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A generalized PID-type control scheme with simple tuning for the global regulation of robot manipulators with constrained inputs

Marco Mendoza[a], Arturo Zavala-Río[b], Víctor Santibáñez[c] & Fernando Reyes[d]

[a] Universidad Autónoma de San Luis Potosí, Facultad de Ciencias, San Luis Potosí, Mexico

[b] Instituto Potosino de Investigación Científica y Tecnológica, División de Matemáticas Aplicadas, San Luis Potosí, Mexico

[c] Instituto Tecnológico de la Laguna, Torreón, Mexico

[d] Benemérita Universidad Autónoma de Puebla, Facultad de Ciencias de la Electrónica, Puebla, Mexico

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Marco Mendoza[a], Arturo Zavala-Río[b*], Víctor Santibáñez[c] and Fernando Reyes[d]

[a] *Universidad Autónoma de San Luis Potosí, Facultad de Ciencias, San Luis Potosí, Mexico*
[b] *Instituto Potosino de Investigación Científica y Tecnológica, División de Matemáticas Aplicadas, San Luis Potosí, Mexico*
[c] *Instituto Tecnológico de la Laguna, Torreón, Mexico*
[d] *Benemérita Universidad Autónoma de Puebla, Facultad de Ciencias de la Electrónica, Puebla, Mexico*

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In this paper, a globally stabilizing PID-type control scheme with a generalized saturating structure for robot manipulators under input constraints is proposed. It gives rise to various families of bounded PID-type controllers whose implementation is released from the exact knowledge of the system parameters and model structure. Compared to previous approaches of the kind, the proposed scheme is not only characterized by its generalized structure but also by its very simple tuning criterion, the simplest hitherto obtained in the considered analytical framework. Experimental results on a 3-degree-of-freedom direct-drive manipulator corroborate the efficiency of the proposed approach.

Keywords: PID control; global regulation; robot manipulators; bounded inputs; simple tuning

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1. Introduction

Over recent years, sophisticated control schemes have been proposed. However, in actual applications where robot manipulators are involved, Proportional-Integral-Derivative (PID) algorithms seem to be a regular practice (Rocco, 1996; Visioli & Legnani, 2002). Since it has not been possible to develop a global proof of the stability properties provided through their classical linear form, alternative versions of these controllers using nonlinear structures, mainly oriented to guarantee global stabilization, have been proposed for instance in (Arimoto, 1995; Kelly, 1998; Santibáñez & Kelly, 1998; Sun, Songyu, Shao & Liu, 2009). However, these modified algorithms consider that actuators can provide any required torque value which is impossible in practice.

In order to ensure a proper operation of the robots, it is important to design control schemes that take into account the natural constraints of real actuators. Otherwise, unexpected behaviors and related risks could take place in view of the saturation nonlinearity that generally relates the controller outputs to the plant inputs in actual feedback systems (Chen & Wang, 1988; Kapasouris & Athans, 1990; Krikelis & Barkas, 1984; Zavala-Río, Aguilera-González, Martínez-Sibaja, Astorga-Zaragoza & Adam-Medina, 2013). Under the consideration of such a restriction, various schemes have been proposed in the literature. For instance, state-feedback controllers with Saturating-Proportional (SP) and Saturating-Derivative (SD) actions (Kelly, Santibáñez & Berghuis, 1997; Santibáñez & Kelly, 1996; Santibáñez, Kelly & Reyes, 1998), as well as output-feedback schemes (Loría, Kelly, Ortega & Santibáñez, 1997; Santibáñez & Kelly, 1997), were some of the earlier proposals. These algorithms involve exact gravity compensation, accurate position and velocity vectors (or the latter replaced by the *dirty derivative* of the former), and the use of specific saturation func-

*Corresponding author. Email: azavala@ipicyt.edu.mx

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tions. Alternative PD-type controllers, involving *generalized* saturation functions, were presented in (Zavala-Río & Santibáñez, 2006) and (Zavala-Río & Santibáñez, 2007) in order to obtain alternative saturating structures and/or give rise to improved closed-loop performances. Parametric dependency has been further alleviated through adaptive approaches (Colbaugh, Barany & Glass, 1997; Laib, 2000; López-Araujo, Zavala-Río, Santibáñez & Reyes, 2013a,b; Zergeroglu, Dixon, Behal & Dawson, 2000). The proposal in (Colbaugh et al., 1997) consists of an algorithm with a switching (variable) structure from a non-adaptive SP-SD type controller (without gravity compensation) to a (linear) PD regulator with discontinuous adaptive gravity compensation. Those in (Zergeroglu et al., 2000) and (Laib, 2000) are state and output feedback (respectively) regulators that keep the structure of the SP-SD controller of (Kelly et al., 1997) but involve adaptive gravity compensation, where the region of attraction of the desired equilibrium can be enlarged by increasing the control gain values. The works in (López-Araujo et al., 2013a) and (López-Araujo et al., 2013b) present a generalized state-feedback scheme and an output-feedback algorithm (respectively), both achieving global regulation —avoiding input saturation— through a fixed (non-varying) continuous structure. The above-cited adaptive approaches prove to be useful under parameter uncertainty but remain partially model dependent by involving the regression matrix implicated in the linear structural characterization of the gravity force vector with respect to its parametric coefficient set.

On the other hand, semiglobal regulation has been achieved through bounded PID controllers with different saturating structures in (Alvarez-Ramírez, Kelly & Cervantes, 2003) and (Alvarez-Ramírez, Santibáñez & Campa, 2008). The stability analysis in these works is carried out through the singular perturbation methodology which shows the existence of an appropriate tuning mainly characterized by the requirement of sufficiently small integral action gains and high enough proportional and derivative ones. As far as the authors are aware, the first saturating PID-type controller for global regulation was developed in (Gorez, 1999). Nevertheless, the structure of the proposed algorithm is quite complex. Other works have devoted efforts to solve the global PID regulation problem for manipulators with bounded inputs through simpler structures, giving rise to the SP-SI-SD type algorithm developed in (Meza, Santibáñez & Hernández, 2005) *via* passivity theory and later on in (Su, Müller & Zheng, 2010) through Lyapunov stability analysis, and to the SPD-SI type scheme presented in (Santibáñez, Kelly, Zavala-Río & Parada, 2008). Furthermore, in addition to the actuator torque constraints, recent studies have incorporated the saturation effects of the electronic control devices of practical PID regulators (Orrante-Sakanassi, Santibáñez & Campa, 2010; Santibáñez, Camarillo, Moreno-Valenzuela & Campa, 2010; Yarza, Santibáñez & Moreno-Valenzuela, 2011). Exponential and/or global asymptotic stabilization conditions were established under these natural restrictions for various implementation structures that are common in industrial robots.

The above cited bounded PID-type approaches solve the formulated problem under input and data restrictions. However, some design particularities and/or the developed closed-loop analyses have generally conducted to restrictive tuning criteria that include a set of conditions that are either not all necessary or more strict than really needed, which hinders the tuning task, and whose applicability remains valid for the particular saturating structure considered in the control design.

In this paper, a globally stabilising PID-type control scheme with a generalized saturating structure for robot manipulators under input constraints is proposed. It gives rise to various families of bounded PID-type controllers that include the SP-SI-SD and SPD-SI structures as particular cases, among others. In addition to the achievement of the global regulation objective without the need for the exact knowledge of the system parameters and model structure, the proposed scheme is not designed or analysed using a particular sigmoidal function to cope with the input constraints but may involve any one within a well-characterized set of saturation functions. Moreover, and very importantly, the developed closed-loop analysis gives rise to a very simple control gain selection criterion, which proves to be an important progress over previous approaches of the kind. Simplification of the tuning conditions for PID-type controllers has been a research subject for several years (Hernández-Guzmán, Santibáñez & Silva-Ortigoza, 2008; Kelly, 1995; Orrante-Sakanassi, Santibáñez & Hernández-Guzmán, 2014a,b) and had never been achieved to be as simple as it is shown in this paper. Experimental results on a 3-degree-of-freedom (DOF) direct-drive manipulator corroborate the proposed contribution.

2. Preliminaries

Let $X \in \mathbb{R}^{m\times n}$ and $y \in \mathbb{R}^n$. Throughout this work, X_{ij} represents the element of X at its i^{th} row and j^{th} column, and y_i denotes the i^{th} element of y. 0_n stands for the origin of $\mathbb{R}^n$ and I_n represents the $n \times n$ identity matrix. $\|\cdot\|$ stands for the standard Euclidean norm for vectors, *i.e.* $\|y\| = \sqrt{\sum_{i=1}^n y_i^2}$, and induced norm for matrices, *i.e.* $\|X\| = \sqrt{\lambda_{\max}\{X^T X\}}$ where $\lambda_{\max}\{X^T X\}$ represents the maximum eigenvalue of $X^T X$. The image of $\mathcal{B} \subset \mathbb{R}^n$ under $\psi : \mathbb{R}^n \to \mathbb{R}^m$ is denoted $\psi(\mathcal{B})$. For a continuous scalar function $\psi : \mathbb{R} \to \mathbb{R}$, ψ' denotes its derivative, when differentiable, $D^+\psi$ its upper right-hand (Dini) derivative, *i.e.* $D^+\psi(\varsigma) = \limsup_{h\to 0^+} \frac{\psi(\varsigma+h)-\psi(\varsigma)}{h}$, with $D^+\psi = \psi'$ at points of differentiability (Khalil, 2002, Appendix C.2), and ψ^{-1} its inverse, when invertible.

Consider the n-DOF serial rigid manipulator dynamics with viscous friction (Arimoto, 1996, §2.1), (Sciavicco & Siciliano, 2000, §6.2), (Lewis, Dawson & Abdallah, 2004, §7.2)

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau \tag{1}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity, and acceleration vectors, $H(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, and $C(q,\dot{q})\dot{q}$, $F\dot{q}$, $g(q)$, $\tau \in \mathbb{R}^n$ are respectively the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with $F \in \mathbb{R}^{n\times n}$ being a positive definite constant diagonal matrix whose entries $f_i > 0$, $i = 1, \dots, n$, are the viscous friction coefficients, and $g(q) = \nabla\mathcal{U}(q)$, with $\mathcal{U}(q)$ being the gravitational potential energy, or equivalently

$$\mathcal{U}(q) = \mathcal{U}(q_0) + \int_{q_0}^{q} g^T(r)dr \tag{2a}$$

with

$$\int_{q_0}^{q} g^T(r)dr = \int_{q_{01}}^{q_1} g_1(r_1, q_{02}, \dots, q_{0n})dr_1 + \int_{q_{02}}^{q_2} g_2(q_1, r_2, q_{03}, \dots, q_{0n})dr_2 + \dots + \int_{q_{0n}}^{q_n} g_n(q_1, \dots, q_{n-1}, r_n)dr_n \tag{2b}$$

for any[1] $q, q_0 \in \mathbb{R}^n$. Some well-known properties characterizing the terms of such a dynamical model are recalled here (Arimoto, 1996, §2.1), (Lewis et al., 2004, §3.3), (Kelly, Santibáñez & Loría, 2005, Chap. 4). Subsequently, we denote $\dot{H}$ the rate of change of H, *i.e.* $\dot{H} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n} : (q,\dot{q}) \mapsto \left[\frac{\partial H_{ij}}{\partial q}(q)\dot{q}\right]$.

Property 1: $H(q)$ is a continuously differentiable matrix function being positive definite, symmetric, and bounded on $\mathbb{R}^n$, *i.e.* such that $\mu_m I_n \le H(q) \le \mu_M I_n$, $\forall q \in \mathbb{R}^n$, for some constants $\mu_M \ge \mu_m > 0$.

[1]Since $g(q)$ is the gradient of the gravitational potential energy $\mathcal{U}(q)$, a scalar function, then, for any $q, q_0 \in \mathbb{R}^n$, the inverse relation $\mathcal{U}(q) = \mathcal{U}(q_0) + \int_{q_0}^{q} g^T(r)dr$ is independent of the integration path (Khalil, 2002, p. 120). Eq. (2b) considers integration along the axes. This way, on every axis (*i.e.* at every integral in the right-hand side of (2b)), the corresponding coordinate varies (according to the specified integral limits) while the rest of the coordinates remain constant.

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Property 2: The Coriolis matrix $C(q,\dot{q})$ satisfies:

2.1. $\|C(q,\dot{q})\| \leq k_C\|\dot{q}\|$, $\forall (q,\dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, for some constant $k_C \geq 0$;
2.2. for all $(q,\dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, $\dot{q}^T \left[\frac{1}{2}\dot{H}(q,\dot{q}) - C(q,\dot{q})\right]\dot{q} = 0$ and actually $\dot{H}(q,\dot{q}) = C(q,\dot{q}) + C^T(q,\dot{q})$.

