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Chaotic Attractors Based on Unstable Dissipative Systems via Third-Order Differential Equation

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In this work, we present an approach how to yield 1D, 2D and 3D-grid multi-scroll chaotic systems in R^3 based on *unstable dissipative systems* via third-order differential equation. This class of systems is constructed by a switching control law changing the equilibrium point of an unstable dissipative system. The switching control law that governs the position of the equilibrium point varies according to the number of scrolls displayed in the attractor.

1. Introduction

Chaos has been an extremely studied area in the last decades, and designing systems with chaotic behavior is of great interest for the scientific community. One of the most remarkable properties is that simpler nonlinear deterministic equations can have unpredictable (chaotic) long-term solution.

The characterization [1], electronic implementation [2], and design of new switched systems with chaotic behavior [3], especially possessing multiple scrolls [4] or wings [5,6], has been of great interest for the scientific community. The methods implemented to generate multi-scroll systems in the literature may be catalogued in two [3-33]: i) systems presenting more equilibrium points than wings or scrolls, ii) systems presenting the same number of equilibrium points and wings or scrolls. This paper is devoted to the second kind of systems. In this work, we present a generalized theory which is capable of explaining different approaches as saturation, threshold and step functions in R^3. This class of systems is constructed with *unstable dissipative systems* (UDS) [7,8] and a control law to display various multi-scroll strange attractors. The multi-scroll strange attractors result from the combination of several unstable "one-spiral" trajectories by means of a switching given by the control law. Without loss of generality we focus our study to the simple jerk equation and a switching control law to generate PWL systems that produce multiscroll attractors.

This paper is organized as follow: In Section 2, we introduce a theory to explain the generation of 1D-grid multi-scroll via UDS system, along with some examples using the jerky equation. In Section 3 we exemplify the theory based on UDS to generate 2D and 3D-grid multi-scroll attractors, and present numerical results. Finally in Section 4 we draw conclusions.

2. Generation of Multi-scroll Attractors by UDS

We consider the class of affine linear system given by

$$\dot{\chi} = \mathbf{A}\chi + \mathbf{B}, \tag{1}$$

where $\chi = [x_1, \ldots, x_n]^T \in R^n$ is the state variable, $\mathbf{B} = [b_1, \ldots, b_n]^T \in R^n$ stands for a real vector, $\mathbf{A} = [\alpha_{ij}] \in R^{n\times n}$ denotes a linear operator and the equilibrium point is located at $\chi^* = -\mathbf{A}^{-1}\mathbf{B}$. The dynamics of the system is given by matrix $\mathbf{A}$ which has a stable manifold E^s and another unstable E^u. According to the above discussion it is possible to define an unstable dissipative system **UDS**, a similar definition is given in [9], as follows:

Definition 1. A system given by (1) in R^n and eigenvalues λ_i, with $i = 1, \ldots, n$. We said that system (1) is a UDS if $\sum_{i=1}^{n} \lambda_i < 0$, and at least two λ_i are complex conjugate eigenvalues with positive real part $Re\{\lambda_i\} > 0$.

Different kinds of behaviors are possible to find in a system given by (1) which satisfies the definition 1 with ordered eigenvalues set $\Lambda = \{\lambda_1 \ldots \lambda_n\}$ and $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$. Due to the system is hyperbolic so it has a stable manifold $E^s = span\{\lambda_1, \ldots \lambda_j\} \subset R^n$ and another unstable $E^u = span\{\lambda_{j+1}, \ldots, \lambda_n\} \subset R^n$ with $1 \leq j \leq n$ and the following statements are true: (a) All initial condition $\chi_0 \in R^n/E^s$ leads to an unstable orbit that goes to infinity. (b) All initial condition $\chi_0 \in E^s$ leads to a stable orbit that settles down at χ^* and the system does not generate oscillations. (c) The basin of attraction B is $E^s \subset R^n$.
Now, we consider a switching system based on the affine linear system (1) given by

$$\dot{\chi} = \mathbf{A}\chi + \mathbf{B}(\chi),$$

$$\mathbf{B}(\chi) = \begin{cases} \mathbf{B}_1, \text{ if } \chi \in D_1; \\ \vdots \quad \vdots \\ \mathbf{B}_k, \text{ if } \chi \in D_k. \end{cases} \tag{2}$$

