Electronic version of an article published as International Journal of Bifurcation and Chaos, 22 (01), 1250011 (2012)
https://doi.org/10.1142/S0218216517500559

International Journal of Bifurcation and Chaos

A multivibrator circuit based on chaos generation

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Received (to be inserted by publisher)

We present a parameterized method to design multivibrator circuits via piecewise-linear (PWL) chaotic systems, which can exhibit double-scroll oscillations. The circuit is conformed exploiting a parametric modulation that manipulates the equilibrium stability of each linear subsystem. Chua's oscillator is used as benchmark to illustrate the effectiveness of the proposed method to design multivibrator circuits. Thus, our proposal allows one the design of the three configurations of a multivibrator: Monostable, Astable, and Bistable. Potential applications are illustrated designing a pulse generator and a full S-R flip flop device based on our all-in-one multivibrator circuit.

Keywords: Multivibrator circuits, PWL Systems, Chaos-based design.

1. Introduction

Chaos is now known to be useful. In fact, under certain conditions, is a desirable feature of systems and circuits. The dynamical richness of chaotic behavior has significant potential applications to real-world problems, including secure communications, persistent excitation, information processing and encryption, to mention but a few [Ott, 2002; Strogatz, 2001; Tam *et al.* , 2007]. Actually, the intentional induction of chaotic behavior into a system, i.e. its chaotification, might be beneficial to achieve a wide variety of alternative goals [Chen & Dong , 1998; Kapitaniak , 2000; Stark & Hardyuand , 2003]. Over the last twenty years, significant efforts have been devoted to the design of simple electronic circuits with chaotic behavior [Carroll & Pecora, 1995; Yu *et al.*, 2010]. In particular, we are interested in using chaos theory to design multivibrator electronic circuits in order to have different configurations dynamically available.

A multivibrator circuit is a simple two-state systems that has only one of three possible configurations, these are: (*i*) *Astable*. In this configuration, the multivibrator circuit is unstable on both of its states. As a

consequence, the circuit spends a determined time-period in one state and flips to the other where spends a time-period, and then moves back to the first one, where the cycle is repeated continuously. Therefore, this configuration is used to generate 0 and 1 sequences. Usually, astable multivibrators are use in phase locked loop (PLL) systems to derive an output signal with its phase and frequency matched to those of the input signal [Sasaki *et al.* , 1996]. (*ii*) *Monostable*. In this configuration, one state of the circuit is stable while the other is unstable. As such, the system may spend some time in the unstable state, but eventually will move into the stable state and remain there afterwards. This configuration can be used, for instance, to define a time-period of activity measured from an event. Monostable multivibrator might be used to implement a pulse waveform with an adjustable width in communication systems, digital systems, PLL circuits, and power electronics systems. Actually, the monostable configuration can be realized using operational transresistance amplifiers [Lo & Chien, 2006]. (*iii*) *Bistable*. In this configuration both states are stable. This implies that the circuit remains in its current state, until being force to change to the other by an external event or trigger. The multivibrator system in bistable configuration can be use as a fundamental building block of a register or memory device. This circuit is usually realized using basic logic gates and can behave as a flip-flop; however, recent efforts have been devoted to study alternative topologies and realizations [Kosta *et al.*, 2003].

We are interested in propounding alternative topologies for these three configurations of multivibrator circuits. Our concern is in regards to the question: *Is there a chaotic system that can be exploited to reproduce any of the three multivibrator configurations?* Intuitively, the answer is affirmative if we think that: (i) The classical chaotic systems by Lorenz or Chen can display two scrolls around two distinct unstable equilibria [Lorenz, 1963; Chen & Ueta, 1999; Ueta & Chen, 2000]. Then, if we associate each scroll to a state in the circuit, the astable configuration might be recovered. (ii) The classical Rössler system exhibits only one scroll around the equilibrium [Rösler, 1976]. Hence, the monostable configuration can be recovered as an electronic circuit that reproduces the Rössler oscillator. And finally, (iii) There are several PWL systems that exhibit what is known as bistable chaos, that is, two stable scrolls coexists for the same system [Bartissol & Chua, 1988; Chua *et al.*, 1993]. Then, if we associate to each scroll a different logical state, the third multivibrator configuration can be directly recovered. Therefore, the challenge is to find a dynamic architecture capable of conciliating the different chaotic behaviors among the three above scenarios towards an alternative general topology of multivibrator circuits. This challenge seems solvable by accounting the fact that chaotic attractors similar to the classical attractors by Lorenz, Chen and Rössler can be obtained from the parametric modulation of a three-dimensional systems [Lü *et al.*, 2002; Lu & Chen, 2002; Campos-Cantón *et al.*, 2007, 2008].

Chua's circuit is one of the best known and more studied examples of chaotic systems [Matsumoto, 1984]. Many applications and designs have been proposed using this chaotic circuit. Particularly, [Cafagna and Grassi, 2005] use it to obtain two logic gates from two state variables, from those chaos-based logic they implemented two NOR gates and build a standard flip-flop device. Other alternatives to yield logic gates using chaos have been reported by [Sinha & Ditto, 1998], in this reference the authors introduced the idea of reconfigurable structures. Nowadays, there is a great interest in developing circuits with different types of dynamically available logic structures, this is usually referred to as chaos computing (see [Campos-Cantón *et al.*, 2010; Murali *et al.*, 2003] and references therein). In this paper, we propose a design methodology to generate a multivibrator circuit based on chaos via parameter modulation in a double-scroll oscillator. Then, the target is to find a bifurcation parameter such that the same double-scroll oscillator behaves as any of the configurations of multivibrator: astable, monostable or bistable. Our approach consists on utilizing a parameterized family of chaotic PWL circuits with different number of scrolls to produced the different multivibrator configurations. The remainder of the paper is organized as follows. In Section 2, we introduce a parametric family of PWL chaotic systems depicting attractors with a different number of scrolls, and formalize the design methodology to generate, departing from chaos, the three multivibrator configurations described above. In Section 3, we show the circuit realization of our proposed all-in-one multivibrator based on chaos. Then, we illustrate its potencial applications proposing realization of a pulse generator and a flip flop device based on our multivibrator circuit. For our flip flop realization, unlike standard flip flops, the entry $(1, 1)$ is allowed, for this reason is called a *full SR flip flop*. Finally, in Section 4, the contribution is closed with some concluding remarks.