Property 3: The viscous friction coefficient matrix satisfies $f_m\|\dot{q}\|^2 \leq \dot{q}^T F\dot{q} \leq f_M\|\dot{q}\|^2$, $\forall \dot{q} \in \mathbb{R}^n$, where $0 < f_m \triangleq \min_i\{f_i\} \leq \max_i\{f_i\} \triangleq f_M$.

Property 4: The gravity force term $g(q)$ is a continuously differentiable bounded vector function with bounded Jacobian matrix[2] $\frac{\partial g}{\partial q}$. Equivalently, every element of the gravity force vector, $g_i(q)$, $i = 1,\ldots,n$, satisfies:

4.1. $|g_i(q)| \leq B_{gi}$, $\forall q \in \mathbb{R}^n$, for some positive constant B_{gi};
4.2. $\frac{\partial g_i}{\partial q_j}$, $j = 1,\ldots,n$, exist and are continuous and such that $\left|\frac{\partial g_i}{\partial q_j}(q)\right| \leq \left\|\frac{\partial g}{\partial q}(q)\right\| \leq k_g$, $\forall q \in \mathbb{R}^n$, for some positive constant k_g, and consequently $|g_i(x) - g_i(y)| \leq \|g(x) - g(y)\| \leq k_g\|x - y\|$, $\forall x, y \in \mathbb{R}^n$.

In this work, we consider the (realistic) case where the absolute value of each input τ_i is constrained to be smaller than a given saturation bound $T_i > 0$, *i.e.*, $|\tau_i| \leq T_i$, $i = 1,\ldots,n$. More precisely, letting u_i represent the control variable (controller output) relative to the i^{th} degree of freedom, we have that

$$\tau_i = T_i \text{sat}(u_i/T_i) \tag{3}$$

where $\text{sat}(\cdot)$ is the standard saturation function, *i.e.* $\text{sat}(\varsigma) = \text{sign}(\varsigma)\min\{|\varsigma|, 1\}$. Let us note, from Eqs. (1) and (3), that $T_i \geq B_{gi}$ (see Property 4.1), $\forall i \in \{1,\ldots,n\}$, is a necessary condition for the robot manipulator to be stabilizable at any desired equilibrium configuration $q_d \in \mathbb{R}^n$. Thus, the following assumption turns out to be important within the analytical setting considered here.

Assumption 1: *$T_i > \alpha B_{gi}$, $\forall i \in \{1,\ldots,n\}$, for some scalar $\alpha \geq 1$.*

The control scheme proposed in this work involves functions fulfilling the following definition.

Definition 1: Given a positive constant M, a nondecreasing Lipschitz-continuous function $\sigma : \mathbb{R} \to \mathbb{R}$ is said to be a *generalized saturation* with bound M if

(a) $\varsigma\sigma(\varsigma) > 0$, $\forall \varsigma \neq 0$;
(b) $|\sigma(\varsigma)| \leq M$, $\forall \varsigma \in \mathbb{R}$.

If in addition

(c) $\sigma(\varsigma) = \varsigma$ when $|\varsigma| \leq L$,

for some positive constant $L \leq M$, σ is said to be a *linear saturation* for (L, M) (Teel, 1992).

Functions satisfying Definition 1 have the following properties.

Lemma 1: *Let $\sigma : \mathbb{R} \to \mathbb{R}$ be a generalized saturation with bound M and let k be a positive constant. Then*

1. $\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) = 0$;
2. $\exists \sigma'_M \in (0,\infty)$ *such that* $0 \leq D^+\sigma(\varsigma) \leq \sigma'_M$, $\forall \varsigma \in \mathbb{R}$;
3. $|\sigma(k\varsigma + \eta) - \sigma(\eta)| \leq \sigma'_M k|\varsigma|$, $\forall \varsigma, \eta \in \mathbb{R}$;
4. $|\sigma(k\varsigma)| \leq \sigma'_M k|\varsigma|$, $\forall \varsigma \in \mathbb{R}$;

[2]Property 4 is satisfied for instance by robot manipulators having only revolute joints (Kelly et al., 2005, §4.3).

5. $\frac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma'_M\varsigma^2}{2}$, $\forall\varsigma \in \mathbb{R}$;
6. $\int_0^\varsigma \sigma(kr)dr > 0$, $\forall\varsigma \neq 0$;
7. $\int_0^\varsigma \sigma(kr)dr \to \infty$ *as* $|\varsigma| \to \infty$;
8. *if* σ *is strictly increasing, then*
   (a) $\varsigma[\sigma(\varsigma+\eta) - \sigma(\eta)] > 0$, $\forall\varsigma \neq 0$, $\forall\eta \in \mathbb{R}$;
   (b) *for any constant* $a \in \mathbb{R}$, $\bar{\sigma}(\varsigma) = \sigma(\varsigma + a) - \sigma(a)$ *is a strictly increasing generalized saturation function with bound* $\bar{M} = M + |\sigma(a)|$;
9. *if* σ *is a linear saturation for* (L, M) *then, for any continuous function* $\nu : \mathbb{R} \to \mathbb{R}$ *such that* $|\nu(\eta)| < L$, $\forall\eta \in \mathbb{R}$, *we have that* $\varsigma\big[\sigma\big(\varsigma + \nu(\eta)\big) - \sigma\big(\nu(\eta)\big)\big] > 0$, $\forall\varsigma \neq 0$, $\forall\eta \in \mathbb{R}$.

Proof. Items 3 and 4 are a direct consequence of the Lipschitz-continuity of σ and item 2 of the statement (as analogously stated for instance in (Khalil, 2002, Lemma 3.3) under continuous differentiability). The rest of the items are proven in (López-Araujo et al., 2013a). □

3. The proposed control scheme

We propose a Proportional-Integral-Dissipative type control scheme with generalized form

$$u(q, \dot{q}, \phi) = -s_d(\bar{q}, \dot{q}, \phi) - s_P(K_P\bar{q}) + s_I(K_I\phi) \tag{4}$$

where $\bar{q} = q - q_d$, for any constant (desired equilibrium position) vector $q_d \in \mathbb{R}^n$; $\phi \in \mathbb{R}^n$ is an auxiliary state vector coming from the integral-action dynamics defined as[3]

$$\dot{\phi} = -\dot{q} - \varepsilon K_P^{-1} s_P(K_P\bar{q}) \tag{5}$$

$K_P \in \mathbb{R}^{n\times n}$ and $K_I \in \mathbb{R}^{n\times n}$ are positive definite diagonal matrices —*i.e.*, $K_P = \mathrm{diag}[k_{P1}, \ldots, k_{Pn}]$ and $K_I = \mathrm{diag}[k_{I1}, \ldots, k_{In}]$ with $k_{Pi} > 0$ and $k_{Ii} > 0$, $\forall i = 1, \ldots, n$— such that

$$k_{Pm} \triangleq \min_i\{k_{Pi}\} > k_g \tag{6}$$

(see Property 4.2); for any $x \in \mathbb{R}^n$, $s_P(x) = \Big(\sigma_{P1}(x_1), \ldots, \sigma_{Pn}(x_n)\Big)^T$ and $s_I(x) = \Big(\sigma_{I1}(x_1), \ldots, \sigma_{In}(x_n)\Big)^T$, with $\sigma_{Pi}(\cdot)$, $i = 1, \ldots, n$, being (suitable) **linear saturation functions** for (L_{Pi}, M_{Pi}) and $\sigma_{Ii}(\cdot)$, $i = 1, \ldots, n$, being **strictly increasing generalized saturation functions** with bounds M_{Ii}, such that

$$L_{Pi} > 2B_{gi} \tag{7a}$$

$$M_{Ii} > B_{gi} \tag{7b}$$

$i = 1, \ldots, n$; $s_d : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector function satisfying

$$s_d(\bar{q}, 0_n, \phi) = 0_n \tag{8}$$

[3]Under time parametrization of the system trajectories, (5) adopts the (equivalent) form $\phi(t) = \phi(0) - \int_0^t \big[\dot{q}(\varsigma) + \varepsilon K_P^{-1} s_P\big(K_P\bar{q}(\varsigma)\big)\big]d\varsigma$, for any initial condition $(q, \dot{q}, \phi)(0) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$.

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$\forall \bar{q} \in \mathbb{R}^n$, $\forall \phi \in \mathbb{R}^n$,

$$\dot{q}^T s_d(\bar{q}, \dot{q}, \phi) > 0 \tag{9}$$

$\forall \dot{q} \neq 0_n$, $\forall \bar{q} \in \mathbb{R}^n$, $\forall \phi \in \mathbb{R}^n$,

$$\| s_d(\bar{q}, \dot{q}, \phi) \| \leq \kappa \|\dot{q}\| \tag{10}$$

$\forall (\bar{q}, \dot{q}, \phi) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$, for some constant $\kappa > 0$, and

$$|u_i(q, \dot{q}, \phi)| = | - s_{di}(\bar{q}, \dot{q}, \phi) - \sigma_{Pi}(k_{Pi}\bar{q}_i) + \sigma_{Ii}(k_{Ii}\phi_i)| < T_i \tag{11}$$

$i = 1, \ldots, n$, $\forall (\bar{q}, \dot{q}, \phi) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$, for suitable bounds M_{Pi} and M_{Ii} of $\sigma_{Pi}(\cdot)$ and $\sigma_{Ii}(\cdot)$; and ε (in (5)) is a positive constant satisfying

$$\varepsilon < \varepsilon_M \triangleq \min\{\varepsilon_1, \varepsilon_2\} \tag{12}$$

where

$$\varepsilon_1 \triangleq \sqrt{\frac{\beta_0 \beta_P \mu_m}{\mu_M^2}} \quad , \quad \varepsilon_2 \triangleq \frac{f_m}{\beta_M + \frac{(f_M+\kappa)^2}{4\beta_0 k_{Pm}}} < \frac{f_m}{\beta_M} \triangleq \varepsilon_3$$

with

$$\beta_0 \triangleq 1 - \max\left\{ \frac{k_g}{k_{Pm}}, \max_i \left\{ \frac{2B_{gi}}{L_{Pi}} \right\} \right\} \quad , \quad \beta_P \triangleq \min_i \left\{ \frac{k_{Pi}}{\sigma'_{PiM}} \right\} \quad , \quad \beta_M \triangleq k_C B_P + \mu_M \sigma'_{PM}$$

$$B_P \triangleq \sqrt{\sum_{i=1}^{n} \left(\frac{M_{Pi}}{k_{Pi}} \right)^2} \quad , \quad \sigma'_{PM} \triangleq \max_i \{\sigma'_{PiM}\}$$

(observe that by inequalities (6) and (7a): $0 < \beta_0 < 1$), σ'_{PiM} being the positive bound of $D^+\sigma_{Pi}(\cdot)$, in accordance to item 2 of Lemma 1, and μ_m, μ_M, k_C, f_m, f_M, B_{gi}, and k_g as defined through Properties 1–4.

Remark 1: In order to preserve the main feature of PID-type controllers, the vector function s_d in (4) shall not involve any term of the open-loop system dynamics (whether as online or desired compensation) or the exact value of any of its parameters. In general, s_d will include a derivative-action term (acting on the derivative of the position error, *i.e.* on the velocity vector) and may involve some form of the proportional and/or the integral ones, as illustrated in Appendix A.

Remark 2: A closed loop analysis under stationary conditions ($\ddot{q} = \dot{q} = \dot{\phi} = 0_n$) reveals the main functions of the integral action. On the one hand, the auxiliary dynamics in (5) forces the desired configuration q_d to be the unique closed-loop equilibrium position, eliminating steady-state configuration errors, consequently overcoming the main limitation of approaches that consider exact (online or desired) gravity compensation (Kelly et al., 1997; Santibáñez & Kelly, 1996; Santibáñez et al., 1998; Zavala-Río & Santibáñez, 2006, 2007). Such an equilibrium position is additionally guaranteed to have the required stability properties, in accordance to the global regulation objective, under the satisfaction of inequalities (6) and (7a), as can be concluded from the closed-loop analysis developed in Section 4. On the other hand, the saturating integral (rightmost) term in the right-hand side of (4) shall compensate for the steady-state gravity forces. This is the reason

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why inequalities (7b) are required, since they ensure that $g(\mathbb{R}^n)$ be the image under s_I of some subset $\Phi \in \mathbb{R}^n$ (in the subspace related to the auxiliary state variable ϕ), thus guaranteeing suitable steady-state gravity compensation for all $q_d \in \mathbb{R}^n$. The relation stated from Φ to $g(\mathbb{R}^n)$ under s_I is further rendered injective (one-to-one) through the strictly increasing requirement on the generalized saturations σ_{Ii}, $i = 1,\ldots,n$, which guarantees their invertibility. Furthermore, input saturation is avoided through (11). In this direction, it is important to note that, depending on the specific choice of the vector function s_d, Assumption 1 may be required to be satisfied with some α strictly greater than unity in order to guarantee the feasibility of the simultaneous fulfillment of (11) and inequalities (7). For instance, in the particular control structure cases presented in Appendix A, such a feasibility is achieved by requiring $\alpha = 3$, as pointed out in Remark 6. A similar condition on the control input bounds has been required by other approaches where input constraints have been considered (Colbaugh et al., 1997). In saturating PID-type schemes from previous references, a similar or analog condition on the control input bounds remains implicit by requiring corresponding parameters to be high enough to satisfy conditions coming from the stability analysis and simultaneously low enough to fulfill the *input-saturation-avoidance* inequalities.