Where $R^n = \cup_{i=1}^{k} D_i$. Thus, the equilibria of the system (2) are $\chi_i^* = -\mathbf{A}^{-1}\mathbf{B}_i$, with $i = 1, \ldots, k$. So the goal is to define vectors $\mathbf{B}_i$ which can generate a class of dynamical systems in R^n with oscillations into an attractor, that is, the flow $\Phi(\chi(0))$ of the system (2) is trapped into an attractor A by means of defining at least two vectors $\mathbf{B}_1$ and $\mathbf{B}_2$. This class of systems can display various multi-scroll strange attractors as a result of the combination of several unstable "one-spiral" trajectories by using $\mathbf{B}(\chi)$, i.e., we are interested in a vector field which can yield multi-scroll attractors constitute by a commuted vector, $\mathbf{B}_i$ with $i = 1, \ldots, k$ and

$k \geq 2$. Each domain $\mathcal{D}_i \subset R^n$, contains an equilibrium point $\chi_i^* = -\mathbf{A}^{-1}\mathbf{B}_i$. As a summary, a multi-scroll chaotic system based on UDS can be generated by (2) in R^n and equilibrium points χ_i^*, with $i = 1, \ldots, k$ and $k > 2$. The special characteristic of this multi-scroll chaotic system is that each χ_i^* may contain oscillations around if its domain D_i is large enough to support the scroll, so the flow $\phi(\chi_0)$ generates an attractor $A \subset R^n$.

The general case of the linear ordinary differential equation(ODE) with constant coefficients is given by the following form:

$$\frac{d^n x}{dt^n} + \alpha_1 \frac{d^{n-1}x}{dt^{n-1}} + \ldots + \alpha_{n-1}\frac{dx}{dt} + \alpha_n x = 0, \tag{3}$$

This equation determines a linear system (1) with equilibrium point at the origin. So by means of controlling the equilibria in different domains $\chi_i^* \in D_i$ it is necessary to commute to different values of the vector $\mathbf{B} = \mathbf{B}_i$, with $i = 1, \ldots, k$.

In order to illustrate our approach we consider the particular case of the linear ordinary differential equation written in the jerky form as $\dddot{x} + \alpha_{33}\ddot{x} + \alpha_{32}\dot{x} + \alpha_{31}x = 0$, representing the state space equations of (1), where the matrix $\mathbf{A}$ is described as follows:

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_{31} & -\alpha_{32} & -\alpha_{33} \end{pmatrix}, \tag{4}$$

where the coefficients $\alpha_{31}, \alpha_{32}, \alpha_{33} \in R$ may be any arbitrary scalar satisfying the Definition 1. The characteristic polynomial of matrix $\mathbf{A}$ given by (4) takes the following form:

$$\lambda^3 + \alpha_{33}\lambda^2 + \alpha_{32}\lambda + \alpha_{31}. \tag{5}$$

For simplicity, we are defining the coefficients as $\alpha_{31} = 0.6,\ \alpha_{32} = 0.6,\ \alpha_{33} = 0.6$, with these values the eigenvalues result in $\lambda_1 = -0.7948,\ \lambda_{2,3} = 0.0974 \pm 0.8634i$, which satisfy Definition 1. The stable and unstable manifolds are determined by the eigenvector of the matrix $\mathbf{A}$, as follows:

$$E^s = Span\{(-0.7017, 0.5577, -0.4433)^T\}; \tag{6}$$

$$E^u = Span\{(0.6559, 0.0639, -0.4826)^T, (0, 5662, 0.1103)^T\}. \tag{7}$$

The location of the equilibria χ_i^* of the system (4) are determined by the vectors $\mathbf{B}_i$, with $i = 1, \ldots, k$, which can be defined as follows:

$$\mathbf{B}_i = \begin{pmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{pmatrix}, \tag{8}$$

$$\mathbf{B}_i = -A\chi_i^*. \tag{9}$$

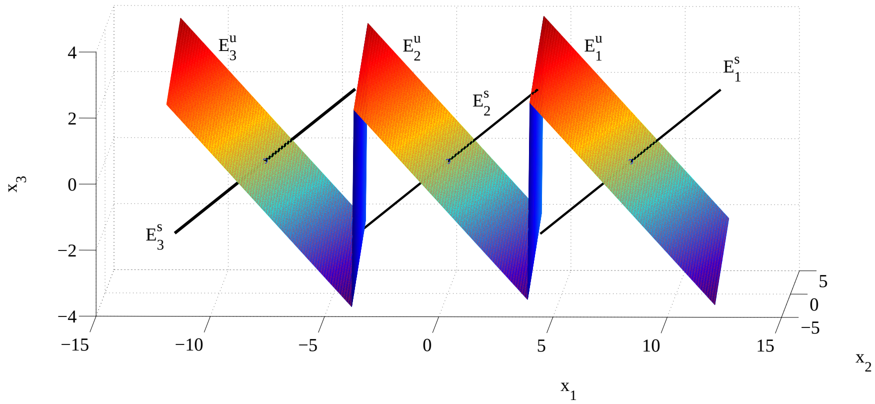

Fig. 1. Unstable and stable manifolds for 1D-grid multi-scroll attractor by UDS.

Considering the matrix $\mathbf{A}$ given by (4), then the vectors $\mathbf{B}_i$ are expressed as follows

$$\begin{pmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{pmatrix} = \begin{pmatrix} -\beta_1 \\ -\beta_2 \\ \alpha_{31}\beta_3 + \alpha_{32}\beta_1 + \alpha_{33}\beta_2 \end{pmatrix}, \tag{10}$$

where $\beta_1 = x^*_{2i}$, $\beta_2 = x^*_{3i}$ and $\beta_3 = x^*_{1i}$. Note that β_1, β_2 and β_3 are step functions.

2.1. *1D-grid multi-scroll attractors by UDS*

In this subsection, 1D-grid multi-scroll attractor is considered, thus the equilibria will be considered on the x_1-axis, so they have the following form $(x^*_{1i}, 0, 0)$. This determines that $b_{1i} = b_{2i} = 0$ and $b_{3i} = \alpha_{31}x^*_{1i}$, then the vectors $\mathbf{B}_i$ are given as follows:

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ \alpha_{31}\beta_3 \end{pmatrix}. \tag{11}$$

Now the aim is to generate a triple-scroll attractor, therein the phase space needs to be partitioned in three domains D_i, with $i = 1, \ldots, 3$, such that $R^3 = \cup_{i=1}^3 D_i$ and $\cap_{i=1}^3 D_i = \emptyset$. Considering the following equilibria $\chi^*_1 = (8, 0, 0)^T \in D_1$, χ^*_2 at the origin $(0, 0, 0)^T \in D_2$, and $\chi^*_3 = (-8, 0, 0)^T \in D_3$. The vectors $\mathbf{B}_i$ are determined by the parameter α_{31} and the first component x^*_{1i} of each equilibria χ^*_i, so the β_3 is given as follows

$$\beta_3 = \begin{cases} x^*_{11}, \text{ if } \chi \in D_1; \\ x^*_{12} \ \text{ if } \chi \in D_2; \\ x^*_{13}, \text{ if } \chi \in D_3. \end{cases} \tag{12}$$