2. A PWL Family Of Chaotic Systems

Let us depart from basic notions by considering a PWL dynamical system given by:

$$\dot{\mathbf{x}} = \begin{cases} A_1\mathbf{x} + B_1, & \text{if } x_\sigma > d \\ A_2\mathbf{x} + B_2, & \text{if } |x_\sigma| \leq d \\ A_3\mathbf{x} + B_3, & \text{if } x_\sigma < -d \end{cases} \tag{1}$$

where $\mathbf{x} = [x_1, x_2, x_3]^\top \in \mathbb{R}^3$ is the state vector of the system; x_σ is a scalar component of $\mathbf{x}$; the matrices $A_k = \{a_{ijk}\} \in \mathbb{R}^{3\times3}$ and the vectors $B_k = [b_{1k}, b_{2k}, b_{3k}]^T \in \mathbb{R}^3$ for $k = 1, 2, 3$; and d is a real scalar. The system's description divides its state space into sections, where locally the trajectories are those of the corresponding linear vector field $(A_k\mathbf{x} + B_k)$. Each linear subsystem of (1) has its own equilibrium point located at $\chi_k = -A_k^{-1}B_k$, with the stability of the system in its vicinity given by the eigenvalues of A_k. Letting the eigenvalue spectrum of A_k be $\Lambda = \{\lambda_1, \lambda_2, \lambda_3\}$, we have two possibilities, either all λ are real, or we have one real and two complex conjugate eigenvalues. Only the second case can yield a chaotic attractor [Lorenz, 1963; Chen & Ueta, 1999; Bartissol & Chua, 1988]. Then, the eigenvalues of a PWL chaotic system are one purely real and one complex conjugate pair, denoted in the remainder as λ_{PR} and λ_{CC}, respectively. Depending on the signs of the real parts of λ_{PR} and λ_{CC}, different trajectories are generated in the vicinity of χ_k. We have that a positive λ_{PR} throws the trajectories away from χ_k, while a negative λ_{PR} moves the trajectories towards it. In regards to λ_{CC}, the imaginary part of λ_{CC} induces spinning motion around χ_k. Hence, if we have a negative λ_{PR} and the real part of λ_{CC} is positive, the trajectories around χ_k behave like an unstable focus, *i.e.*, the trajectories spin away from the equilibrium. Contrarily, if the real part of λ_{CC} is negative, the trajectories spin towards the equilibrium.

The concern in this contribution is about of finding a parametric modulation for the PWL chaotic system in (1) such that a multivibrator circuit can be designed. Thus, motivated by facts in the previous paragraph, we have that the stability properties and location of the equilibrium points of (1) can be changed modulating the entries of the pair (A_k,B_k) for each linear subsystem ($k = 1, 2, 3$). In fact, it is sufficient to modulate a single entry $a_{i\sigma k}$ or b_{ik} of A_k or B_k, respectively, to provoke significant changes in the dynamics of the system due to rejection or attraction of trajectories to the corresponding equilibrium points. Henceforth, motivated by this observation, we have the possibility to generate alternative chaotic attractor parameterizing the linear functions $a_{i\sigma k}x_\sigma + b_{ik}$ at each subsystem. In what follows, our observations about the dynamical features of the PWL chaotic systems are generalized into a methodology to design the three configurations of our multivibrator chaotic circuit.

The dynamical features of the PWL chaotic systems give us some basic requirements for the operation of (1) as a multivibrator circuit. First, it is necessary that the overall system be dissipative such that the PWL trajectories will be bounded. Aside from this basic requirement, we found that for the emergence of chaos all the linear subsystems must have a purely real (λ_{PR}) and a pair of complex conjugate (λ_{CC}) eigenvalues. By taking care of attending these basic requirements, alternative chaotic attractors with different numbers of scrolls can be generated as described in Appendix A. In particular, a multivibrator circuit is designed based on chaos via parametric modulation in a double-scroll attractor. As a summary, the next two steps define our proposed design algorithm.

Step 1: Consider a PWL system of the form:

$$\dot{x} = A_k x + B_k, \text{ for } x_\sigma \in S_k, \ k = 1, 2, 3. \tag{2}$$

the state space of (2) is divided in terms of the σth component of state variable ($\sigma = 1, 2$ or 3). For each section $S_k \subset \mathbf{R}$, the entries $a_{i\sigma k}$ of $A_k \in \mathbf{R}^{3\times3}$ and b_{ik} of $B_k \in \mathbf{R}^3$ change their values while the rest of the entries of A_k and B_k remain constant. As the trajectories switch from one linear subsystem to the next, they move into the vicinity of equilibriums with different locations and stability features, which are determined by the current values of $a_{i\sigma k}$ and b_{ik}.

Step 2: Add a modulation parameter ϵ_k such that the linear segment $a_{i\sigma k}x_\sigma + b_{ik}$, gives the desired equilibrium point location and stability features for each subsystem of (2):

$$\epsilon_k(a_{i\sigma k}x_\sigma + b_{ik}), \text{ for } k = 1, 2, 3. \tag{3}$$