4. Closed-Loop Analysis

Consider system (1),(3) taking $u = u(q,\dot{q},\phi)$ as defined through Eqs. (4)-(5). Observe that the satisfaction of (11), under the consideration of (3), shows that

$$T_i > |u_i(q,\dot{q},\phi)| = |u_i| = |\tau_i| \qquad i = 1,\ldots,n \qquad \forall (q,\dot{q},\phi) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \tag{13}$$

Hence, the closed-loop dynamics takes the form

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = -s_d(\bar{q},\dot{q},\phi) - s_P(K_P\bar{q}) + \bar{s}_I(\bar{\phi}) + g(q_d) \tag{14a}$$

$$\dot{\bar{\phi}} = -\dot{q} - \varepsilon K_P^{-1} s_P(K_P\bar{q}) \tag{14b}$$

where $\bar{\phi} = \phi - \phi^*$ and

$$\bar{s}_I(\bar{\phi}) = s_I(K_I\bar{\phi} + K_I\phi^*) - s_I(K_I\phi^*) \tag{15}$$

with $\phi^* = (\phi_1^*,\ldots,\phi_n^*)^T$ such that $s_I(K_I\phi^*) = g(q_d)$, or equivalently $\phi_i^* = \sigma_{Ii}^{-1}\big(g_i(q_d)\big)/k_{Ii}$, $i = 1,\ldots,n$ (recall Remark 2). Observe that, by point 8b of Lemma 1, the elements of $\bar{s}_I(\bar{\phi})$ in Eq. (15), *i.e.*

$$\bar{\sigma}_{Ii}(\bar{\phi}_i) = \sigma_{Ii}(k_{Ii}\bar{\phi}_i + k_{Ii}\phi_i^*) - \sigma_{Ii}(k_{Ii}\phi_i^*)$$

$i = 1,\ldots,n$, turn out to be strictly increasing generalized saturation functions.

Proposition 1: *Consider the closed-loop system in Eqs.* (14)*, under the satisfaction of inequalities* (7)*, the conditions on the vector function s_d stated through expressions* (8)–(11)*, and Assumption 1 with suitable value of α. Thus, for any positive definite diagonal matrices K_I and K_P such that inequality* (6) *is satisfied, and any ε fulfilling inequality (12), global asymptotic stability of the closed-loop trivial solution $(\bar{q},\bar{\phi})(t) \equiv (0_n, 0_n)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1,\ldots,n$, $\forall t \geq 0$.*

Proof. By (13), one sees that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $\forall t \geq 0$. This proves that, under the proposed scheme, the input saturation values, T_i, are never attained. Now, in order to carry out the stability analysis, the following scalar function is defined[4]

$$V(\bar{q},\dot{q},\bar{\phi}) = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}H(q)\dot{q} + \mathcal{U}(q) - \mathcal{U}(q_d) - g^T(q_d)\bar{q} + \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr \tag{16}$$

where $\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i$, $\int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr = \sum_{i=1}^n \int_0^{\bar{\phi}_i} \bar{\sigma}_{Ii}(r_i)dr_i$ and recall that $\mathcal{U}$ represents the gravitational potential energy. Note, by recalling Eqs. (2), that the defined scalar function can be rewritten as

$$V(\bar{q},\dot{q},\bar{\phi}) = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}H(q)\dot{q} + \gamma_0\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + \mathcal{U}_{\gamma_0}^c(\bar{q}) + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr$$

where

$$\begin{aligned}\mathcal{U}_{\gamma_0}^c(\bar{q}) &= \int_{0_n}^{\bar{q}} \left[g(r+q_d) - g(q_d) + (1-\gamma_0)s_P(K_P r)\right]^T dr \\ &= \sum_{i=1}^n \int_0^{\bar{q}_i} \left[\bar{g}_i(r_i) - g_i(q_d) + (1-\gamma_0)\sigma_{Pi}(k_{Pi}r_i)\right] dr_i\end{aligned} \tag{17}$$

with

$$\begin{aligned}\bar{g}_1(r_1) &= g_1(r_1+q_{d1}, q_{d2}, \ldots, q_{dn}) \\ \bar{g}_2(r_2) &= g_2(q_1, r_2+q_{d2}, q_{d3}, \ldots, q_{dn}) \\ &\vdots \\ \bar{g}_n(r_n) &= g_n(q_1, q_2, \ldots, q_{n-1}, r_n+q_{dn})\end{aligned}$$

and γ_0 is a constant satisfying

$$\beta_0\frac{\varepsilon^2}{\varepsilon_1^2} < \gamma_0 < \beta_0 \tag{18}$$

(observe, from inequality (12) and the definition of β_0, that $0 < \beta_0\varepsilon^2/\varepsilon_1^2 < \beta_0 < 1$). Under this consideration, $\mathcal{U}_{\gamma_0}^c(\bar{q})$ turns out to be lower-bounded by

$$W_{10}(\bar{q}) = \sum_{i=1}^n w_i^{10}(\bar{q}_i) \tag{19a}$$

where

$$w_i^{10}(\bar{q}_i) \triangleq \begin{cases} \frac{k_{li}}{2}\bar{q}_i^2 & \text{if } |\bar{q}_i| \le \bar{q}_i^* \\ k_{li}\bar{q}_i^*\left(|\bar{q}_i| - \frac{\bar{q}_i^*}{2}\right) & \text{if } |\bar{q}_i| > \bar{q}_i^* \end{cases} \tag{19b}$$

with $0 < k_{li} \le (1-\gamma_0)k_{Pi} - k_g$ and $\bar{q}_i^* = [L_{Pi} - 2B_{gi}/(1-\gamma_0)]/k_{Pi}$ (note that by inequality (18) and the definition of β_0: $0 < (1-\gamma_0)k_{Pi} - k_g$ and $\bar{q}_i^* > 0$); this is proven in Appendix B. From this,

[4]Note that, in the error variable space, $q = \bar{q}+q_d$ and consequently $H(q) = H(\bar{q}+q_d)$, $C(q,\dot{q}) = C(\bar{q}+q_d,\dot{q})$ and $g(q) = g(\bar{q}+q_d)$. However, for the sake of simplicity, $H(q)$, $C(q,\dot{q})$, and $g(q)$ are used throughout the paper.

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Property 1 and item 5 of Lemma 1, we have that

$$\begin{aligned} V(\bar{q},\dot{q},\bar{\phi}) &\geq \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| + \gamma_0\sum_{i=1}^{n}\frac{\sigma_{Pi}^2(k_{Pi}\bar{q}_i)}{2k_{Pi}\sigma'_{PiM}} + W_{10}(\bar{q}) + \int_{0_n}^{\bar{\phi}}\bar{s}_I^T(r)dr \\ &\geq W_{11}(\bar{q},\dot{q}) + W_{10}(\bar{q}) + \int_{0_n}^{\bar{\phi}}\bar{s}_I^T(r)dr \end{aligned} \tag{20}$$

with

$$\begin{aligned} W_{11}(\bar{q},\dot{q}) &= \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| + \frac{\gamma_0\beta_P}{2}\|K_P^{-1}s_P(K_P\bar{q})\|^2 \\ &= \frac{1}{2}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}^T\begin{pmatrix}\gamma_0\beta_P & -\varepsilon\mu_M \\ -\varepsilon\mu_M & \mu_m\end{pmatrix}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix} \end{aligned} \tag{21}$$

By inequality (18), $W_{11}(\bar{q},\dot{q})$ is positive definite (since with $\varepsilon < \varepsilon_M \leq \varepsilon_1$, in accordance to inequality (12), any γ_0 satisfying (18) renders the matrix in the right-hand side of (21) positive definite) and observe that $W_{11}(0_n,\dot{q}) \to \infty$ as $\|\dot{q}\| \to \infty$, while from Eqs. (19) and items 6 and 7 of Lemma 1, it is clear that W_{10} and the integral term in the right-hand side of (20) are radially unbounded positive definite functions of $\bar{q}$ and $\bar{\phi}$ respectively. Thus, $V(\bar{q},\dot{q},\bar{\phi})$ is concluded to be positive definite and radially unbounded. Its upper right-hand derivative along the system trajectories, $\dot{V} = D^+V$ (Michel, Hou & Liu, 2008, §6.1A), is given by

$$\begin{aligned} \dot{V}(\bar{q},\dot{q},\bar{\phi}) =\ & \dot{q}^TH(q)\ddot{q} + \frac{1}{2}\dot{q}^T\dot{H}(q,\dot{q})\dot{q} + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}H(q)\ddot{q} + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}\dot{H}(q,\dot{q})\dot{q} \\ & + \varepsilon\dot{q}^Ts'_P(K_P\bar{q})H(q)\dot{q} + g^T(q)\dot{q} - g^T(q_d)\dot{q} + s_P^T(K_P\bar{q})\dot{q} + \bar{s}_I^T(\bar{\phi})\dot{\bar{\phi}} \\ =\ & \dot{q}^T\big[-C(q,\dot{q})\dot{q} - F\dot{q} - g(q) - s_d(\bar{q},\dot{q},\phi) - s_P(K_P\bar{q}) + \bar{s}_I(\bar{\phi}) + g(q_d)\big] + \frac{1}{2}\dot{q}^T\dot{H}(q,\dot{q})\dot{q} \\ & + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}\big[-C(q,\dot{q})\dot{q} - F\dot{q} - g(q) - s_d(\bar{q},\dot{q},\phi) - s_P(K_P\bar{q}) + \bar{s}_I(\bar{\phi}) + g(q_d)\big] \\ & + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}\dot{H}(q,\dot{q})\dot{q} + \varepsilon\dot{q}^Ts'_P(K_P\bar{q})H(q)\dot{q} + g^T(q)\dot{q} - g^T(q_d)\dot{q} + s_P^T(K_P\bar{q})\dot{q} \\ & + \bar{s}_I^T(\bar{\phi})\big[-\dot{q} - \varepsilon K_P^{-1}s_P(K_P\bar{q})\big] \\ =\ & -\dot{q}^TF\dot{q} - \dot{q}^Ts_d(\bar{q},\dot{q},\phi) - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}F\dot{q} - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}\big[g(q) + s_P(K_P\bar{q}) - g(q_d)\big] \\ & - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}s_d(\bar{q},\dot{q},\phi) + \varepsilon\dot{q}^TC(q,\dot{q})K_P^{-1}s_P(K_P\bar{q}) + \varepsilon\dot{q}^Ts'_P(K_P\bar{q})H(q)\dot{q} \end{aligned}$$

where $H(q)\ddot{q}$ and $\dot{\bar{\phi}}$ have been replaced by their equivalent expressions from the closed-loop dynamics in Eqs. (14), Property 2.2 has been used and $s'_P(K_P\bar{q}) \triangleq \mathrm{diag}[D^+\sigma_{P1}(k_{P1}\bar{q}_1),\ldots,D^+\sigma_{Pn}(k_{Pn}\bar{q}_n)]$. The resulting expression can be rewritten as

$$\begin{aligned} \dot{V}(\bar{q},\dot{q},\bar{\phi}) =\ & -\dot{q}^Ts_d(\bar{q},\dot{q},\phi) - \dot{q}^TF\dot{q} - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}F\dot{q} - \varepsilon\gamma_1 s_P^T(K_P\bar{q})K_P^{-1}K_PK_P^{-1}s_P(K_P\bar{q}) \\ & - \varepsilon\mathcal{W}_{\gamma_1}(\bar{q}) - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}s_d(\bar{q},\dot{q},\phi) + \varepsilon\dot{q}^TC(q,\dot{q})K_P^{-1}s_P(K_P\bar{q}) + \varepsilon\dot{q}^Ts'_P(K_P\bar{q})H(q)\dot{q} \end{aligned}$$

where

$$\begin{aligned} \mathcal{W}_{\gamma_1}(\bar{q}) &= s_P^T(K_P\bar{q})K_P^{-1}\big[(1-\gamma_1)s_P(K_P\bar{q}) + g(q) - g(q_d)\big] \\ &= \sum_{i=1}^{n}\left[\frac{(1-\gamma_1)}{k_{Pi}}\sigma_{Pi}^2(k_{Pi}\bar{q}_i) + \frac{\sigma_{Pi}(k_{Pi}\bar{q}_i)}{k_{Pi}}\big[g_i(q) - g_i(q_d)\big]\right] \end{aligned} \tag{22}$$