The phase space can be partitioned by defining orthogonal planes at the x_1-axis. Therein, an embodiment can be given as $D_3 = \{\chi | x_1 < \sigma_2\}$, $D_2 = \{\chi | \sigma_2 \leq x_1 \leq \sigma_1\}$

and $D_1 = \{\chi|\sigma_1 < x_1\}$. There are different ways to select the parameters σ_1 and σ_2. For example, if the parameters $\sigma_1 = 3.8$ and $\sigma_2 = -3.8$, then the parameter β_3 is governed by the following switching control law (SCL):

$$\beta_3 = \begin{cases} 8, & \text{if } x_1 \geq 3.8; \\ 0 & \text{if } -3.8 < x_1 < 3.8; \\ -8, & \text{if } x_1 \leq -3.8. \end{cases} \tag{13}$$

Figure 1 shows the stable E^s and unstable E^u subspace of each affine linear system given by (2) with (4) and (13). The equilibria are located at the intersection of these two manifolds $\chi_i^* = E_i^s \cap E_i^u$, with $i = 1, 2, 3$. The stretching and folding behavior required for chaotic dynamics is given by means of the stable and unstable manifolds in each domain D_i, i.e., a trajectory escapes from each domain D_i due to the E_i^u unstable manifold and can be attracted again due to the E^s stable manifold, as it can be seen in Fig. 1.

A 1D-grid triple-scroll attractor is generated by the β_3 SCL (13) under equations (2)-(4). Figure 2 shows the projections of the 1D triple-scroll chaotic attractor onto the planes: a)(x_1, x_2), b)(x_1, x_3), and c)(x_2, x_3). The initial condition used is $(0, 0.1, 0.1)^T$, and is the same for the following cases. Figure 3 shows with blue dots the basin of attraction B onto the (x_1, x_2)-plane. B was obtained by first selecting a grid onto the (x_1, x_2)-plane given by $-60 \leq x_1 \leq 60$ and $-60 \leq x_2 \leq 60$. We then varied the values of x_1 and x_2 with increments of 1, observing the convergence of each trajectory, under the condition $|\chi(t)| < 100$.

The maximal Lyapunov exponent was computed and is equal to 0.1261, thus this demonstrates the chaotic behavior of the dynamical system. Figure 4 shows the points capture in the intersection of the chaotic attractor with the Poincaré section defined by conditions $x_1 = 0$ and $\dot{x}_1 > 0$. In this figure is possible to see the shape of a horseshoe in the points capture in the middle, so the map given by the Poincaré section may be studied as a discrete dynamical system. Introducing more equilibrium points to the system, along with the corresponding switching control law and the nonlinearity[30], one can create any number of scrolls inside the 1D-grid.

So, n-scroll chaotic attractors can be yielded by controlling the β_3 parameter. A quadtuple and quintuple scroll attractors can be generating as follows:

$$\beta_3 = \begin{cases} 16, & \text{if } x_1 \geq 11.4; \\ 8, & \text{if } 3.8 < x_1 < 11.4; \\ 0 & \text{if } -3.8 < x_1 < 3.8; \\ -8, & \text{if } x_1 \leq -3.8. \end{cases} \tag{14}$$

$$\beta_3 = \begin{cases} 16, & \text{if } x_1 \geq 11.4; \\ 8, & \text{if } 3.8 < x_1 < 11.4; \\ 0 & \text{if } -3.8 < x_1 < 3.8; \\ -8 & \text{if } -11.4 < x_1 < -3.8; \\ -16, & \text{if } x_1 \leq -11.4. \end{cases} \tag{15}$$