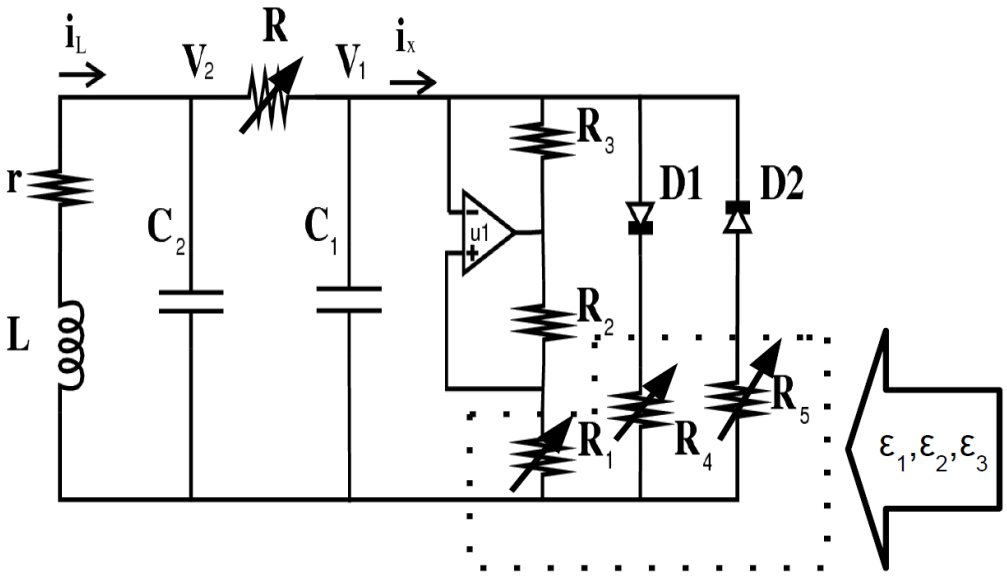

Fig. 1. Schematic diagram of the parameterized Chua's circuit.

where, by tuning the modulation parameter $\epsilon_k \in [-E, E] \subset \mathbf{R}$, one can derive different chaotic attractor features that correspond to the three multivibrator configurations.

Following the above steps, construction of multivibrator circuits can be realized controlling the parameters ϵ_k. Next, we illustrate the parametric modulation design using as example the benchmark circuit by Chua [Bartissol & Chua, 1988; Chua *et al.*, 1993].

3. Parametric Modulation Of Chua's Circuit

The effectiveness of the proposed methodology is illustrated by using Chua's chaotic systems [Bartissol & Chua, 1988; Chua *et al.*, 1993], one possible electronic realization of this system is shown in Figure 1. In its adimensional form, Chua's system can be written as a PWL system in the form (1) given by [Barajas-Ramirez *et al.*, 2003]:

$$A_k = \begin{pmatrix} a_{11k} & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -15 & -0.0385 \end{pmatrix}, \quad B_k = \begin{pmatrix} b_{1k} \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

for $k = 1, 2, 3$; with $\sigma = 1$, $d = 1$. Some typical values are the following: $a_{111} = a_{113} = -3.2$, $a_{112} = 2.7$, $b_{11} = 5.9$, $b_{12} = 0$, and $b_{13} = -5.9$. The proposed parametric modulation, $(\epsilon_k a_{11k}, \epsilon_k b_{1k})$, results on the following linear segments: $\epsilon_1(-3.2x_1 + 5.9)$, $\epsilon_2(2.7x_1)$, and $\epsilon_3(-3.2x_1 - 5.9)$. This modulation can be electronically realized as a combination of the resistance values R_1, R_4, and R_5 on the circuit in Figure 1.

Tuning the parameters ϵ_k three different chaotic regimes can be produced by the system in (4). In particular, if the modulating parameters are set to the following values $\epsilon_1 = 0.35$, $\epsilon_2 = 0.8$, and $\epsilon_3 = 0.35$, a double-scroll attractor is generated. Figure 2(a) shows its projection onto the plane (x_1, x_2). In Figure 2(b), the time evolution of the state x_1 for this chaotic regime is shown. Notice that the value of x_1 flips from positive to negative continuously. Next, the modulating parameters are set to $\epsilon_1 = 0.35$, $\epsilon_2 = 0.5$, and $\epsilon_3 = 0.35$. With these values the system in (4) enters into a regime where two stable chaotic attractors coexist, this is usually called bistable chaos. That is, for a set of initial conditions, the system evolves oscillating chaotically around one equilibrium point. Alternatively, for another set of initial conditions, the system evolves on a different chaotic attractor around the opposite equilibrium point. This behavior is illustrated in the Figures 2(c) and 2(e). Specifically, the projection on the plane (x_1, x_2) of the chaotic attractor in Figure 2(c) corresponds to a positive initial condition, while the projection on Figure 2(e)

is obtain from a negative initial condition. In this case, the time evolution of the state x_1, shown in Figures 2(d) and 2(f), remains with the same sign as the corresponding initial condition. Finally, setting the modulating parameters to $\epsilon_1 = 0.35$, $\epsilon_2 = 0.5$, and $\epsilon_3 = 0.4$ the system in (4) enters a chaotic regime with only one stable single-scroll attractor. As shown in Figure 2(g), for this set of modulating parameters the stable attractor is located on the positive side of the plane (x_1, x_2). Observing the time evolution of the state x_1, shown on Figure 2(h), one can see that, if the initial condition is negative after a transitory period, the trajectory moves into the positive side where it remains oscillating on the stable chaotic attractor.

An analogy between the three multivibrator circuits configurations and the chaotic regimes of Chua's system is summarized as follow.

i) ***Astable***: This corresponds to the double-scroll chaotic regime, in it the sign of x_1 oscillates continuously from one state (positive) to the other (negative). The modulating parameters are set to the following values $\epsilon_1 = 0.35$, $\epsilon_2 = 0.8$, and $\epsilon_3 = 0.35$. Figure 2(a) shows its projection onto the plane (x_1, x_2). In Figure 2(b), the time evolution of the state x_1 for this chaotic regime is shown. Notice that the value of x_1 flips from positive to negative continuously.

ii) ***Bistable***: This corresponds to the case when the system exhibits bistable chaos, in it the sign of x_1 remains oscillating around one of the states (positive or negative) indefinitely. This behavior is illustrated in the Figures 2(c) and 2(e). Specifically, the projection on the plane (x_1, x_2) of the chaotic attractor in Figure 2(c) corresponds to a positive initial condition, while the projection on Figure 2(e) is obtain from a negative initial condition. In this case, the time evolution of the state x_1, shown in Figures 2(d) and 2(f), remains with the same sign as the corresponding initial condition. The modulating parameters are set to $\epsilon_1 = 0.35$, $\epsilon_2 = 0.5$, and $\epsilon_3 = 0.35$.