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and γ_1 is a constant satisfying

$$\beta_0 \frac{\varepsilon}{\varepsilon_2} \left[\frac{\varepsilon_3 - \varepsilon_2}{\varepsilon_3 - \varepsilon} \right] < \gamma_1 < \beta_0 \tag{23}$$

(from inequality (12) and the definition of β_0, one verifies, after simple developments, that $0 < \beta_0 \varepsilon(\varepsilon_3 - \varepsilon_2)/[\varepsilon_2(\varepsilon_3 - \varepsilon)] < \beta_0 < 1$; in particular $\varepsilon\varepsilon_2/\varepsilon_3 < \varepsilon < \varepsilon_2 \iff \varepsilon\varepsilon_2 < \varepsilon\varepsilon_3 < \varepsilon_2\varepsilon_3 \iff 0 < \varepsilon(\varepsilon_3 - \varepsilon_2) < \varepsilon_2(\varepsilon_3 - \varepsilon) \iff 0 < \varepsilon(\varepsilon_3 - \varepsilon_2)/[\varepsilon_2(\varepsilon_3 - \varepsilon)] < 1$). Under this consideration, $\mathcal{W}_{\gamma_1}(\bar{q})$ turns out to be lower-bounded by

$$W_{20}(\bar{q}) = \sum_{i=1}^{n} w_i^{20}(\bar{q}_i) \tag{24a}$$

where

$$w_i^{20}(\bar{q}_i) = \begin{cases} a_i \bar{q}_i^2 & \text{if } |\bar{q}_i| \leq L_{Pi}/k_{Pi} \\ \frac{b_i}{k_{Pi}} \left(|\sigma_{Pi}(k_{Pi}\bar{q}_i)| - L_{Pi} \right) + a_i \left(\frac{L_{Pi}}{k_{Pi}} \right)^2 & \text{if } |\bar{q}_i| > L_{Pi}/k_{Pi} \end{cases} \tag{24b}$$

with $b_i = (1-\gamma_1)L_{Pi} - 2B_{gi}$, $a_i = \min\left\{ d, \frac{b_i k_{Pi}}{L_{Pi}} \right\}$ and $d = (1-\gamma_1)k_{Pm} - k_g$ (notice, from inequality (23) and the definition of β_0, that $b_i > 0$ and $d > 0$, hence $a_i > 0$); this is proven in Appendix C. From this, Properties 1, 2.1 and 3, inequality (10), Definition 1, item 2 of Lemma 1 and the positive definite character of K_P, we have that

$$\begin{aligned} \dot{V}(\bar{q}, \dot{q}, \bar{\phi}) \leq & \; -\dot{q}^T s_d(\bar{q}, \dot{q}, \phi) - f_m \|\dot{q}\|^2 + \varepsilon f_M \|K_P^{-1} s_P(K_P \bar{q})\| \|\dot{q}\| - \varepsilon \gamma_1 k_{Pm} \|K_P^{-1} s_P(K_P \bar{q})\|^2 \\ & + \varepsilon\kappa \|K_P^{-1} s_P(K_P \bar{q})\| \|\dot{q}\| + \varepsilon k_C B_P \|\dot{q}\|^2 + \varepsilon \mu_M \sigma'_{PM} \|\dot{q}\|^2 - \varepsilon \mathcal{W}_{\gamma_1}(\bar{q}) \\ \leq & -\dot{q}^T s_d(\bar{q}, \dot{q}, \phi) - \varepsilon W_{21}(\bar{q}, \dot{q}) - \varepsilon W_{20}(\bar{q}) \end{aligned} \tag{25}$$

where

$$\begin{aligned} W_{21}(\bar{q}, \dot{q}) &= \gamma_1 k_{Pm} \|K_P^{-1} s_P(K_P \bar{q})\|^2 - (f_M + \kappa) \|K_P^{-1} s_P(K_P \bar{q})\| \|\dot{q}\| + \left(\frac{f_m}{\varepsilon} - \beta_M \right) \|\dot{q}\|^2 \\ &= \begin{pmatrix} \|K_P^{-1} s_P(K_P \bar{q})\| \\ \|\dot{q}\| \end{pmatrix}^T Q_{21} \begin{pmatrix} \|K_P^{-1} s_P(K_P \bar{q})\| \\ \|\dot{q}\| \end{pmatrix} \end{aligned}$$

with

$$Q_{21} = \begin{pmatrix} \gamma_1 k_{Pm} & -\frac{f_M + \kappa}{2} \\ -\frac{f_M + \kappa}{2} & \frac{f_m}{\varepsilon} - \beta_M \end{pmatrix} = \begin{pmatrix} \gamma_1 k_{Pm} & -\sqrt{k_{Pm} \beta_M \beta_0 \left(\frac{\varepsilon_3 - \varepsilon_2}{\varepsilon_2} \right)} \\ -\sqrt{k_{Pm} \beta_M \beta_0 \left(\frac{\varepsilon_3 - \varepsilon_2}{\varepsilon_2} \right)} & \beta_M \left(\frac{\varepsilon_3 - \varepsilon}{\varepsilon} \right) \end{pmatrix}$$

By inequality (23), $W_{21}(\bar{q}, \dot{q})$ is positive definite (since with $\varepsilon < \varepsilon_M \leq \varepsilon_2 < \varepsilon_3$, in accordance to inequality (12), any γ_1 satisfying (23) renders Q_{21} positive definite), while from Eqs. (24) and inequality (9), it is clear that W_{20} is a positive definite function of $\bar{q}$ and the first term in the right-hand side of (25) is negative definite with respect to $\dot{q}$ (uniformly in $\bar{q}$ and $\bar{\phi}$). Hence, $\dot{V}(\bar{q}, \dot{q}, \bar{\phi}) \leq 0$ with $\dot{V}(\bar{q}, \dot{q}, \bar{\phi}) = 0 \iff (\bar{q}, \dot{q}) = (0_n, 0_n)$. Further, from the closed-loop dynamics in Eqs. (14), we see that $\bar{q}(t) \equiv \dot{q}(t) \equiv 0_n \implies \ddot{q}(t) \equiv 0_n \implies \bar{s}_I\big(\bar{\phi}(t)\big) \equiv 0_n \iff \bar{\phi}(t) \equiv 0_n$ (at any $(\bar{q}, \dot{q}, \bar{\phi})$ on $Z = \{(x, y, z) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n : x = y = 0_n\}$ with $\bar{\phi} \neq 0_n$, the resulting unbalanced force term $\bar{s}_I(\bar{\phi})$ acts on the closed-loop dynamics forcing the system trajectories to leave Z). Therefore, by the invariance theory (Michel et al., 2008, §7.2) —more precisely by[5] (Michel et al., 2008, Corollary 7.2.1)—, the closed-loop trivial solution $(\bar{q}, \bar{\phi})(t) \equiv (0_n, 0_n)$ is concluded to be globally asymptotically stable, which completes the proof. □

Remark 3: Let us note that the fulfillment of inequality (12) is not necessary but only sufficient for the closed-loop analysis to hold. As a matter of fact, proving Proposition 1 through inequality (12) is tantamount to show the existence of some $\varepsilon^* \geq \varepsilon_M$ such that, for any $\varepsilon \in (0, \varepsilon^*)$, global stabilization is guaranteed. Hence, the proposed scheme permits successful implementations with values of ε higher than ε_M (up to certain limit, ε^*).

Remark 4: Multiple control structures arise from the generalized formulation presented here; details in this direction are given in Appendix A. This is already an important feature of the proposed scheme since it does not only provide a unifying approach that includes particular algorithms from previous references, but it further permits the construction of alternative controllers with innovative saturating structures, thus providing a wide range of possibilities for performance improvement. Beyond such a general character, an important distinction of the proposed approach is on its very simple control-gain tuning criterion. Indeed, observe that the control gains in the approach proposed in this work are not tied to the satisfaction of any additional tuning restriction apart from inequality (6), and condition (12) concerning the integral-action-related parameter ε. This is mainly a result of two original aspects (with respect to previous saturating PID-type approaches). The first of these is on the developed closed-loop analysis through the lower bound finding (and respective proofs) of $\mathcal{U}^c_{\gamma_0}(\bar{q})$ in $V(\bar{q}, \dot{q}, \bar{\phi})$ (Eq. (17)) and $\mathcal{W}_{\gamma_1}(\bar{q})$ in $\dot{V}(\bar{q}, \dot{q}, \bar{\phi})$ (Eq. (22)). The second one is on the design of the algorithm through the structuring of its terms, by involving a *generalized* type of saturation functions whose bound is explicitly considered in their definition and which are permitted to keep an identity relation with their argument in a region around zero, and using them to limit the (P/I/D) control actions, *i.e.* the control-gain-by-scaled closed-loop (error) variables.[6] Such a structuring has been further crucial to state the (strictly) *passive* character of the *desired-gravity-compensation error* $\bar{s}_I(\bar{\phi})$ in Eq. (15) (such that $\bar{\phi}^T \bar{s}_I(\bar{\phi}) > 0$, $\forall \bar{\phi} \neq 0_n$), which permitted the inclusion of the corresponding *controller-induced-potential* energy term in the Lyapunov function, namely the last one in the right-hand side of Eq. (16).

Remark 5: Let us note that the consideration of the viscous friction term in the open-loop dynamics (1) has rendered possible the proof of the global character of the closed-loop stability properties. This has been a common practice since the earliest PID-type bounded approach of (Gorez, 1999). The same practice is observed in the subsequent PID-type bounded scheme proposals of (Meza et al., 2005; Santibáñez et al., 2008) and those reported later on. Other problem formulations in the constrained input framework have been similarly solved (Aguiñaga-Ruiz, Zavala-Río, Santibáñez & Reyes, 2009; Santibáñez & Kelly, 2001; Zavala-Río, Aguiñaga-Ruiz & Santibáñez, 2011). Of course, global controllers —in the referred constrained context— whose stability proof could be carried out in the absence of friction forces would constitute analytically stronger approaches. However, solutions involving viscous friction are meaningful in practice since such a friction component is an ever-present phenomenon in the considered type of systems (Armstrong, 1991).

[5]Corollary 7.2.1 in (Michel et al., 2008) states a version of the global Barbashin-Krasovskii's theorem that considers autonomous systems with continuous dynamics and makes use of continuous scalar (Lyapunov) functions and their upper-right derivative along the system trajectories (in contrast, for instance, with the statement presented in (Khalil, 2002, Corollary 4.2), which is addressed to autonomous state equations with locally Lipschitz-continuous vector fields and makes use of continuously differentiable scalar functions).

[6]Some previous approaches use the control gains to scale the saturation functions (externally) which are in turn used to limit the closed-loop variables either directly or scaled by other coefficients, giving the (external) control gains the additional role of control-action bounds.

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5. Experimental Results

In order to corroborate the efficiency of the proposed scheme, several real-time control tests were implemented on a 3-DOF robot manipulator. The experimental setup, shown in Fig. 1,

is a 3-revolute-joint anthropomorphic-type arm located at the *Benemérita Universidad Autónoma de Puebla*, Mexico. The robot actuators are direct-drive brushless servomotors operated in torque mode, *i.e.* they act as torque sources (without gear reduction) and receive an analog voltage as a torque reference signal. Joint positions are obtained using incremental encoders on the motors and the standard backwards difference algorithm was used to obtain the velocity signals. In order to get the encoder data and generate reference voltages, the robot includes a motion control board based on FPGA's and an electronic interface (more precisely, the MFIO3A model from *Precision MicroDynamic Inc*). The control algorithm is executed at a 2.5 millisecond sampling period on a PC-host computer. A more detailed technical description of this robot is given in (Chávez-Olivares, Reyes, González-Galván, Mendoza & Bonilla, 2012; Reyes & Rosado, 2005).

For the experimental manipulator, Property 4 is satisfied with $B_{g1} = 0$, $B_{g2} = 40.29$ Nm, $B_{g3} = 1.825$ Nm and $k_g = 40.37$ Nm/rad.[7] The maximum allowed torques (input saturation bounds) are $T_1 = 50$ Nm, $T_2 = 150$ Nm and $T_3 = 15$ Nm for the first, second and third links, respectively. From these data, one easily corroborates that Assumption 1 is fulfilled with $\alpha = 3$.