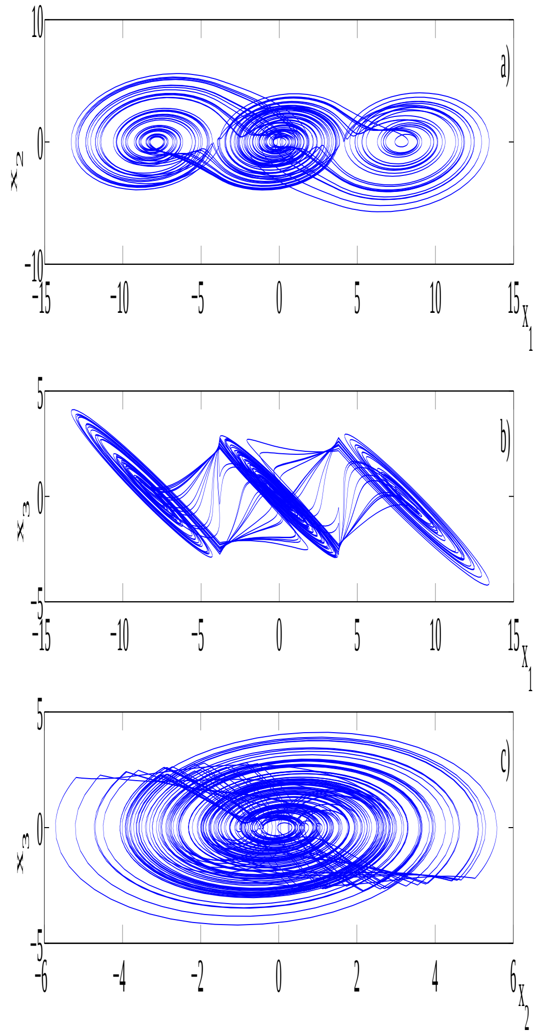

Fig. 2. The projections of the 1D-triple-scroll chaotic attractor onto the plane: A)(x_1, x_2), B)(x_1, x_3), and C)(x_2, x_3).

The β_3, given by the SCL's (14) and (15), introduce other equilibrium points located at $\chi_4^* = (16, 0, 0)^T$ and $\chi_5^* = (-16, 0, 0)^T$, respectively.
The shape of the attractor depends on the commutation surfaces determine by the parameters σ_1 and σ_2. For example, like double-scroll attractor can be generated if the parameters $\sigma_1 = 2.6$ and $\sigma_2 = -2.6$, see the projection of the attractor onto the (x_1, x_2) plane in Fig 5 a). Fig 5 b)shows the projection of the attractor onto the

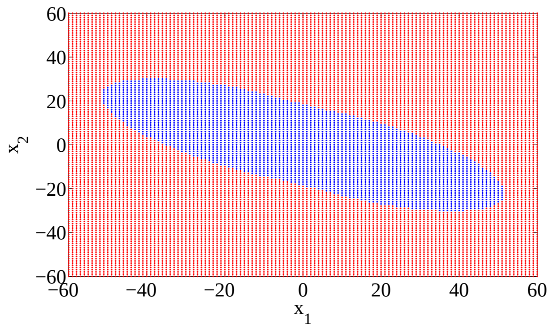

Fig. 3. The Basin of attraction onto the (x_1, x_2)-plane is marked with blue dots.

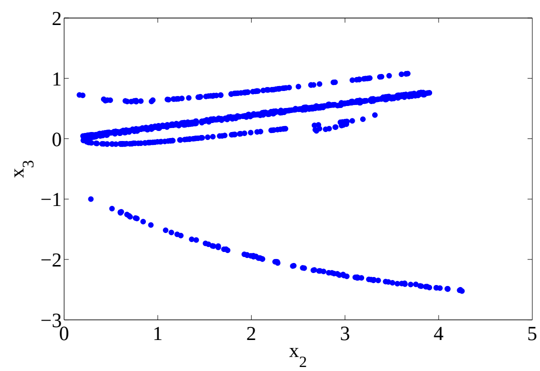

Fig. 4. The intersection of the chaotic attractor with the Poincaré section defined by $x_1 = 0$.

(x_1, x_2) plane for $\sigma_1 = 4$ and $\sigma_2 = -4.6$. These attractors are generated by different β_3 and equations (2)-(4).