iii) ***Monostable***: This corresponds to the single-scroll chaotic regime, in this case one of the states (positive x_1) is stable, while the other (negative x_1) is not. Then, the ultimately x_1 will oscillate around only one of the equilibrium points despite initial conditions. Setting the modulating parameters to $\epsilon_1 = 0.35$, $\epsilon_2 = 0.5$, and $\epsilon_3 = 0.4$ the system in (4) enters a chaotic regime with only one stable single-scroll attractor. As shown in Figure 2(g), for this set of modulating parameters the stable attractor is located on the positive side of the plane (x_1, x_2). Observing the time evolution of the state x_1, shown on Figure 2(h), one can see that, if the initial condition is negative after a transitory period, the trajectory moves into the positive side where it remains oscillating on the stable chaotic attractor.

The transient behavior produced by the initial conditions on the corresponding multivibrator configurations is expected because the proposed designs are based on chaotic systems. This chaotic feature is actually useful to show that addition to astable and bistable configurations, the same circuit can behave as monostable by simply adjusting the modulating parameters as indicated.

4. Designing the multivibrator circuit

The chaos-based multivibrator can be realized using the Chua's circuit as is shown in Figure 1. A mathematical model of the Chua's circuit is given by [Bartissol & Chua, 1988; Chua *et al.*, 1993]:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} \alpha(x_2 - x_1 - f(x_1)) \\ x_1 - x_2 + x_3 \\ -\beta x_2 + \gamma x_3 \end{pmatrix} \tag{5}$$

where $f(x_1)$ is the so-called Chua's nonlinear negative resistance, which is describe as follows:

$$f(x_1) = \begin{cases} b_1 x_1 - c_1, \text{ if } x_1 > 1; \\ a x_1, \qquad \text{if } |x_1| < 1; \\ b_2 x_1 + c_2, \text{ if } x_1 < -1. \end{cases}$$

with $c_i = b_i - a$, $i = 1, 2$.

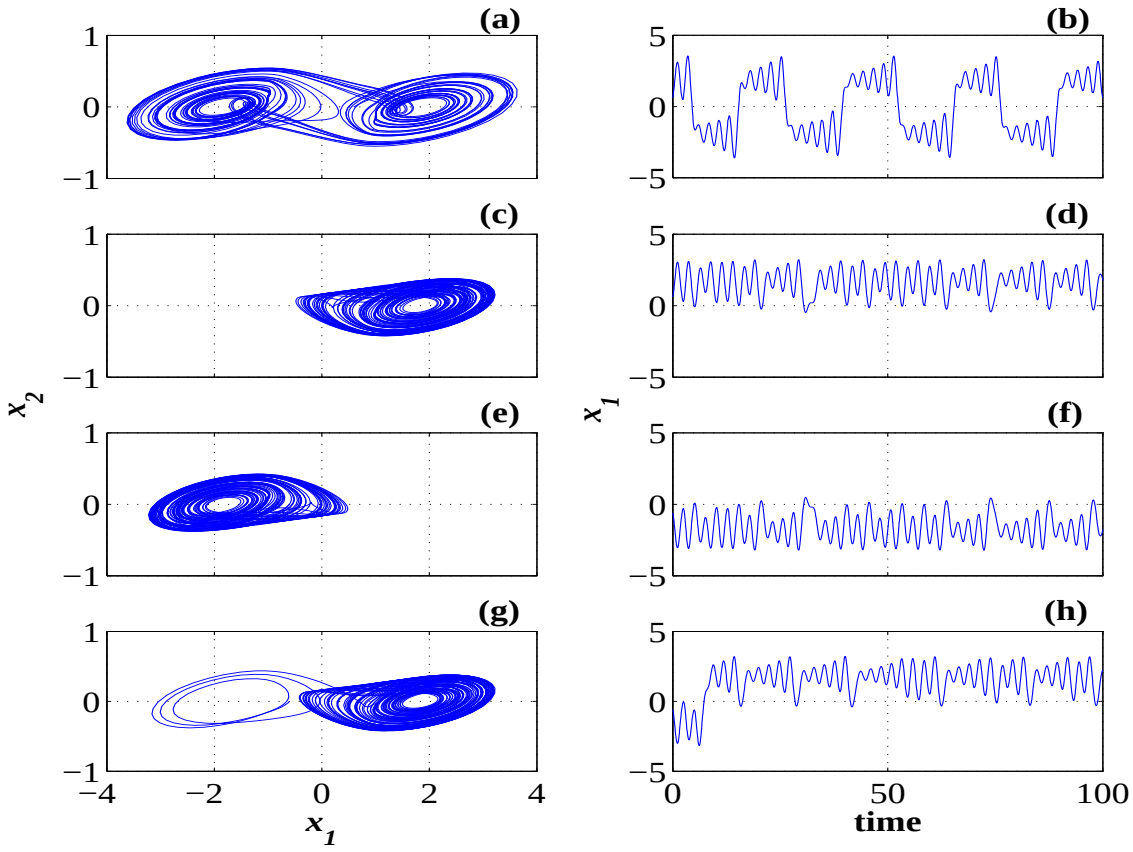

Fig. 2. Chaotic attractors of the parameterized Chua's system: (a) Projection onto (x_1, x_2) and (b) x_1 time series. (c) Projection onto (x_1, x_2) and (d) x_1 time series with positive initial condition. (e) Projection onto (x_1, x_2) and (f) x_1 time series with negative initial condition. (g) Projection onto (x_1, x_2) and (h) x_1 time series negative initial condition.