The proposed scheme in Eqs. (4)-(5) was tested in its SP-SI-SD, SPD-SI, SPID-like and SP-SID forms, under the respective consideration of expressions (A1)-(A2), (A3)-(A4), (A5)-(A6) and (A7)-(A8). Letting

$$\sigma_h(\varsigma; M) = M\text{sat}(\varsigma/M)$$

(observe that this is a linear saturation with $L = M$) and

$$\sigma_s(\varsigma; L, M) = \begin{cases} \varsigma & \text{if } |\varsigma| \leq L \\ \text{sign}(\varsigma)L + (M - L)\tanh\left(\frac{\varsigma - \text{sign}(\varsigma)L}{M-L}\right) & \text{if } |\varsigma| > L \end{cases}$$

with $0 < L < M$, the saturation functions used for the implementation were defined as

$$\sigma_{Pi}(\varsigma) = \sigma_h(\varsigma; M_{Pi}) \quad , \quad \sigma_{Di}(\varsigma) = \sigma_h(\varsigma; M_{Di}) \quad , \quad \sigma_{Ii}(\varsigma) = \sigma_s(\varsigma; L_{Ii}, M_{Ii})$$

$i = 1, 2, 3$, in the SP-SI-SD case,

$$\sigma_{Pi}(\varsigma) = \sigma_s(\varsigma; L_{Pi}, M_{Pi}) \quad , \quad \sigma_{Ii}(\varsigma) = \sigma_s(\varsigma; L_{Ii}, M_{Ii})$$

$i = 1, 2, 3$, in the SPD-SI case,

$$\sigma_{0i}(\varsigma) = \sigma_h(\varsigma; M_{0i}) \quad , \quad \sigma_{Pi}(\varsigma) = \sigma_h(\varsigma; M_{Pi}) \quad , \quad \sigma_{Ii}(\varsigma) = \sigma_s(\varsigma; L_{Ii}, M_{Ii})$$

$i = 1, 2, 3$, in the SPID-like case and

$$\sigma_{Pi}(\varsigma) = \sigma_h(\varsigma; M_{Pi}) \quad , \quad \sigma_{Ii}(\varsigma) = \sigma_s(\varsigma; L_{Ii}, M_{Ii})$$

$i = 1, 2, 3$, in the SP-SID case. Let us note that with these saturation functions, we have $\sigma'_{PiM} = \sigma'_{IiM} = \sigma'_{DiM} = \sigma'_{0iM} = 1$, $\forall i \in \{1, 2, 3\}$, and that in consequence, for all the four controllers,

[7] A step-by-step derivation of the 3-DOF anthropomorphic robot arm model is developed in (López-Araujo, 2013, Appendix A). For the particular experimental manipulator considered in this work, the gravity vector with precise values of the involved parameters is presented in (Reyes & Rosado, 2005). Further results on the parameter identification of this robot are presented in (Chávez-Olivares et al., 2012).

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inequality (10) is satisfied with $\kappa = \max_i\{k_{Di}\}$ (see Eqs. (A9)).

For comparison purposes, additional experimental tests were implemented using the bounded PID-type scheme presented in (Su et al., 2010) (choice made taking into account the analog nature of the compared algorithms: globally stabilizing in a bounded-input context, and the recent appearance of (Su et al., 2010)), *i.e.*

$$u = -K_P \mathrm{Tanh}(\bar{q}) - K_I \mathrm{Tanh}(\phi) - K_D \mathrm{Tanh}(\dot{q}) \tag{26a}$$

$$\dot{\phi} = \eta^2 \dot{q} + \eta \mathrm{Tanh}(\bar{q}) \tag{26b}$$

where η is a (sufficiently large) positive constant and $\mathrm{Tanh}(x) = \big(\tanh x_1, \ldots, \tanh x_n\big)^T$ for any $x \in \mathbb{R}^n$.[8] For the sake of simplicity, this algorithm is subsequently referred to as the S10 controller.

At all the experiments, the desired joint positions were fixed at $q_d = \big(q_{d1}, q_{d2}, q_{d3}\big)^T = \big(\pi/4, \pi/4, \pi/2\big)^T$ [rad]. The initial conditions were $q(0) = \dot{q}(0) = 0_3$, and $\phi(0) = 0_3$ was taken for the algorithms obtained through the proposed design methodology, while $\phi(0) = \eta^2\bar{q}(0)$ was taken for the S10 controller in view of the way how it is presented in (Su et al., 2010) (recall Footnote 8).[9]

The control and saturation function parameter values were set so as to achieve pre-specified performance requirements. Three cases were considered respectively referred to as Test 1, 2 and 3. Test 1 consisted in achieving $\|\bar{q}(t_1)\| = 0.3\|\bar{q}(0)\|$, for some $t_1 > 0$, with the same time instant t_1 for all the implemented algorithms (thus reaching 70% of the initial distance —in the standard Euclidean norm sense— to the desired point in the configuration space at the same time instant); this was accomplished with $t_1 = 0.52$ s. Test 2 consisted in getting closed-loop responses with as-small-as-possible overshoot (under the conditions derived from the closed-loop analysis) within a tolerance margin level of 20% of the desired position value at every link. Test 3 consisted in forcing all the controllers to produce an underdamped closed-loop response characterized by the same overshoot level at every link; this was achieved with an overshoot level of $1.2\, q_{di}$, $i = 1, 2, 3$ (*i.e.* 20% of the desired position value at every link). For the algorithms obtained from the proposed design methodology, care was taken to set P/I/D gains and saturation function parameters fulfilling the corresponding conditions arisen from the closed-loop analysis, taking $\varepsilon = 0.001$ s^{-1} for every tested controller.[10] As for the S10 algorithm, care was also taken to adhere to the input-saturation-avoidance inequalities and stability conditions (some of which had to be verified numerically) presented in (Su et al., 2010) (values characterizing Properties 1–3, obtained through the results presented in (Chávez-Olivares et al., 2012), were used to verify the stability conditions). The resulting control and saturation function parameter values are shown in Tables C1–C3, whence one can corroborate that inequalities (6)-(7) are fulfilled by all the controllers obtained through the proposed scheme, as well as the corresponding saturation-avoidance inequalities (A2), (A4), (A6) and (A8), through which (11) is guaranteed.

[8]In place of Eq. (26b), the work in (Su et al., 2010) defines $\phi(t) = \eta^2\bar{q}(t) + \eta\int_0^t \mathrm{Tanh}\big(\delta_P\bar{q}(\varsigma)\big)d\varsigma$, which imposes the auxiliary variable initial condition $\phi(0) = \eta^2\bar{q}(0)$. Instead, Eq. (26b) —or its (equivalent) time representation $\phi(t) = \phi(0) + \int_0^t \big[\eta^2\dot{q}(\varsigma) + \eta\mathrm{Tanh}\big(\delta_P\bar{q}(\varsigma)\big)\big]d\varsigma$— keeps the required auxiliary dynamics while permitting any initial condition for ϕ. This proves to be more appropriate in the global stabilization framework considered in (Su et al., 2010) (and what is generally expected from an approach developed within such a framework).

[9]For the sake of fairness, $\phi(0) = 0_3$ was initially considered for all the algorithms including the S10 controller. Nevertheless, preliminary simulation implementations of the S10 controller with such auxiliary variable initial conditions gave rise to extremely slow closed-loop responses, while with $\phi(0) = \eta^2\bar{q}(0)$ the stabilization times were comparable to those arisen with the algorithms obtained through the proposed design methodology. Thus, in order to avoid controversy, the experimental tests were implemented taking $\phi(0) = \eta^2\bar{q}(0)$ for the S10 controller, while leaving $\phi(0) = 0_3$ for the rest of the tested algorithms.

[10]Based on the results presented in (Chávez-Olivares et al., 2012), values characterizing Properties 1–3 (*i.e.* μ_m, μ_M, k_C, f_m, f_M) were calculated for the considered experimental manipulator. These were used to perform alternative tests under the fulfillment of inequality (12). Since no substantial differences were observed with respect to similar tests carried out using a (sufficiently small) common value of ε, the authors opted for these latter case —namely $\varepsilon = 0.001$ s^{-1} for all the implemented controllers— to perform the previously described Tests 1, 2 and 3, which emphasizes the relevance/coherence of Remark 3.

Figs. 2–4, 5-6 and 7-8 respectively show the experimental results of Tests 1, 2 and 3.

One sees from the graphs that at all the experiments the control objective is achieved avoiding input saturation. In particular, Figure 3 shows the accomplishment of the performance requirement characterizing Test 1: all the tested controllers achieve to reduce (not necessarily each $\bar{q}_i$, $i = 1, 2, 3$, but) the position-error norm to 30% of its initial value at $t_1 = 0.52$ s. Observe further that in Test 2, all the algorithms obtained from the proposed methodology achieved responses avoiding —or with negligible— overshoot at all the links, while this was not possible with the S10 controller —under the saturation-avoidance and stability conditions presented in (Su et al., 2010)— (particularly seen through the response of $\bar{q}_2$). In order to establish comparison criteria, three performance indices were evaluated for every controller at every test: the Integral of the Square of the position Error (ISE), *i.e.* $\int_{t_0}^{t_f}\left[\sum_{i=1}^{3}\bar{q}_i^2(t)\right]dt$ (with t_f the final time of the experiment and t_0 the initial time of the criterion evaluation), the Integral of the Square of the Input torques (ISI), *i.e.* $\int_{t_0}^{t_f}\left[\sum_{i=1}^{3}u_i^2(t)\right]dt$, and the stabilization time, taken as $t_s = \inf\left\{t_e \geq 0 : \|\bar{q}(t)\| \leq 0.02\|q_d\|,\ \forall t \geq t_e\right\}$. Tables C4–C6 show the resulting values of such performance index evaluations.

One sees from the obtained values that the SPD-SI is the controller with the highest number of lowest performance index evaluations (6 of the 11 evaluations, indicated by a check mark), thus resulting the best evaluated algorithm. On the other hand, the S10 controller is the one with the highest number of highest performance index evaluations (8 of the 11 evaluations, indicated by an asterisk), thus resulting the worst evaluated algorithm.

6. Conclusions

In this paper, a generalized PID-type control scheme for the global regulation of robot manipulators with bounded inputs has been proposed. With respect to previous approaches of the kind, the developed scheme achieves the global stabilization objective avoiding input saturation through a very simple tuning criterion, the simplest hitherto obtained in the considered analytical framework. Moreover, through its generalized form, the proposed scheme gives rise to multiple particular PID-type controllers with diverse saturating structures, permitting further innovation in their construction. This yields a wide range of possibilities for performance improvement, rendering the proposed design methodology attractive for practical applications. The efficiency of the proposed scheme was verified through actual implementations on a 3-DOF direct-drive manipulator. The experimental results showed smooth closed-loop responses with brief transient and minimized steady-state errors obtained through control signals that remain within the input ranges, thus avoiding saturation. The ability of the proposed scheme to achieve the control objective in spite of system parameter inaccuracies and/or model imprecisions was thus corroborated.

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Appendix A.