3. 2D and 3D-grid multi-scroll attractors by UDS

In this section we expand the concept of 1D-grid to 2D and 3D-grid multi-scroll attractors based on UDS. We start with 2D-grid multi-scroll chaotic attractor generated by introducing new equilibria to the system. Now the equilibria are onto the (x_1, x_2) plane, thus they have the following form $(x^*_{1i}, x^*_{2i}, 0)$. Therefore, accordingly to eq. (10) the vectors $\mathbf{B}_i$ are given by $b_{1i} = -\beta_1$, $b_{2i} = 0$ and $b_{3i} = \alpha_{31}\beta_3 + \alpha_{32}\beta_1$. In the case 1D-grid of triple-scroll attractor, the equilibria were given as $(8, x^*_{21}, 0)$, $(0, x^*_{22}, 0)$ and $(-8, x^*_{23}, 0)$ with $x^*_{21} = x^*_{22} = x^*_{23} = 0$, as shown in Fig. 6 a). Now, if $x^*_{21}, x^*_{22}, x^*_{23} \in \{-8, 0, 8\}$ then there are nine equilibria, as shown in Fig. 6 b). Thus,

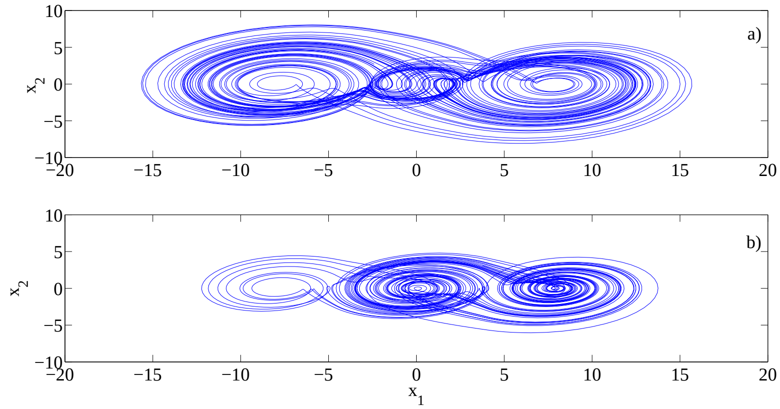

Fig. 5. Projection of the attractor onto the (x_1, x_2) plane generated by considering the parameters: a) $\sigma_1 = 2.6$ and $\sigma_2 = -2.6$; b) $\sigma_1 = 4$ and $\sigma_2 = -4.6$.

the vector $\mathbf{B}$ is defined as follows:

$$\mathbf{B} = \begin{pmatrix} -\beta_1 \\ 0 \\ 0.6\beta_1 + 0.6\beta_3 \end{pmatrix}, \tag{16}$$

The parameter β_3 is considered the same as in the previous case given by (13) and the parameter β_1 is governed by the following switching control law (SCL):

$$\beta_1 = \begin{cases} 8, & \text{if } x_2 \geq 4; \\ 0 & \text{if } -4 < x_2 < 4; \\ -8, & \text{if } x_2 \leq -4. \end{cases} \tag{17}$$

Thus, a 2D-grid 3×3-scroll chaotic attractor is generated by introducing new equilibria to the system. Fig. 7 shows the projections of the 2D 3×3-scroll chaotic attractor onto the planes: a)(x_1, x_2), b)(x_1, x_3), and c)(x_2, x_3), with initial condition $(0, 0.1, 0.1)^T$. For the case of 3D-grid multiscroll chaotic attractor, now the equilibria are in R^3, thus they have the following form $(x^*_{1i}, x^*_{2i}, x^*_{3i})$. Therefore, accordingly to eq. (10) the vectors $\mathbf{B}_i$ are given by $b_{1i} = -\beta_1$, $b_{2i} = -\beta_2$ and $b_{3i} = \alpha_{31}\beta_3 + \alpha_{32}\beta_1 + \alpha_{33}\beta_2$. The equilibria are shown in Fig. 6 c) and there are 27 equilibria. In this similar way a 3D-grid 3×3×3-scroll attractor is generated, so the location of the equilibria of the system are determined by the vector $\mathbf{B}$ which can be defined as follows:

$$\mathbf{B} = \begin{pmatrix} -\beta_1 \\ -\beta_2 \\ 0.6\beta_1 + 0.6\beta_2 + 0.6\beta_3 \end{pmatrix}, \tag{18}$$

