parameters (n_0, τ_0, c_0) and unknown network parameters (n, τ, c) satisfying

$$n \geq n_0, \tau \leq \tau_0 \text{ and } c \leq c_0, \tag{9}$$

then $\Phi_{(n_0,\tau_0,c_0)} \subseteq \Phi_{(n,\tau,c)}$ holds.

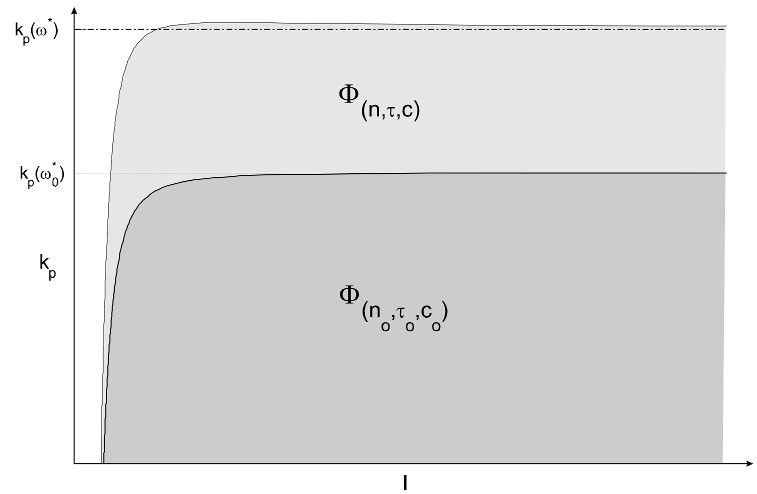

Fig. 1. Stability Regions $\Phi_{(n,\tau,c)}$ and $\Phi_{(n_0,\tau_0,c_0)}$ for (n_0, τ_0, c_0) and (n, τ, c) satisfying (9).

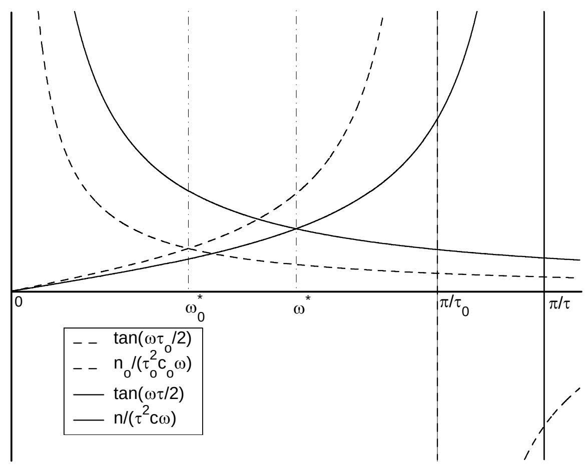

Fig. 2. Numerical solutions ω_0^* and ω^* of (6) for (n_0, τ_0, c_0) and (n, τ, c) satisfying (9).

Proof. The remark follows directly from Proposition 2 in [3], which states that stabilizing against the largest expected values of τ and c, and the smallest expected value of n yields a robust stabilizing controller.

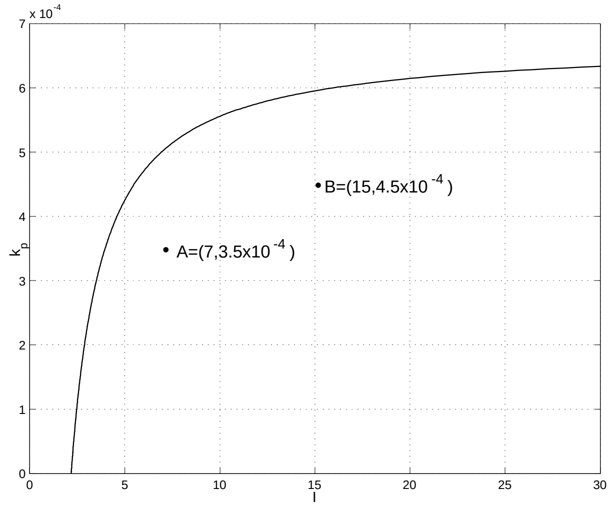

Fig. 3. Stability region $\Phi_{(n_0,\tau_0,c_0)}$ for $n_0 = 40$, $\tau = 0.7$ and $c = 300$