On the basis of bounded algorithms from previous references, several particular control structures arise through the proposed generalized scheme. For instance, let $K_D \in \mathbb{R}^n$ be a positive definite diagonal matrix. An *SP-SI-SD* algorithm (Meza et al., 2005; Santibáñez & Kelly, 1996) is obtained by defining

$$s_d(\bar{q}, \dot{q}, \phi) = s_D(K_D\dot{q}) \tag{A1}$$

giving rise to a control law of the form

$$u(q, \dot{q}, \phi) = -s_P(K_P\bar{q}) - s_D(K_D\dot{q}) + s_I(K_I\phi)$$

where, for any $x \in \mathbb{R}^n$, $s_D(x) = \big(\sigma_{D1}(x_1), \ldots, \sigma_{Dn}(x_n)\big)^T$, with $\sigma_{Di}(\cdot)$, $i = 1, \ldots, n$, being *generalized saturation functions* with bounds M_{Di}, and the involved bound values, M_{Pi}, M_{Di} and M_{Ii}, satisfying

$$M_{Pi} + M_{Di} + M_{Ii} < T_i \tag{A2}$$

An *SPD-SI* scheme (López-Araujo et al., 2013a; Santibáñez et al., 2008; Zavala-Río & Santibáñez, 2006) is obtained by defining

$$s_d(\bar{q}, \dot{q}, \phi) = s_P(K_P\bar{q} + K_D\dot{q}) - s_P(K_P\bar{q}) \tag{A3}$$

resulting in a controller of the form

$$u(q, \dot{q}, \phi) = -s_P(K_P\bar{q} + K_D\dot{q}) + s_I(K_I\phi)$$

with the linear saturations $\sigma_{Pi}(\cdot)$ being *strictly increasing* and bound values fulfilling

$$M_{Pi} + M_{Ii} < T_i \tag{A4}$$

An *SPID-like* scheme (López-Araujo et al., 2013a; Yarza et al., 2011; Zavala-Río & Santibáñez, 2006) is obtained by defining

$$s_d(\bar{q}, \dot{q}, \phi) = s_0\big(s_I(K_I\phi) - s_P(K_P\bar{q})\big) - s_0\big(s_I(K_I\phi) - s_P(K_P\bar{q}) - K_D\dot{q}\big) \tag{A5}$$

where, for any $x \in \mathbb{R}^n$, $s_0(x) = \big(\sigma_{01}(x_1), \ldots, \sigma_{0n}(x_n)\big)^T$, with $\sigma_{0i}(\cdot)$, $i = 1, \ldots, n$, being *linear saturation functions* for (L_{0i}, M_{0i}), and the involved linear/generalized saturation function parameters satisfying

$$M_{Pi} + M_{Ii} < L_{0i} \le M_{0i} < T_i \tag{A6}$$

whence, by virtue of item (c) of Definition 1, we have that $s_0\big(s_I(K_I\bar{\phi}) - s_P(K_P\bar{\phi})\big) \equiv s_I(K_I\bar{\phi}) - s_P(K_P\bar{\phi})$, giving rise to a control law of the form

$$u(q, \dot{q}, \phi) = s_0\big(- s_P(K_P\bar{q}) - K_D\dot{q} + s_I(K_I\phi)\big)$$

Furthermore, the general character of the proposed scheme permits the generation of control laws with innovative saturating structure. For instance, an *SP-SID* controller can be obtained by defining

$$s_d(\bar{q}, \dot{q}, \phi) = s_I(K_I\phi) - s_I(K_I\phi - K_D\dot{q}) \tag{A7}$$

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resulting in a controller of the form

$$u(q, \dot{q}, \phi) = -s_P(K_P\bar{q}) + s_I(-K_D\dot{q} + K_I\phi)$$

with bound values fulfilling

$$M_{Pi} + M_{Ii} < T_i \tag{A8}$$

One can verify that in all the above cases the expressions in (8)-(11) are satisfied. In particular, the input-saturation-avoidance requirement stated through (11) is accomplished through the fulfillment of inequalities (A2), (A4), (A6) and (A8). Furthermore, from items 3 and 4 of Lemma 1, one sees that $s_d(\bar{q}, \dot{q}, \phi)$ in every one of the above cases in (A1), (A3), (A5) and (A7) satisfies inequality (10) with

$$\kappa = \max_i\{\sigma'_{iM}k_{Di}\} \tag{A9a}$$

where

$$\sigma'_{iM} = \begin{cases} \sigma'_{DiM} & \text{in the SP-SI-SD case} \\ \sigma'_{PiM} & \text{in the SPD-SI case} \\ \sigma'_{0iM} & \text{in the SPID-like case} \\ \sigma'_{IiM} & \text{in the SP-SID case} \end{cases} \tag{A9b}$$

σ'_{DiM}, σ'_{PiM}, σ'_{0iM} and σ'_{IiM} respectively being the positive bounds of $D^+\sigma_{Di}(\cdot)$, $D^+\sigma_{Pi}(\cdot)$, $D^+\sigma_{0i}(\cdot)$ and $D^+\sigma_{Ii}(\cdot)$, in accordance to item 2 of Lemma 1.

Remark 6: Observe that the *input-saturation-avoidance* conditions for the particular control structures presented in this appendix, *i.e.* inequalities (A2), (A4), (A6) and (A8), imply (all of them) that $M_{Pi}+M_{Ii} < T_i$, while the satisfaction of inequalities (7) implies that $M_{Pi}+M_{Ii} > 3B_{gi}$. Hence, for the specific choices of s_d presented in equations (A1), (A3), (A5) and (A7), the feasibility of the simultaneous fulfillment of inequalities (7) and the corresponding input-saturation-avoidance condition —(A2), (A4), (A6) or (A8), respectively— is ensured by requiring the satisfaction of Assumption 1 with $\alpha = 3$. Other particular choices of s_d in the generalized scheme (4) could require different values of $\alpha \geq 1$.

Appendix B.

Let $\bar{q}_i^* \triangleq [L_{Pi} - 2B_{gi}/(1-\gamma_0)]/k_{Pi}$,

$$\varrho_i(\bar{q}_i) \triangleq \bar{g}_i(\bar{q}_i) - g_i(q_d) + (1-\gamma_0)\sigma_{Pi}(k_{Pi}\bar{q}_i) \quad , \quad \rho_i(\bar{q}_i) \triangleq \bar{g}_i(\bar{q}_i) - g_i(q_d) + (1-\gamma_0)k_{Pi}\bar{q}_i$$

$\mathcal{S}_i^* \triangleq \{\bar{q}_i \in \mathbb{R} : |\bar{q}_i| \leq \bar{q}_i^*\}$, $\mathcal{S}_i \triangleq \{\bar{q}_i \in \mathbb{R} : |\bar{q}_i| \leq L_{Pi}/k_{Pi}\}$ and

$$w_i^{10}(\bar{q}_i) \triangleq \begin{cases} \frac{k_{li}}{2}\bar{q}_i^2 & \forall|\bar{q}_i| \leq \bar{q}_i^* \\ k_{li}\bar{q}_i^*\left(|\bar{q}_i| - \frac{\bar{q}_i^*}{2}\right) & \forall|\bar{q}_i| > \bar{q}_i^* \end{cases}$$

where k_{li} and γ_0 are constants satisfying $0 < k_{li} \leq (1-\gamma_0)k_{Pi} - k_g$ and $0 < \gamma_0 < \beta_0 < 1$ (which is fulfilled under the satisfaction of (18)), respectively.

Claim 1: *$\rho_i(\bar{q}_i)$ is a strictly increasing function satisfying $|\rho_i(\bar{q}_i)| \geq k_{li}|\bar{q}_i|$, $\forall \bar{q}_i \in \mathbb{R}$.*

Proof. Observe that $\rho_i(0) = 0 = [k_{li}\bar{q}_i]_{\bar{q}_i=0}$ and notice from Property 4.2 that $\frac{d\rho_i}{d\bar{q}_i}(\bar{q}_i) = (1-\gamma_0)k_{Pi} + \frac{d\bar{g}_i}{d\bar{q}_i}(\bar{q}_i) \geq (1-\gamma_0)k_{Pi} - k_g \geq k_{li} = \frac{d}{d\bar{q}_i}[k_{li}\bar{q}_i] > 0$, $\forall \bar{q}_i \in \mathbb{R}$, which proves the claim. □

Claim 2: $|\varrho_i(\bar{q}_i)| \geq |k_{li}\bar{q}_i^* \text{sat}(\bar{q}_i/\bar{q}_i^*)|$, $\forall \bar{q}_i \in \mathbb{R}$.

Proof. First, note from item (a) of Definition 1 that $\sigma_{Pi}(k_{Pi}\bar{q}_i) = k_{Pi}\bar{q}_i$, $\forall \bar{q}_i \in \mathcal{S}_i$. Furthermore, observe that $\mathcal{S}_i^* \subset \mathcal{S}_i$. Consequently, $|\varrho_i(\bar{q}_i)| = |\rho_i(\bar{q}_i)| \geq k_{li}|\bar{q}_i| = |k_{li}\bar{q}_i^* \text{sat}(\bar{q}_i/\bar{q}_i^*)|$, $\forall \bar{q}_i \in \mathcal{S}_i^*$ (where Claim 1 has been considered). On the other hand, from the strictly increasing character of $\rho_i(\bar{q}_i)$ (according to Claim 1), notice that $|\varrho_i(\bar{q}_i)| = |\rho_i(\bar{q}_i)| \geq k_{li}\bar{q}_i^* = |k_{li}\bar{q}_i^* \text{sat}(\bar{q}_i/\bar{q}_i^*)|$, $\forall \bar{q}_i \in \mathcal{S}_i \setminus \mathcal{S}_i^*$. Finally, observe from the definition of $\varrho_i(\bar{q}_i)$ that on $\mathbb{R} \setminus \mathcal{S}_i$, we have that $|\varrho_i(\bar{q}_i)| \geq (1-\gamma_0)L_{Pi} - 2B_{gi} = (1-\gamma_0)k_{Pi}\bar{q}_i^* > [(1-\gamma_0)k_{Pi} - k_g]\bar{q}_i^* \geq k_{li}\bar{q}_i^* = |k_{li}\bar{q}_i^* \text{sat}(\bar{q}_i/\bar{q}_i^*)|$ $\forall \bar{q}_i \in \mathbb{R} \setminus \mathcal{S}_i$. □

Claim 3: $\int_0^{\bar{q}_i} \varrho_i(r_i)dr_i \geq w_i^{10}(\bar{q}_i)$, $\forall \bar{q}_i \in \mathbb{R}$.

Proof. The proof follows directly from Claim 2 by noting that $\varrho_i(0) = 0 = [k_{li}\bar{q}_i^* \text{sat}(\bar{q}_i/\bar{q}_i^*)]_{\bar{q}_i=0}$, $\bar{q}_i\varrho_i(\bar{q}_i) \geq \bar{q}_i[k_{li}\bar{q}_i^* \text{sat}(\bar{q}_i/\bar{q}_i^*)] > 0$, $\forall \bar{q}_i \neq 0$, and $w_i^{10}(\bar{q}_i) = \int_0^{\bar{q}_i} k_{li}\bar{q}_i^* \text{sat}(r_i/\bar{q}_i^*)dr_i$. □

Appendix C.

For any $\bar{q} \in \mathbb{R}^n$, let $\ell \in \{0, \ldots, n\}$ be the number of elements of $\bar{q}$ that satisfy $|\bar{q}_i| \leq L_{Pi}/k_{Pi}$. Without loss of generality, suppose that the first ℓ elements of $\bar{q}$ are those fulfilling such inequality, and let $\bar{q}^\ell = (\bar{q}_1, \ldots, \bar{q}_\ell)^T$, $g^\ell(q) = \big(g_1(q), \ldots, g_\ell(q)\big)^T$ and $K_P^\ell = \text{diag}[k_{P1}, \ldots, k_{P\ell}]$. Under such considerations, $\mathcal{W}_{\gamma_1}(\bar{q})$ can be written as

$$\mathcal{W}_{\gamma_1}(\bar{q}) = \mathcal{W}_{\gamma_1}^s(\bar{q}) + \mathcal{W}_{\gamma_1}^g(\bar{q})$$

where

$$\mathcal{W}_{\gamma_1}^s(\bar{q}) = \sum_{i=1}^{\ell}(1-\gamma_1)k_{Pi}\bar{q}_i^2 + \sum_{i=\ell+1}^{n}\frac{(1-\gamma_1)}{k_{Pi}}\sigma_{Pi}^2(k_{Pi}\bar{q}_i)$$

$$\mathcal{W}_{\gamma_1}^g(\bar{q}) = \sum_{i=1}^{\ell}\bar{q}_i\big[g_i(\bar{q}+q_d) - g_i(q_d)\big] + \sum_{i=\ell+1}^{n}\frac{\sigma_{Pi}(k_{Pi}\bar{q}_i)}{k_{Pi}}\big[g_i(\bar{q}+q_d) - g_i(q_d)\big]$$

with $\sum_{i=k_1}^{k_2}(\cdot)_i = 0$ when $k_2 < k_1$. Note that $\min_{i=1,\ldots,\ell}\{k_{Pi}\} \geq \min_{i=1,\ldots,n}\{k_{Pi}\} = k_{Pm}$ while $|\sigma_{Pi}(k_{Pi}\bar{q}_i)| \geq L_{Pi}$, $\forall |\bar{q}_i| \geq L_{Pi}/k_{Pi}$, and consequently

$$\mathcal{W}_{\gamma_1}^s(\bar{q}) \geq \sum_{i=1}^{\ell}(1-\gamma_1)k_{Pm}\bar{q}_i^2 + \sum_{i=\ell+1}^{n}\frac{(1-\gamma_1)L_{Pi}}{k_{Pi}}|\sigma_{Pi}(k_{Pi}\bar{q}_i)|$$