The relationship between the electronic components of Figure 1 and the parameters of equation (5) are:

$$\alpha = \frac{C_2}{C_1}, \quad \beta = \frac{C_2 R^2}{L}, \quad \gamma = \frac{C_2 R r}{L},$$

$$a = -\frac{R R_2}{R_1 R_3}, \quad b_i = -\frac{R R_2}{R_1 R_3} + \frac{R}{R_j}$$

where the pair of indexes (i, j) are (1,4) or (2,5), exclusively. The double-scroll chaotic attractor is generated in this circuit with the following component values: $C_1 = 100$ nF, $C_2 = 1$, μF, L=67.1 mH, r=2.57 Ω, R_2=R_3= 220 Ω. The resistors R, R_1, R_4 and R_5 are 5 kΩ potentiometers tune to the following values: $R = 1003$ Ω, $R_1 = 790$ Ω, and $R_4 = R_5 = 1700$ Ω. Adjustments of R_1, R_4 and R_5 give us the possibility to restructure the Chua's circuit in order to accomplish the modulating parameter ϵ_k $(k = 1, 2, 3)$, and in turn the multivibrator configurations as discussed previously. Explicitly, the parameterized Chua's circuit is given by:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} \alpha x_2 - \alpha(\epsilon_k x_1 + \epsilon_k f(x_1)) \\ x_1 - x_2 + x_3 \\ -\beta x_2 + \gamma x_3 \end{pmatrix} \tag{6}$$

Notice that parametric modulation algorithm described in subsection 2.1, corresponds to altering the entire linear segment of the PWL function. That is, the modulating parameter directly affects both the

	Astable	**Bistable**	**Monostable**
$\epsilon_1, \epsilon_2, \epsilon_3$ Values:	0.35, 0.8, 0.35	0.35, 0.5, 0.35	0.35, 0.5, 0.4
R_1 Tune to:	825 Ω	884 Ω	884 Ω
R_4 Tune to:	3.058 kΩ	4.062 kΩ	4.062 kΩ
R_5 Tune to:	3.058 kΩ	4.062 kΩ	3.515 kΩ
Circuit's attractor Figure:	3(a)	3(b) and (c)	3(d)

state x_1, and the nonlinear negative resistance $f(x_1)$. In order to be written as in (5), the parameterized Chua's circuit is rewritten as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} \alpha(x_2 - x_1 - G(x_1)) \\ x_1 - x_2 + x_3 \\ -\beta x_2 + \gamma x_3 \end{pmatrix} \tag{7}$$

where

$$G(x_1) = \begin{cases} (\epsilon_1(1+b_1)-1)x_1 - \epsilon_1 c_1, \text{ if } x_1 > 1; \\ (\epsilon_2(1+a)-1)x_1, \qquad \text{if } |x_1| < 1; \\ (\epsilon_3(1+b_2)-1)x_1 + \epsilon_3 c_2, \text{ if } x_1 < -1. \end{cases} \tag{8}$$

Equation (8) is the parameterized representation of the Chua's nonlinear negative resistor, which can be obtained in the circuit of Figure 1 tuning the potentiometers R_1, R_4, and R_5 as follows:

$$\begin{aligned} R_1 &= \frac{-R}{\epsilon_2(1+a)-1}\ ; \\ R_4 &= \frac{-R}{\epsilon_2(1+a)-\epsilon_1(1+b_1)}\ ; \\ R_5 &= \frac{-R}{\epsilon_2(1+a)-\epsilon_3(1+b_2)} \end{aligned} \tag{9}$$

The Table 1 shows the potentiometer values, corresponding to the modulating parameters (ϵ_k), such that the Chua's circuit behaves as each of the multivibrator configurations. The simulation of the multivibrator circuit based on chaos, was carried out on a standard electronic circuit design/simulation software. The second column of Table 1 corresponds to the astable configuration, with the potentiometer values shown, the parameterized Chua's circuit has the double-scroll attractor in Figure 3 (a). The third column corresponds to the bistable multivibrator, in this case, since two single-scroll attractors coexist, the resulting attractors are shown in Figures 3 (b) and 3 (c), for positive and negative initial conditions, respectively. Lastly, the fourth column shows the potentiometer values for a monostable configuration, the resulting chaotic attractor of is presented in Figure 3 (d).

4.1. *Potential Applications*

The proposed all-in-one multivibrator circuit based on chaos can be used to realized different devices. As an illustration, we present two of its potential applications, namely, a pulse generator and a full SR flip-flop. In order to realize these devices we construct two auxiliar circuits to control the input and output of the parameterized Chua's circuit. Figure 4 shows the schematic diagram of the input control circuit, in it realization all resistors are equal to $1k\Omega$, except for R_{14}, which is set to $100k\Omega$. The schematic diagram for the output circuit is shown in Figure 5, this circuit basically consists of a low-pass RC filter and a voltage comparator that gives a two level output.

The pulse generator and flip-flop devices can be realized as illustrated in the block diagram shown in Figure 6. In the first case, with the parameterized Chua's circuit in an astable configuration and with

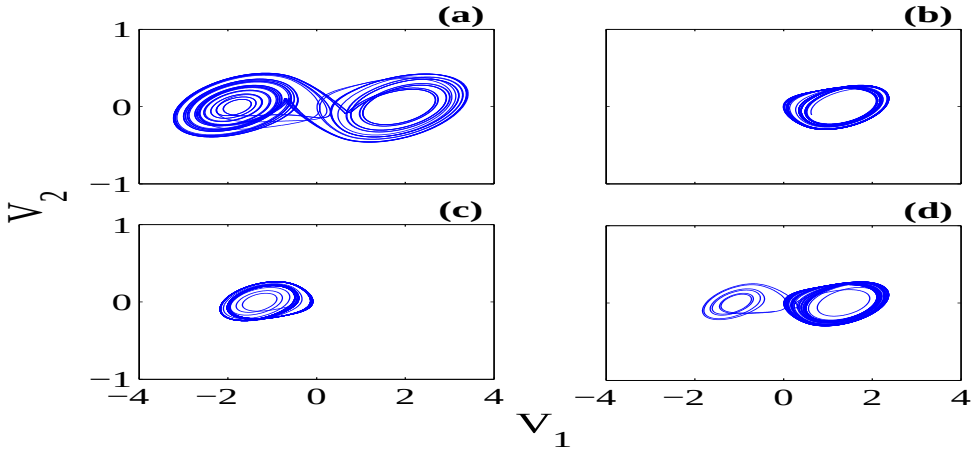

Fig. 3. Attractors for the three configurations of the multivibrator circuit based on chaos: (a) astable, (b)&(c) bistable, and (d) monostable.