3.2 Example

Let us consider nominal network parameters $n_0 = 40$ TCP sessions, $\tau_0 = 0.7$ secs and $c_0 = 300$ packets/secs. As real network parameters we take the following values: $n = 50$ TCP sessions, $\tau = 0.533$ secs and $c = 250$ packets/secs. In Fig. 3 we plot the stability region $\Phi_{(n_0,\tau_0,c_0)}$ in the controller's gains space. In Fig. 4 we plot the response of $q(t)$ for the two pairs of gains $A = (7, 3.5 \times 10^{-4})$ and $B = (15, 4.5 \times 10^{-4})$ inside of $\Phi_{(n_0,\tau_0,c_0)}$, see Fig. 3. The simulations were carried out on the nonlinear model (3). The operation point was chosen as $q_0 = 200$ packets.

It can be seen that the robust stabilization is reached. On the other hand, the responses obtained for the two different pairs of gains show the importance of knowing the complete set of controller parameter values that locally stabilize the closed-loop system.

4 Delay-Dependent AQM Controllers

As it can be seen from the previous section, a PI AQM stabilizing controller results in a closed-loop system governed by a retarded delay differential equation for which controlling its spectrum is not practically feasible. When in addition to stabilization, a desired closed-loop dynamic is required, delay-dependent controllers could result more convenient if certain knowledge of the previous dynamic information and delay are assumed. A delay-dependent AQM controller has been recently proposed in [4]. However, the well-known

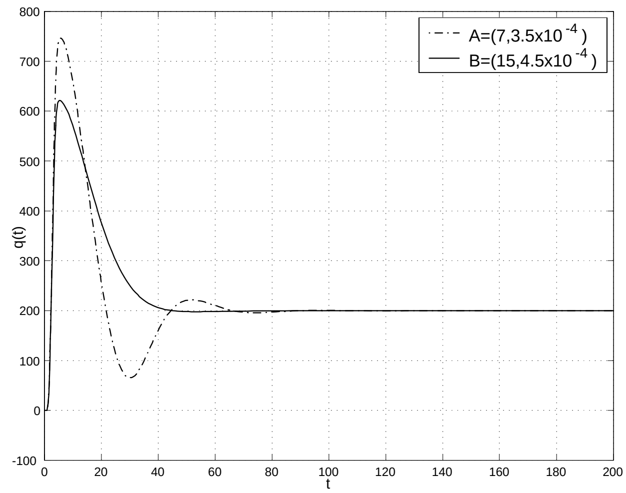

Fig. 4. The response of $q(t)$ for the A and B gains, respectively.

instability mechanism of the numerical implementation of such kind of controllers was not addressed.

In this section, we suggest the design of a delay-dependent AQM controller based on the feedback control laws proposed by Manitus and Olbrot in [5] for finite spectrum assignment of time-delay systems, and provide stability conditions for a safe numerical implementation of the controller.

4.1 Finite Spectrum Assignment

Let us briefly discuss the feedback control laws for finite spectrum assignment of time-delay systems and their numerical implementation problem.

Consider the linear system with delayed input

$$\dot{x}(t) = Ax(t) + Bu(t-h), \tag{10}$$

where $h > 0$, $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ represent the state and control vectors, and A, B are real constant matrices of appropriate dimensions. The control law

$$u(t) = x(t) + K \int_{-h}^{0} e^{-A(h+\theta)} Bu(t+\theta) d\theta \tag{11}$$

assigns a finite spectrum to the closed-loop system (10)-(11) which coincides with the spectrum of the matrix $A + e^{-Ah}BK$, see [5].

The practical value of such a result is limited by the instability mechanism of the numerical approximation of the integral term in (11), see [1, 8, 14] and the references therein. It has been shown there that if the integral is approximated by a finite sum, then the closed-loop system may become unstable if the controller (11) is not internally stable. So, the internal stability of (11) is an essential condition for its successful implementation.