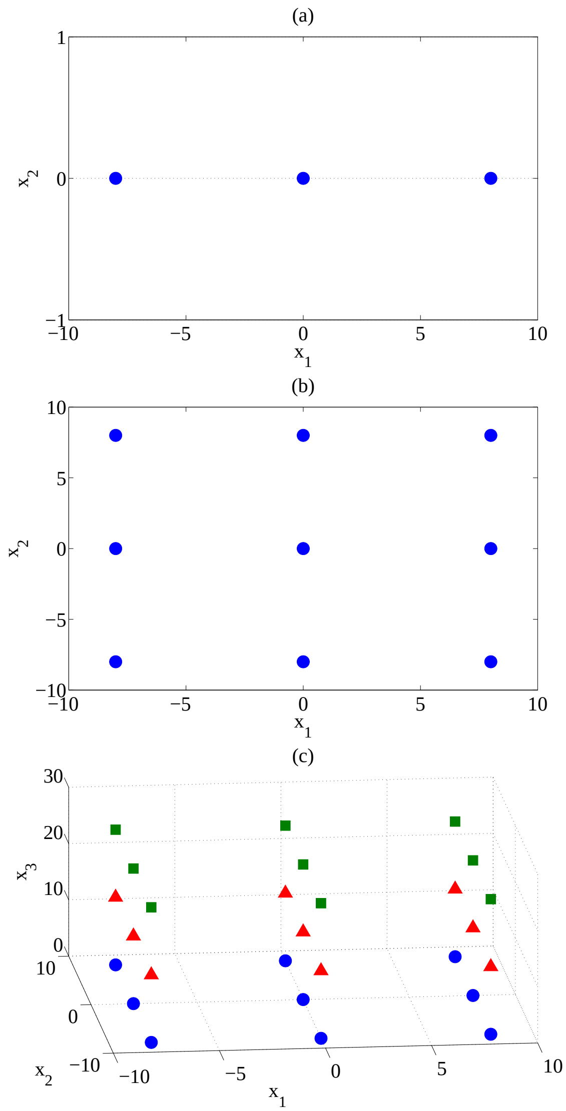

Fig. 6. Location of the equilibria of multi-scrolls attractors for: (a) 1D-grid, (b) 2D-grid and (c) 3D-grid.

The parameters β_1 and β_3 are given by (17) and (13), respectively, and the parameter β_2 is governed by the following switching control law (SCL):

$$\beta_2 = \begin{cases} 24, \text{ if } x_3 \geq 18; \\ 12 \ \text{ if } 6 < x_3 < 18; \\ 0, \ \ \text{if } x_3 \leq 6. \end{cases} \tag{19}$$