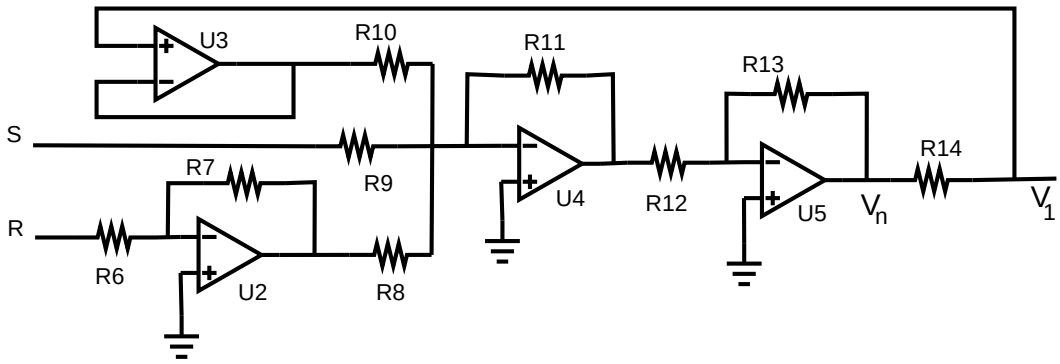

Fig. 4. Schematic diagram of the input circuit for devices based on parameterized Chua's circuit.

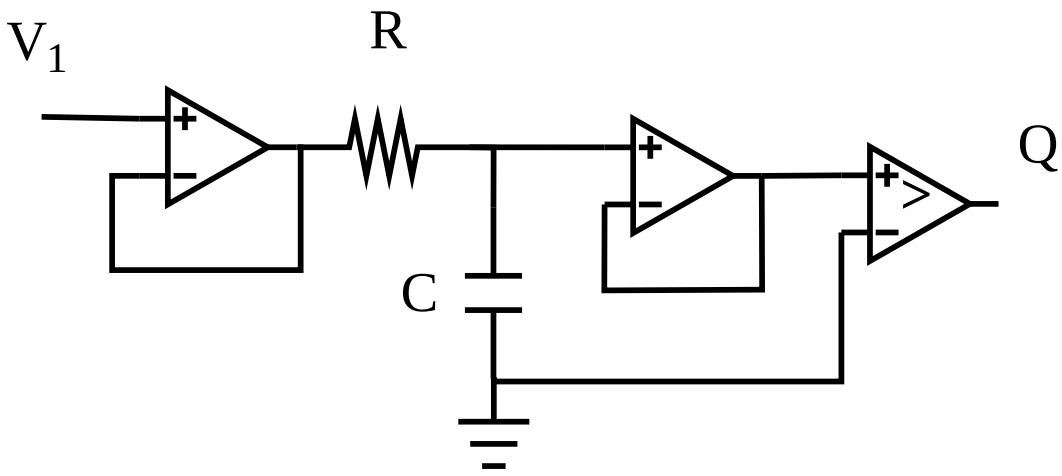

Fig. 5. Schematic diagram of the output circuit for devices based on parameterized Chua's circuit.

the switch SW open, the oscillations around the equilibria points are converted by the output circuit into pulses between zero or five volts. In the second case, adjusting the parameters such that the Chua's circuit be in its bistable configuration and with the switch SW in its closed position, the oscillations will remain positive or negative according to the values of the R and S terminal. In what follows, the operation of these proposed applications is described with more detail:

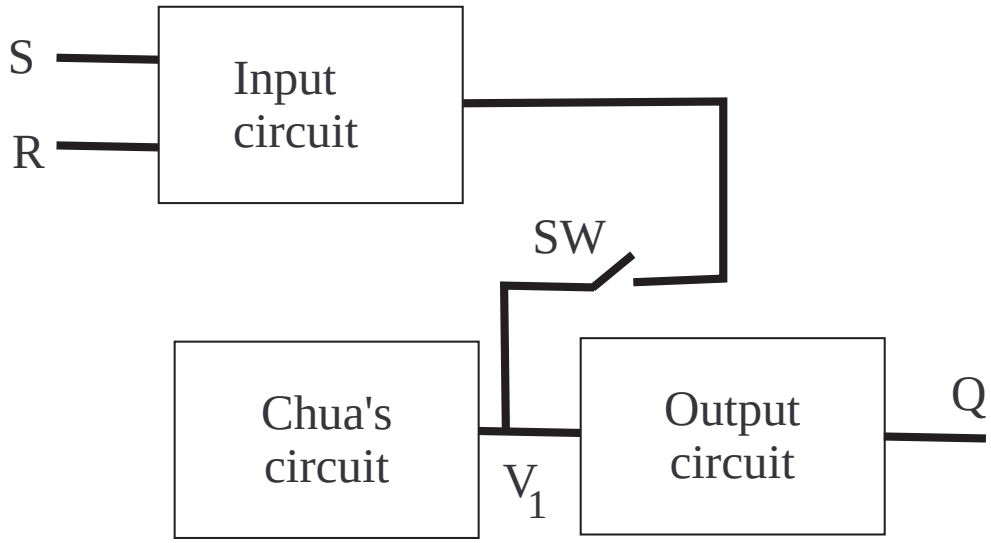

Fig. 6. Block diagram of the realization of pulse generator (SW=open) and flip-flop (SW=closed) based on the parameterized Chua's circuit.