Motivated from the limitations imposed by the internal stability requirement of (11), the introduction of a low-pass filter (implicitly and/or explicitly) in the control loop has been proposed as remedy to overcome the implementation problems, see for instance [9]. However, such a solution could make the implementation unnecessarily complicated in the case of those parameters of (10) for which the internal dynamics of (11) are stable, see [6].

The internal dynamics of (11) are described by the following integral delay system:

$$z(t) = K \int_{-h}^{0} e^{-A(h+\theta)} B z(t+\theta) d\theta. \tag{12}$$

The characteristic function associated to (12) is

$$f(s) = \det \left(I - K \int_{-h}^{0} e^{-A(h+\theta)} B e^{s\theta} d\theta \right).$$

Here I denote the identity matrix of appropriate dimension. The following result provides simple-to-check stability conditions for (12).

Proposition 1. *System (12) is stable if*

$$\max_{\theta \in [-h,0]} \left\| K e^{-A(h+\theta)} B \right\| h < 1. \tag{13}$$

Proof. First observe that for any $h > 0$ and any s with $\mathrm{Re}(s) \geq 0$ the following inequality holds:

$$\left| \frac{1 - e^{-hs}}{s} \right| \leq h.$$

Now assume that $f(s)$ has a zero, s_0, with nonnegative real part. Then there exists a complex vector $\nu \neq 0$ such that $\left(I - K \int_{-h}^{0} e^{-A(h+\theta)} B e^{s_0 \theta} d\theta \right) \nu = 0$.

It follows that

$$1 \leq \max_{\theta \in [-h,0]} \left\| K e^{-A(\theta+h)} B \right\| \left| \frac{1 - e^{-hs_0}}{s} \right| \leq \max_{\theta \in [-h,0]} \left\| K e^{-A(\theta+h)} B \right\| h.$$

The last inequality contradicts the condition of the proposition.

4.2 Controller Design

As mentioned before, in order to get basic results for more general cases we develop the controller design for the linearization of the simplified model (2). The linearization of (2) about the equilibrium (w_0, q_0, p_0) is

$$\dot{\xi}(t) = A\xi(t) + b\tilde{p}(t - \tau), \tag{14}$$

where $\xi(t) = \begin{pmatrix} \tilde{w}(t) \\ \tilde{q}(t) \end{pmatrix}$, $A = \begin{pmatrix} -\frac{2n}{\tau^2 c} & 0 \\ \frac{n}{\tau} & 0 \end{pmatrix}$, $b = \begin{pmatrix} -\frac{\tau c^2}{2n^2} \\ 0 \end{pmatrix}$, $\tilde{w}(t) = w(t) - w_0$, $\tilde{q}(t) = q(t) - q_0$ and $\tilde{p}(t) = p(t) - p_0$. The corresponding control law (11) which assigns a closed-loop finite spectrum to (14) has the following form:

$$\begin{aligned}\tilde{p}(t) &= k_1\tilde{w}(t) + k_2\tilde{q}(t) - \frac{\tau c^2}{2n^2}k_1\int_{-\tau}^{0} e^{\frac{2n}{\tau^2 c}(\tau+\theta)}\tilde{p}(t+\theta)d\theta \\ &\quad -\frac{\tau^2 c^3}{4n^2}k_2\int_{-\tau}^{0}\left(1 - e^{\frac{2n}{\tau^2 c}(\tau+\theta)}\right)\tilde{p}(t+\theta)d\theta. \qquad (15)\end{aligned}$$

Here we assume that the whole state is accessible and that network parameters are known. The closed-loop ideal spectrum is determined by the zeros of the polynomial

$$m(\lambda) = \lambda^2 + \left[\frac{2n}{\tau^2 c} + \frac{\tau c^2}{2n^2}k_1 e^{\frac{2n}{\tau c}} + \frac{\tau^2 c^3}{4n^2}k_2\left(1 - e^{\frac{2n}{\tau c}}\right)\right]\lambda + \frac{c^2}{2n}k_2.$$

The internal dynamics of (15) are governed by the integral delay system

$$z(t) = -\frac{\tau c^2}{2n^2}k_1\int_{-\tau}^{0} e^{\frac{2n}{\tau^2 c}(\tau+\theta)}z(t+\theta)d\theta - \frac{\tau^2 c^3}{4n^2}k_2\int_{-\tau}^{0}\left(1 - e^{\frac{2n}{\tau^2 c}(\tau+\theta)}\right)z(t+\theta)d\theta. \qquad (16)$$

Thus, a successful implementation of (15) can be achieved if there exists a pair (k_1, k_2) such that both the polynomial $m(\lambda)$ and system (16) are stable.