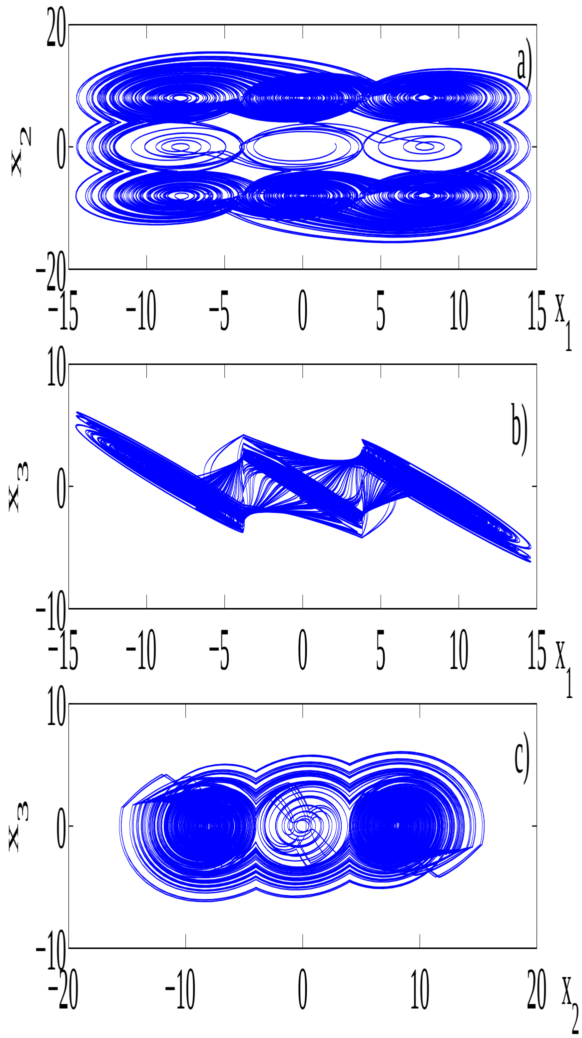

Fig. 7. The projections of the 2D-grid 3×3-scroll chaotic attractor onto the planes: a)(x_1, x_2), b)(x_1, x_3), and c)(x_2, x_3).

Figure 8 shows the projections of the 3D-grid $3{\times}3{\times}3$-scroll chaotic attractor onto the planes: a)(x_1, x_2), b)(x_1, x_3), and c)(x_2, x_3), with initial condition $(0, 0.1, 0.1)^T$.

4. Conclusion

By means of the UDS definition, one can assure the generation of multiscroll chaotic attractors in 1D, 2D and 3D-grids. So the UDS approach is an easy method to yield

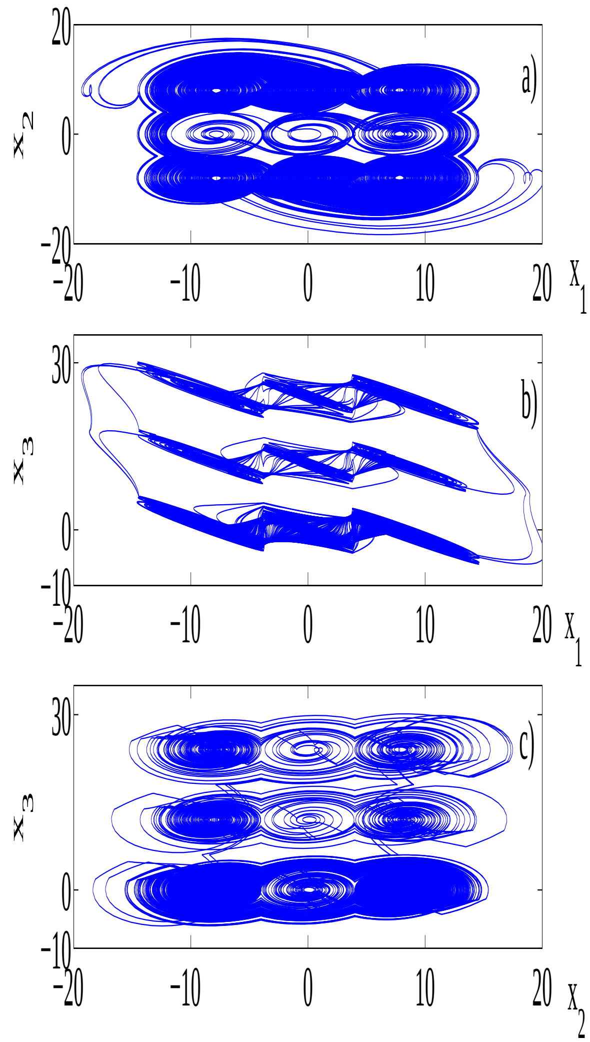

Fig. 8. The projections of the 3D-triple-scroll chaotic attractor onto the plane: A)(x_1, x_2), B)(x_1, x_3), and C)(x_2, x_3).

multiscroll attractors. Controlling the vector **B** with a switching control law it is possible to generate any number of scrolls in whatever direction. Also, the UDS approach can been extended to generate hyperchaotic multiscroll attractors. The future work is to extend the method to yield multiscroll attractors based on UDS to only hyperbolic system without being dissipative, another interest is to extend this work to the generation of families in 1D, 2D and 3D-grid multiscroll chaotic attractors.

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