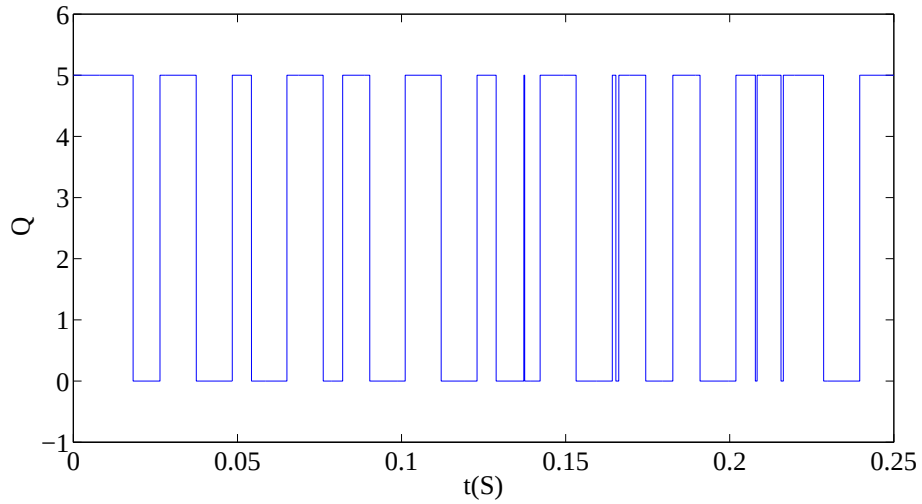

Fig. 7. Irregular pulse train generated by an astable multivibrator based on chaos.

Pulses Generator: The parameterized Chua's circuit in astable configuration serves as a pulses generator. It is possible to yield regular or irregular pulses trains. Using values in the second column of Table 1, the state x_1 through a comparator and filtered by a low pass filter is use to generate pulses. The low pass filter serves to smooth the state x_1 in the vicinity of zero. Then, if the state x_1 is less than zero the output correspond to a zero logic state, otherwise the logic state is set to one. Figure 7 shows a irregular pulses train obtained from the chaotic evolution of Chua's circuit in astable configuration. A regular pulse train, can be yield from this setup via a feedback controller that stabilize the Chua's circuit on a limit circle orbit.

Full SR Flip-Flop: The parameterized Chua's circuit in bistable configuration serves as a flip-flip device. In order to realize this potential application, we construct two auxiliary circuits to control de input and output of the full SR flip-flop device. The interconnection of the control circuits and the bistable Chua's circuit are illustrated in Figure 6. The control input circuit shown in Figure 4 can be used to induce

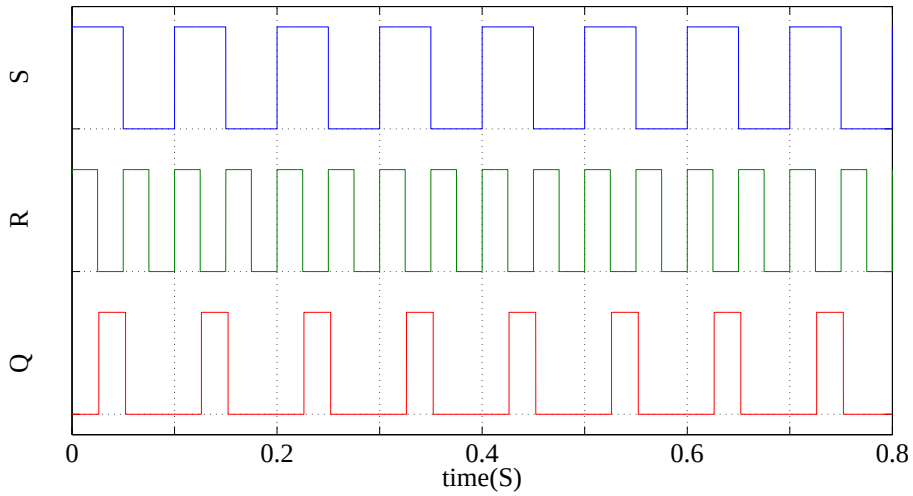

Fig. 8. Flip-flop response generated by a bistable multivibrator based on chaos.

the trajectories to oscillate around χ_1 or χ_3 of the bistable Chua's circuit. Notice that the operation of the input control circuit does not affect the dynamics of the Chua's circuit in the case of $S = R$. In fact, the voltage in the node V_n of Figure 4 is given by $V_n = V_1 + S - R$ Thus, when $S = R$, the current through resistor R_{14} is equal to zero and there is no input to the Chua's circuit. The flip-flop control is achieved restricting the voltage S and R to two voltage levels (low and high). So that when S is high and R is low, oscillations occur only on the positive side (around χ_1). On the other hand, when S is low and R is high, the oscillations occur around χ_3 on the negative side. The output signal Q is taken to be the state x_1 (voltage V_1). If x_1 is greater than zero, we get the one logic state, otherwise the output is a zero logic state. The truth table for this circuit is given as follows: If both S and R have the same level, the output Q retains its previous state. On the other hand, if S is set to zero and R to one, the output is zero; conversely, if R is zero and S is set to one, the output is one. In this way, the bistable multivibrator operates as a full SR flip-flop. Figure 8 shows the input and output signals of the bistable multivibrator operating as a flip-flop. The first signal was introduced in the terminal labeled as S in Figure 4 and the second signal was at R. The third signal is the output given by the circuit shown in Figure 5.

5. Conclusions

This paper shows a method to design a multivibrator circuit based on chaos generation. Our method consists on adding modulation parameters which modify the stability properties of the linear subsystems. Because the chaos generation is exploited, the multivibrator circuit recovers dynamical features onto the logic-gate architecture. Thus, the proposed (dynamical) logic structure is more adaptable than static logic-gates because it can be reconfigurable by parametric modulation. The reconfiguration allows us to achieve distinct tasks with same circuit. That is, the proposed architecture might serve as components of general purpose computing devices with a more flexible structure.

In summary, we introduce a chaos-based multivibrator design adaptable to the three classical configurations (astable, monostable, and bistable) via parametric modulation. As a consequence of the parametric modulation, diverse scrolls might be generated or inhibited around the equilibrium points of a PWL system. In this way, although our results are illustrated using a classical benchmark, alternative system can be exploited in designing multivibrators (as, for example, the PWL version of Dimitrev's system in Appendix). Potential applications are shown as (i) a pulse generators circuit and (ii) a full SR flip-flop device.