The polynomial $m(\lambda)$ is stable if and only if

$$k_2 > 0 \text{ and } \frac{2n}{\tau^2 c} + \frac{\tau c^2}{2n^2}k_1 e^{\frac{2n}{\tau c}} + \frac{\tau^2 c^3}{4n^2}k_2\left(1 - e^{\frac{2n}{\tau c}}\right) > 0. \qquad (17)$$

From (13) we have that (16) is stable if

$$\max_{\theta\in[-\tau,0]} \frac{\tau^2 c^2}{2n^2}\left| e^{\frac{2n}{\tau^2 c}(\tau+\theta)}k_1 + \frac{\tau c}{2}\left(1 - e^{\frac{2n}{\tau^2 c}(\tau+\theta)}\right)k_2\right| < 1. \qquad (18)$$

In Fig. 5 we plot the stability regions determined by (17) and (18) in the plane (k_1, k_2). We denote by R_s the intersection of the two regions. The intersection points of the lines determining the boundaries of the stability regions with the coordinate axis are defined by $a = -\frac{8n^3}{\tau^4 c^4}\left(\frac{1}{1-e^{\frac{2n}{\tau c}}}\right)$, $b = -\frac{4n^3}{\tau^3 c^3}e^{-\frac{2n}{\tau c}}$, $d = -\frac{4n^2}{\tau^3 c^3}\left(\frac{1}{1-e^{\frac{2n}{\tau c}}}\right)$, $e = \frac{2n^2}{\tau^2 c^2}e^{-\frac{2n}{\tau c}}$ and $f = \frac{2n^2}{\tau^2 c^2}$. The following relationship

$$\frac{a}{b} = \frac{d}{-e} = \frac{2}{\tau c e^{-\frac{2n}{\tau c}}\left(1 - e^{\frac{2n}{\tau c}}\right)}$$

holds, and taking into account that $w_0 \gg 1$ we have $a < d$ and $b > -e$.

Remark 3. For a given set of network parameters (n, τ, c), there is always a pair (k_1, k_2) of controller's gains for which both system (16) and the ideal closed-loop system (14)-(15) are stable.

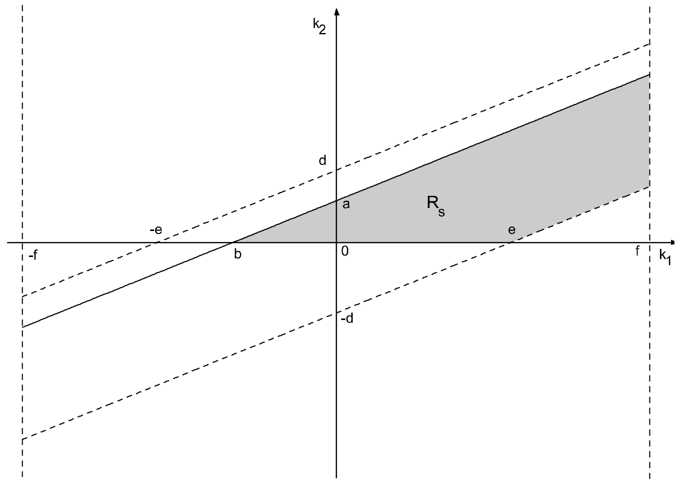

Fig. 5. Stability regions of (16) determined by (18) (- -) and for the ideal closed-loop stability determined by (17) (-). R_s denotes the intersection of the two regions.