On the other hand, from Property 4 we have that, on every (position error) coordinate axis (recall that on axis i, $\bar{q}_j = 0$, $\forall j \neq i$), $\bar{q}_i\big[g_i(\bar{q}+q_d) - g_i(q_d)\big] \leq k_g\bar{q}_i^2$ while $\sigma_{Pi}(k_{Pi}\bar{q}_i)\big[g_i(\bar{q}+q_d) - g_i(q_d)\big] \leq 2B_{g_i}|\sigma_{Pi}(k_{Pi}\bar{q}_i)|$, and consequently

$$\mathcal{W}_{\gamma_1}^g(\bar{q}) \geq -\sum_{i=1}^{\ell}k_g\bar{q}_i^2 - \sum_{i=\ell+1}^{n}\frac{2B_{gi}}{k_{Pi}}|\sigma_{Pi}(k_{Pi}\bar{q}_i)|$$

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From the above expressions, one sees that

$$\mathcal{W}_{\gamma_1}(\bar{q}) \geq \sum_{i=1}^{\ell} \big[(1-\gamma_1)k_{Pm} - k_g\big]\bar{q}_i^2 + \sum_{i=\ell+1}^{n} \frac{\big[(1-\gamma_1)L_{Pi} - 2B_{gi}\big]}{k_{Pi}} |\sigma_{Pi}(k_{Pi}\bar{q}_i)|$$

or more generally (for any ordering on the elements of $\bar{q}$)

$$\mathcal{W}_{\gamma_1}(\bar{q}) \geq \sum_{i=1}^{n} w_i^{\gamma_1}(\bar{q}_i)$$

where

$$w_i^{\gamma_1}(\bar{q}_i) = \begin{cases} \big[(1-\gamma_1)k_{Pm} - k_g\big]\bar{q}_i^2 & \text{if } |\bar{q}_i| \leq L_{Pi}/k_{Pi} \\ \frac{\big[(1-\gamma_1)L_{Pi} - 2B_{gi}\big]}{k_{Pi}} |\sigma_{Pi}(k_{Pi}\bar{q}_i)| & \text{if } |\bar{q}_i| > L_{Pi}/k_{Pi} \end{cases}$$

Finally, by defining $b_i = (1-\gamma_1)L_{Pi} - 2B_{gi}$, $d = (1-\gamma_1)k_{Pm} - k_g$, $a_i = \min\left\{d, \frac{b_i k_{Pi}}{L_{Pi}}\right\}$ and $c_i = \max\left\{\frac{b_i L_{Pi}}{k_{Pi}} - d\left(\frac{L_{Pi}}{k_{Pi}}\right)^2, 0\right\} \geq 0$, it is clear that $d \geq a_i$ and $\frac{b_i}{k_{Pi}}|\sigma_{Pi}(k_{Pi}\bar{q}_i)| \geq \frac{b_i}{k_{Pi}}|\sigma_{Pi}(k_{Pi}\bar{q}_i)| - c_i = \frac{b_i}{k_{Pi}}\big(|\sigma_{Pi}(k_{Pi}\bar{q}_i)| - L_{Pi}\big) + a_i\left(\frac{L_{Pi}}{k_{Pi}}\right)^2$, and consequently

$$\mathcal{W}_{\gamma_1}(\bar{q}) \geq \sum_{i=1}^{n} w_i^{20}(\bar{q}_i)$$

with

$$w_i^{20}(\bar{q}_i) = \begin{cases} a_i\bar{q}_i^2 & \text{if } |\bar{q}_i| \leq L_{Pi}/k_{Pi} \\ \frac{b_i}{k_{Pi}}\big(|\sigma_{Pi}(k_{Pi}\bar{q}_i)| - L_{Pi}\big) + a_i\left(\frac{L_{Pi}}{k_{Pi}}\right)^2 & \text{if } |\bar{q}_i| > L_{Pi}/k_{Pi} \end{cases}$$

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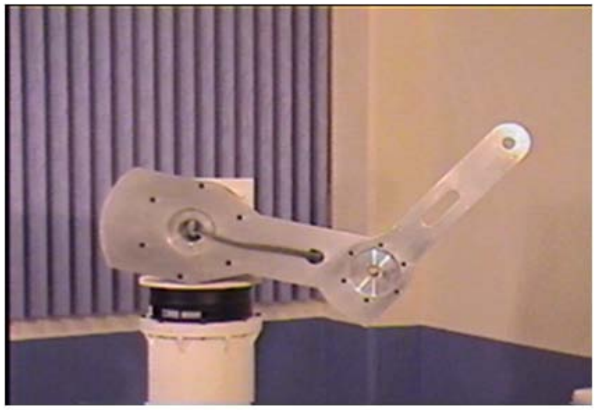

Figure 1. Experimental setup: 3-DOF direct-drive robot manipulator

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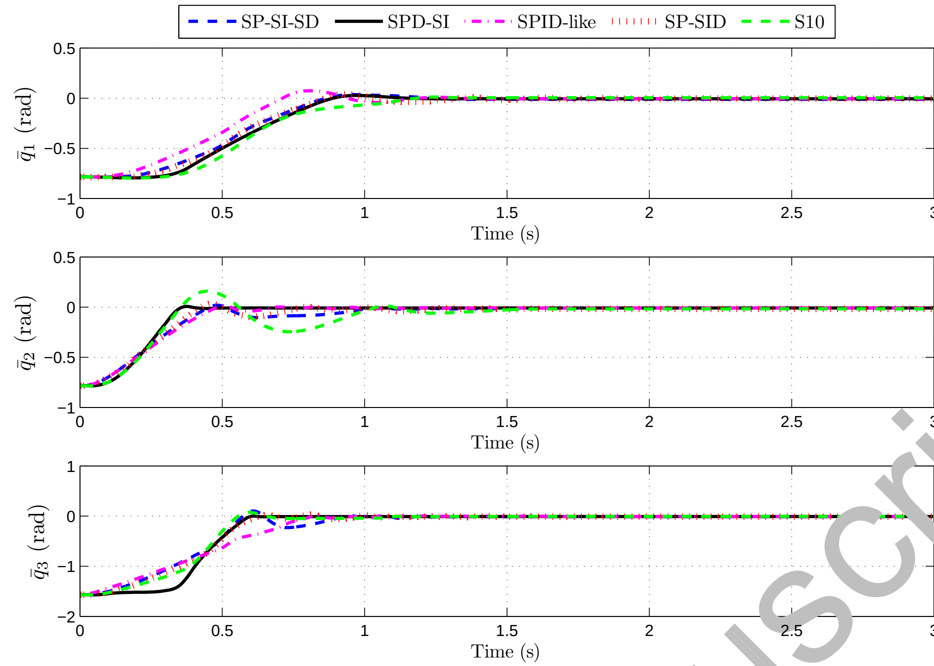

Figure 2. Test 1: position errors

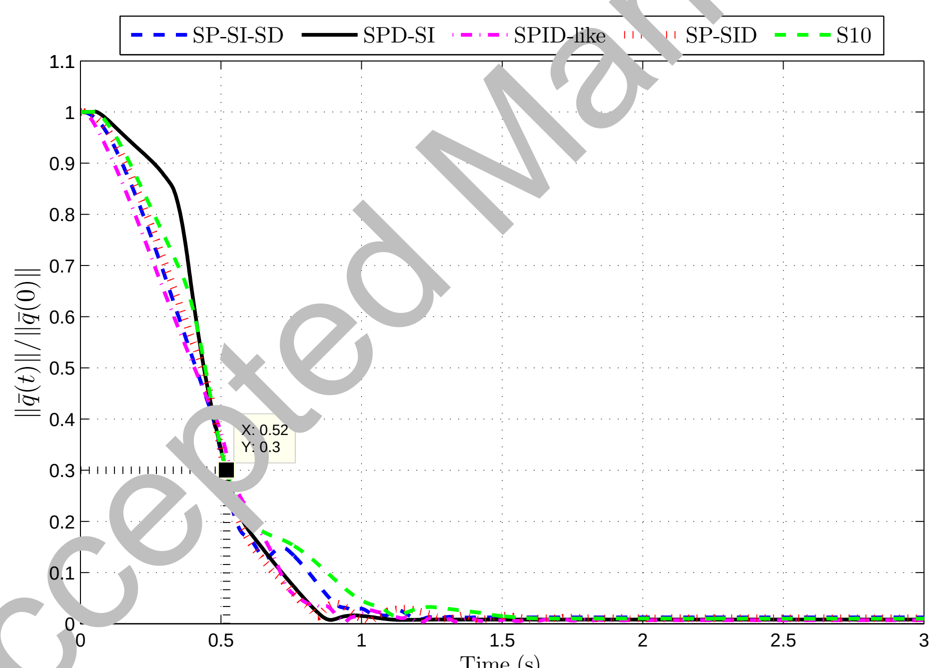

Figure 3. Test 1: position-error norm ratio $\|\bar{q}(t)\|/\|\bar{q}(0)\|$

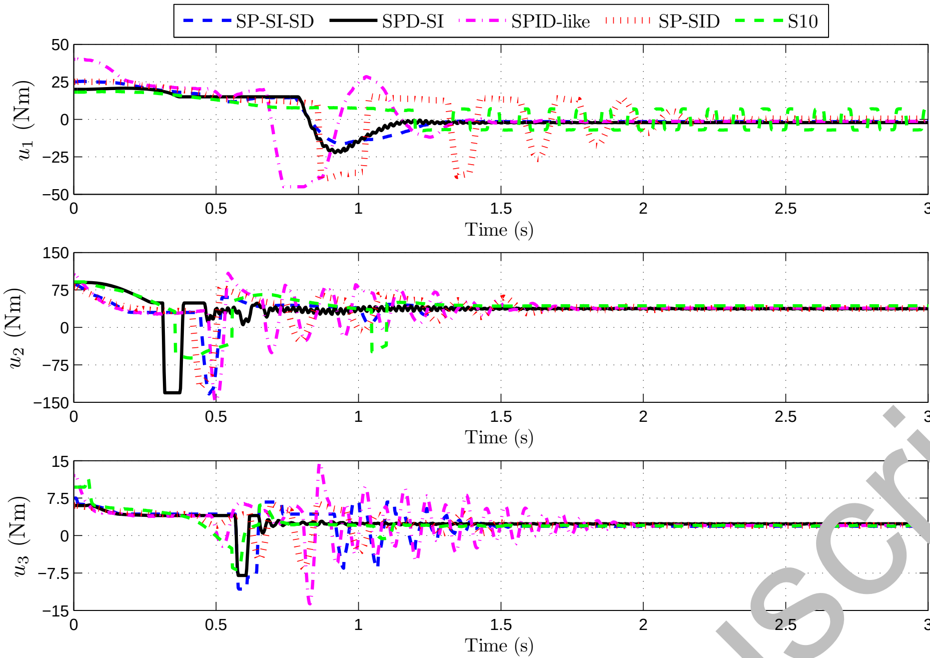

Figure 4. Test 1: control signals

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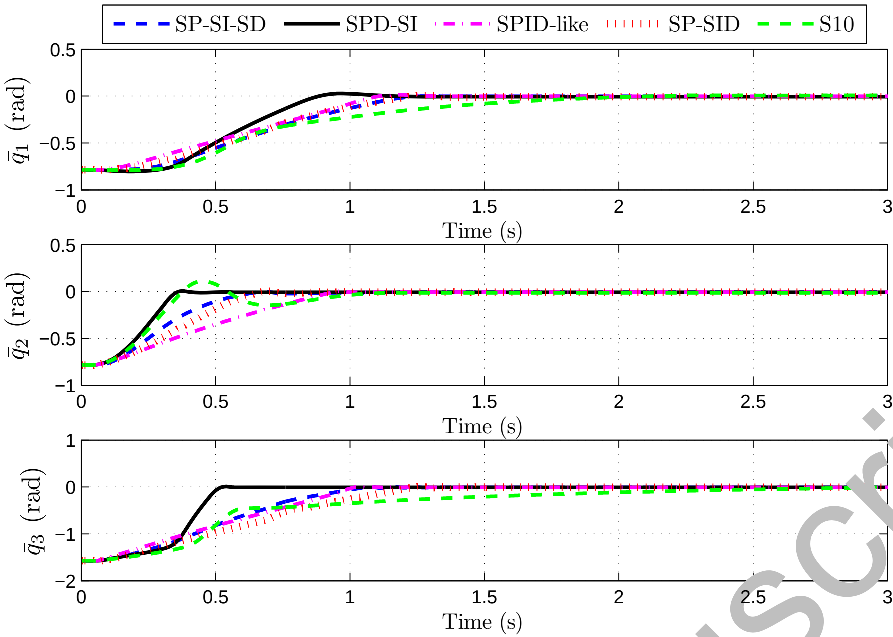