Acknowledgements

This work was supported in part by CONACYT Research Council of México under Grants CB-2007 106915-Y and FMSLP-2008-C01 86513.

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Appendix A: An alternative system

In what follows, we show an alternative system that can be used to design multivibrators. By applying the parametric modulation, chaotic attractors with one, two, or three scrolls are generated with a potential use as multivibrators. The PWL version of the Dmitriev's system, proposed in [Campos-Cantón *et al.*, 2007, 2008], serve us to show our point. The PWL is given by

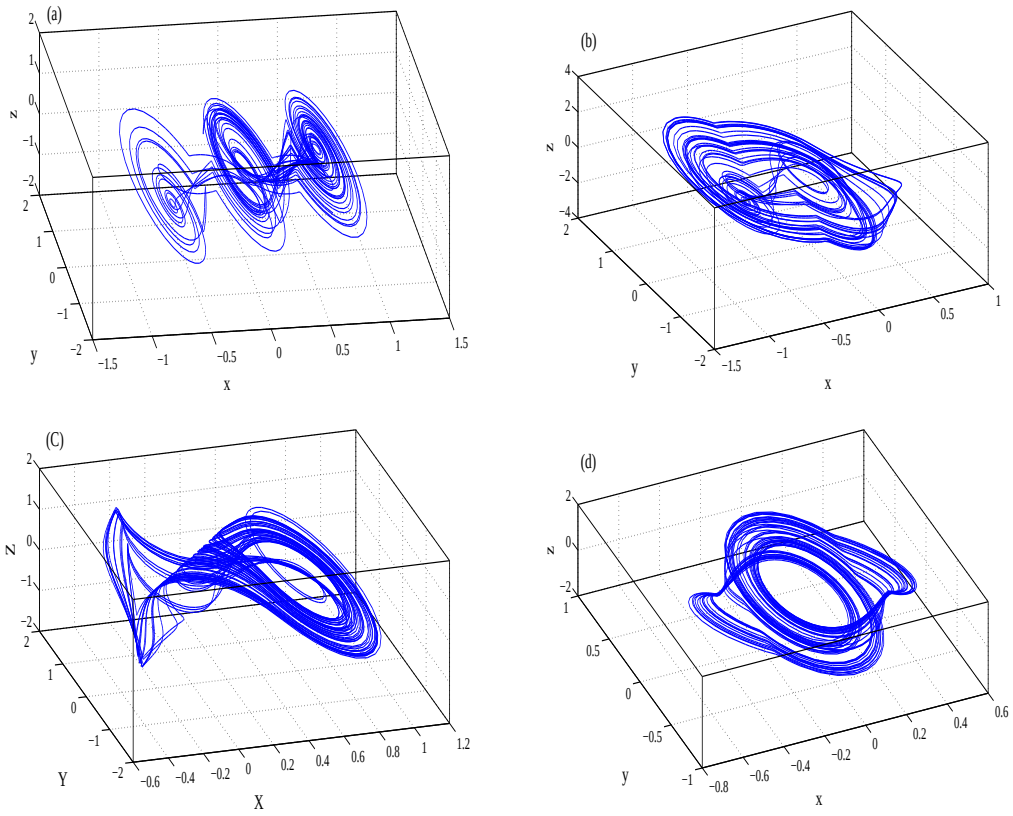

Fig. 9. Gallery of chaotic attractors: (a) Triple-scroll around of χ_1, χ_2, and χ_3, $\epsilon_1 = \epsilon_3 = 1$, $\epsilon_2 = -1$, (b) Double-scroll around of χ_2, and χ_3, $\epsilon_1 = -2$, $\epsilon_2 = -1$, $\epsilon_3 = 1$, (c) Single-scroll around of χ_1, $\epsilon_1 = \epsilon_2 = 1$, $\epsilon_3 = -2$, (d) Single-scroll around of χ_2, $\epsilon_1 = \epsilon_3 = 0.25$, $\epsilon_2 = -0.75$.

$$A_k = \begin{pmatrix} 0 & 1 & 0 \\ -1 & -0.2465 & 1 \\ a_{31k} & -2.0059 & -1.1101 \end{pmatrix} \; ; \; B_k = \begin{pmatrix} 0 \\ 0 \\ b_{3k} \end{pmatrix} \tag{10}$$

for $k = 1, 2, 3$; where $\sigma = 1$, $d = \frac{1}{3}$, $a_{311} = a_{313} = -8.0341$, $a_{312} = 8.5344$, $b_{31} = 5.5228$, $b_{32} = 0$, and $b_{33} = -5.5228$. The system (10) has three equilibrium points χ_1, χ_2, χ_3. As matrices A, B are written in (10), the attractor exhibits scrolls around the equilibrium points χ_1 and χ_3. This is equivalent to use the modulation parameters $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1$. By usign $\epsilon_1 = \epsilon_3 = 1$ and $\epsilon_2 = -1$, an extra scroll is generated around χ_2 resulting in the triple-scroll attractor shown in Figure 9(a). Now, by changing the values of modulation parameters to be $\epsilon_1 = -2$, $\epsilon_2 = -1$, $\epsilon_3 = 1$, an alternative double-scroll attractor is generated around χ_2 and χ_3. The attractor is reached only with negative initial conditions, see Figure 9(b). Complementary, for $\epsilon_1 = 1$, $\epsilon_2 = -1$ and $\epsilon_3 = -2$, a similar attractor is generated around χ_1 and χ_2, which is only reachable from positive initial conditions. In Figure 9(c), we show a single scroll attractor around χ_1, obtained for $\epsilon_1 = \epsilon_2 = 1$, $\epsilon_3 = -2$. In Figure 9(d) is shown a different single scroll atractor around χ_2, the values of modulation parameters are $\epsilon_1 = \epsilon_3 = 0.25$ and $\epsilon_2 = -0.75$.