4.3 Example

Consider the nominal network parameters considered in section 3. In Fig. 8 we plot the corresponding stability region R_s. In Fig. 9 we present the closed-loop response of $q(t)$ with the approximated control law (15) (by using a trapezoidal rule) for a $(k_1, k_2) = (0.07, 0.6 \times 10^{-3})$ inside of R_s. For these particular values the ideal closed-loop eigenvalues are $\lambda_1 = -1.55$ and $\lambda_2 = -0.43$. The operation point was chosen as $q_0 = 300$ packets with initial condition $q(0) = 400$ packets. The simulations were carried out on the nonlinear system (2).

5 Conclusion

In this chapter we addressed the local stability of two AQM controllers supporting TCP flows. We first considered a PI controller for which we derived necessary and sufficient conditions for closed-loop stability of the linearization. This result provides the complete set of PI AQM controllers that locally stabilizes the equilibrium point in counterpart with the existing works in the literature which give only estimates of this set. Then, we proposed a delay-dependent AQM controller based on feedback control laws for finite spectrum assignment of time-delay systems. Stability conditions for a numerically safe implementation of the controller are given. Numerical examples that illustrate the capabilities of the results for determining performance objectives and for sensitivity analyses with respect to perturbations of the system and controller have been performed.

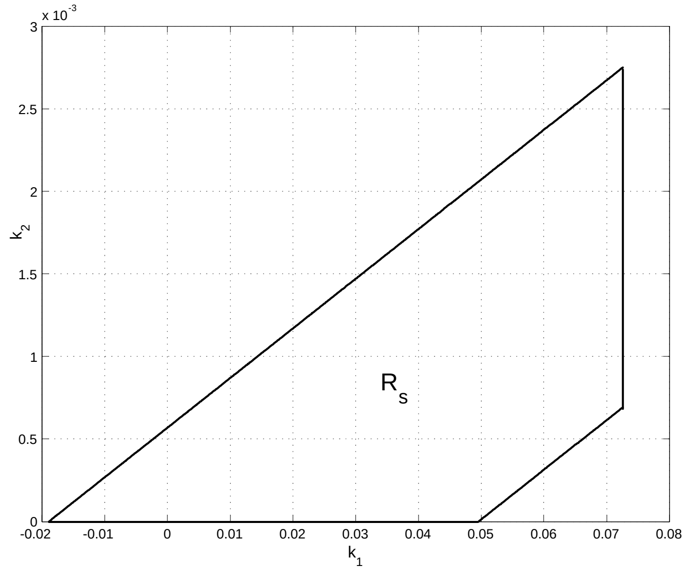

Fig. 6. Stability region R_s for $n = 40$, $\tau = 0.7$ and $c = 300$.

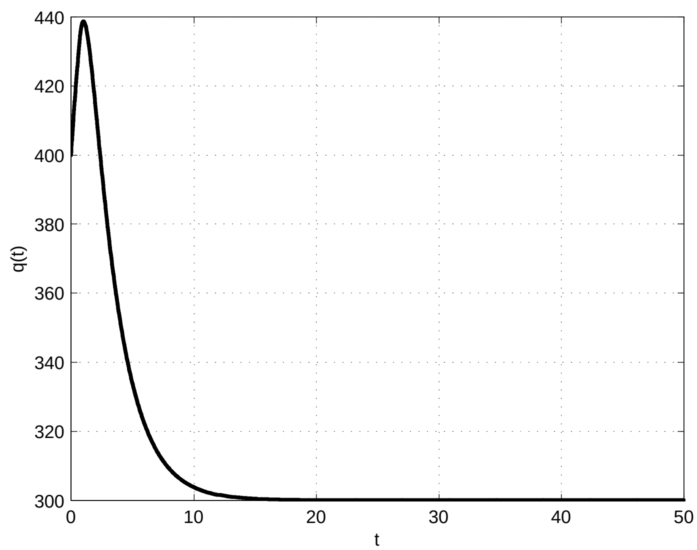

Fig. 7. The numericalsimulation of the closed-loop response of $q(t)$ by approximating the control law (15) with a trapezoidal rule.

6 Acknowledge

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