Figure 5. Test 2: position errors

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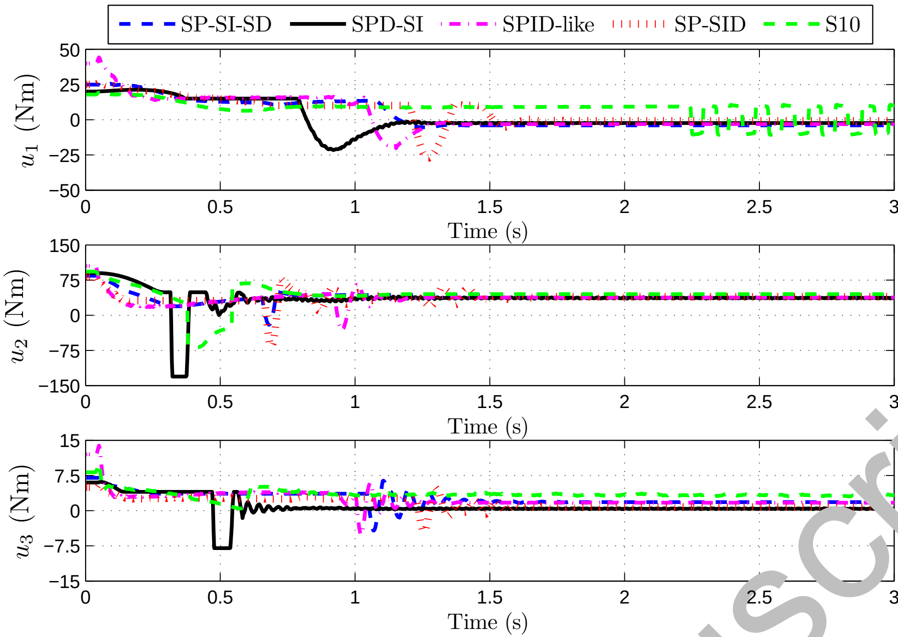

Figure 6. Test 2: control signals

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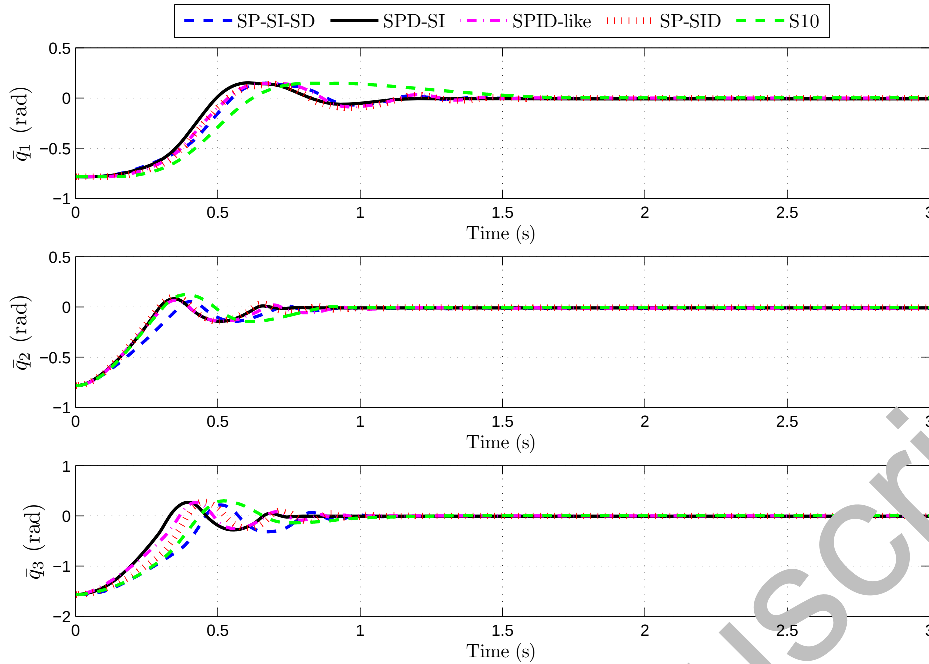

Figure 7. Test 3: position errors

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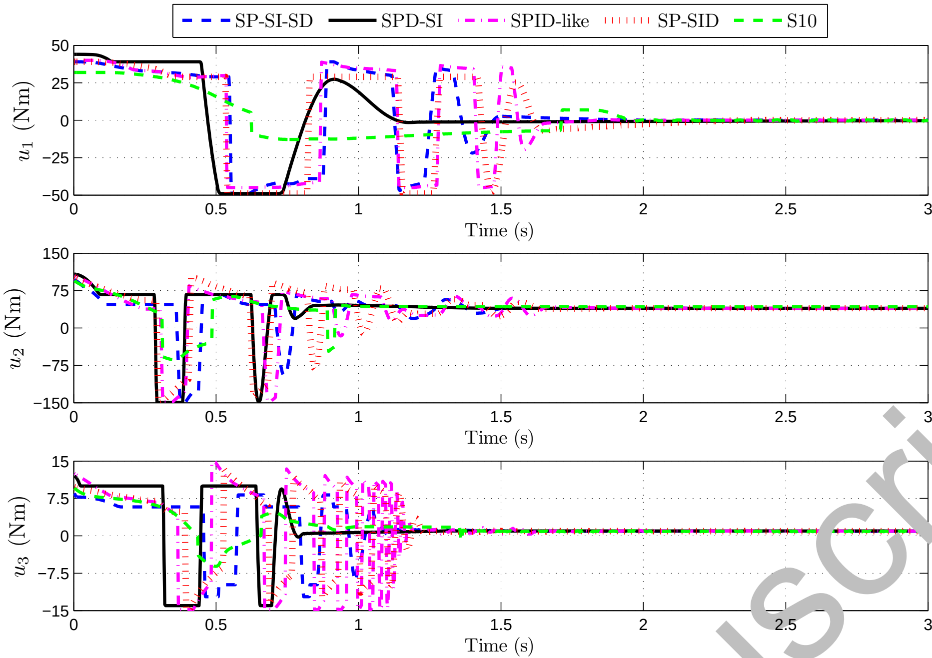

Figure 8. Test 3: control signals

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Table 1. Control and saturation function parameter values for Test 1

parameter	SP-SI-SD	SPD-SI	SPID-like	SP-SID	S10	units
K_P	$\mathrm{diag}\begin{bmatrix}300\\5000\\300\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}450\\8500\\410\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}750\\7500\\490\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}4000\\3500\\220\end{bmatrix}$	$\mathrm{diag}[17, 75, 8.5]$	Nm/rad Nm
K_I	$\mathrm{diag}\begin{bmatrix}10\\150\\12\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}75\\80\\37\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}100\\40\\18\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}15\\30\\0.3\end{bmatrix}$	$\mathrm{diag}[7, 42, 1.9]$	Nm/rad Nm
K_D	$\mathrm{diag}\begin{bmatrix}5\\12\\11\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}20\\250\\9\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}11\\30\\3\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}3\\14\\0.6\end{bmatrix}$	$\mathrm{diag}[3, 10.5, 4.5]$	Nms/rad Nm
ε η	0.001	0.001	0.001	0.001	 100	s^{-1} s/rad
M_{P1}	25	20	40	25		Nm
M_{P2}	86	90	105	81		Nm
M_{P3}	7.5	6	12	6		Nm
$L_{P1/2/3}$		$0.9M_{P1/2/3}$				Nm
M_{I1}	5	5	4.8	15		Nm
M_{I2}	41	41	41	50		Nm
M_{I3}	2	2	2	5		Nm
$L_{I1/2/3}$	$0.9M_{I1/2/3}$	$0.9M_{I1/2/3}$	$0.9M_{I1/2/3}$	$0.9M_{I1/2/3}$		Nm
M_{D1} M_{01}	10		 45			Nm
M_{D2} M_{02}	15		 148			Nm
M_{D3} M_{03}	1.2		 14.5			Nm

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Table 2. Control and saturation function parameter values for Test 2

parameter	SP-SI-SD	SPD-SI	SPID-like	SP-SID	S10	units
K_P	$\mathrm{diag}\begin{bmatrix}320\\9900\\420\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}450\\8500\\300\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}750\\6500\\445\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}2000\\9000\\350\end{bmatrix}$	$\mathrm{diag}[12, 75, 5]$	Nm/rad Nm
K_I	$\mathrm{diag}\begin{bmatrix}10\\100\\10\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}75\\80\\30\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}100\\5\\25\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}20\\35\\0.5\end{bmatrix}$	$\mathrm{diag}[10, 44, 3.6]$	Nm/rad Nm
K_D	$\mathrm{diag}\begin{bmatrix}10\\15\\2\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}20\\250\\10\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}25\\80\\4\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}6\\40\\1.5\end{bmatrix}$	$\mathrm{diag}[10, 20, 5.5]$	Nms/rad Nm
ε η	0.001	0.001	0.001	0.001	 100	s^{-1} s/rad
M_{P1}	25	20	40	25		Nm
M_{P2}	84	90	105	81		Nm
M_{P3}	7	6	12	5		Nm
$L_{P1/2/3}$		$0.9M_{P1/2/3}$				Nm
M_{I1}	5	5	4.8	15		Nm
M_{I2}	41	41	41	50		Nm
M_{I3}	2	2	2	2.5		Nm
$L_{I1/2/3}$	$0.9M_{I1/2/3}$	$0.9M_{I1/2/3}$	$0.9M_{I1/2/3}$	$0.9M_{I1/2/3}$		Nm
M_{D1} M_{01}	10		 45			Nm
M_{D2} M_{02}	24		 148			Nm
M_{D3} M_{03}	1.4		 14.5			Nm

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Table 3. Control and saturation function parameter values for Test 3

parameter	SP-SI-SD	SPD-SI	SPID-like	SP-SID	S10	units
K_P	$\mathrm{diag}\begin{bmatrix}4000\\5500\\300\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}500\\8500\\410\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}9800\\7500\\9950\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}5000\\8500\\1200\end{bmatrix}$	$\mathrm{diag}[38, 82, 8.5]$	Nm/rad Nm
K_I	$\mathrm{diag}\begin{bmatrix}10\\300\\10\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}300\\360\\200\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}120\\70\\7\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}25\\30\\0.3\end{bmatrix}$	$\mathrm{diag}[7, 42, 1.9]$	Nm/rad Nm
K_D	$\mathrm{diag}\begin{bmatrix}5\\12\\11\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}10\\75\\5.5\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}2\\8\\0.7\end{bmatrix}$	$\mathrm{diag}\begin{bmatrix}2\\8\\0.6\end{bmatrix}$	$\mathrm{diag}[2, 15, 2]$	Nms/rad Nm
ε η	0.001	0.001	0.001	0.001	 100	s^{-1} s/rad
M_{P1}	39	44	40	39		Nm
M_{P2}	98	108	105	102		Nm
M_{P3}	9	12	12.6	9.8		Nm
$L_{P1/2/3}$		$0.9M_{P1/2/3}$				Nm
M_{I1}	5	5	4.8	10		Nm
M_{I2}	41	41	41	47		Nm
M_{I3}	2	2	2	5		Nm
$L_{I1/2/3}$	$0.9M_{I1/2/3}$	$0.9M_{I1/2/3}$	$0.9M_{I1/2/3}$	$0.9M_{I1/2/3}$		Nm
M_{D1} M_{01}	5		 45			Nm
M_{D2} M_{02}	10		 148			Nm
M_{D3} M_{03}	1.2		 14.8			Nm

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Table 4. Performance index evaluations for Test 1

perf. index		SP-SI-SD	SPD-SI	SPID-like	SP-SID	S10
ISE	$t_0 = 0$	1.1083	1.3615	1.0517	1.1469	1.2484
	$t_0 = t_1$	0.0302	0.0268	0.0344	0.0230	0.0472 *
ISI	$t_0 = 0$	5872.28	6749.69	7472.93	6876.81	7648.17 *
	$t_0 = t_1$	4072.12	3528.35 ✓	5455.78	4940.91	5286.60
t_s		1.1550	0.8475 ✓	1.0776	1.2753	1.4833 *

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Table 5. Performance index evaluations for Test 2

perf. index	SP-SI-SD	SPD-SI	SPID-like	SP-SID	S10
ISE	1.4329	1.2332 ✓	1.3948	1.5628	1.6204 *
ISI	5934.73	6542.42	4946.21	5221.65	7642.72 *
t_s	1.1500	0.8425 ✓	1.0550	1.2075	2.6925 *

Table 6. Performance index evaluations for Test 3

perf. index	SP-SI-SD	SPD-SI	SPID-like	SP-SID	S10
ISE	0.9367	0.7266 ✓	0.7465	0.8490	0.9382 *
ISI	9288.60	10936.62	11592.94	12852.44	7100.69
t_s	1.0875	1.0525 ✓	1.0975	1.1025	1.4450 *

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