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# Bistable behavior via switching dissipative systems with unstable dynamics and its electronic design

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Abstract
In this work we present a design of a bistable system and its electronic circuit which is generated by a switching system. The switching system is comprised by dissipative subsystems with unstable dynamics based on the jerk equation. For this system with unstable dynamics, it is necessary to use a switching control law in order to change the equilibrium point of the linear part and get bounded trajectories. Also the dynamics of the piecewise linear (PWL) system is illustrated by numerical simulations to depict the bistable states. We present an easy electronic design of the proposed system by employing resistors, capacitors and comparators, to exhibit the capability to generate bistable behavior.



*Keywords:* Multistability, piecewise linear systems, chaos, switching functions, switching control.

## 1. INTRODUCTION

In the context of complex systems, the scientific community has had the task of studying the properties of different dynamical systems. For example, the fact that chaotic systems have a strong dependence on initial conditions. In this sense, a complex system can exhibit various possible (coexisting) final states with a sink which traps the system trajectory depending on its initial state Feudel (2008); Pisarchik and Feudel (2014); Sharma et al. (2013); Blaejczyk-Okolewska and Kapitaniak (1998); Arecchi et al. (1985); Gilardi-Velázquez et al. (2017). This phenomenon is usually called multistability. The occurrence of multistability is very common in various fields of science, such as chemistry Ganapathisubramanian and Showalter (1984); Marmillot et al. (1991), optics Saucedo-Solorio et al. (2003); Brambilla et al. (1991), physics Cai and Feng (2014); Santer and Pellegrino (2011) and biological systems Ozbudak et al. (2004); Zhusubaliyev et al. (2015). Additionally, the importance of generating multistable structures resides in the wide variety of applications that exist: synchronization, complex networks, communication, climate and mainly in what is called chaos computing Sinha and Ditto (1998). In this sense many proposal has been made in chaos-based logic gates as: Cafagna and Grassi (2006) Who present a SR flip-flop based on NOR gates implemented by a Chuas circuit, Munakata et al. (2002) described how implement fundamental logical gates with logistic map, Murali et al. (2005) implement the fundamental NOR gate based on threshold control of chaotic systems, also Murali et al. (2009) show how to obtain logical functions via noisy nonlinear system by changing the nonlinearity, Campos-Cantón et al. (2010) present a circuit with dynamic logic architecture which display NOR, NAND and XOR gates.

In dynamical systems, an attractor is defined as a subset of the phase space toward the trajectories of the dynamical systems converge to it (and attractors can be fixed points, limit cycles or periodic, quasiperiodic, chaotic or hyperchaotic orbits). The basin of attraction is defined as the set of all initial conditions in the phase space whose corresponding trajectories converge to an attractor Kengne (2017); Giesl (2007). Concepts of convergent trajectories and attractor stability are usually associated with an energy-like term called Lyapunov function. Then, with the above concepts it can be said that a multistable dynamical system is a dynamical system that, depending on its initial condition, its trajectories solution can alternate between two or more mutually exclusive Lyapunov stable and convergent states Haddad et al. (2011).

In this work, advantage is taken of the properties of the hybrid dynamical systems, such as the Unstable Dissipative Systems (UDS) theory based on PWL systems whose solution presents chaotic attractors Campos-Cantón et al. (2012). The method proposed here consists in taken systems from the jerk equation, we consider a UDS Type II and designing a switching control law, without changing the linear operator, such the system presents the coexis-

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tence of two stable chaotic attractors. Additionally, the analysis, design and circuit synthesis of a PWL system is presented. As a result the system bistable behavior is displayed by circuit simulation.

## 2. UDS

The system that is considered in this paper is an autonomous nonhomogeneous first order lineal ordinary differential equation system of the form:

$$\dot{x} = f(x) = Ax + g(x), \quad x(0) = x_o, \tag{1}$$

where $x \in \mathbf{R}^n$ is the state vector, $A = \{a_{ij}\}_{i,j=1}^n \in \mathbf{R}^{n\times n}$ is a non-singular linear operator with $a_{ij} \in \mathbf{R}$ and; $g : \mathbf{R}^n \to \mathbf{R}^n$ is a vector which commutes as follows:

$$g(x) = \begin{cases} B_1, & \text{if } \ x \in S_1 = \{x \in \mathbf{R}^n : G_1(x) < \delta_1\}; \\ B_2, & \text{if } \ x \in S_2 = \{x \in \mathbf{R}^n : \delta_1 \le G_2(x) < \delta_2\}; \\ \vdots & \quad \vdots \\ B_m, & \text{if } \ x \in S_m = \{x \in \mathbf{R}^n : \delta_{m-1} \le G_m(x)\}; \end{cases} \tag{2}$$

where $B_i = (b_{i1}, \ldots, b_{in})^T \in \mathbf{R}^n$ for $i = 1, \ldots, m$ are constant vectors with real entries; and $S = \{S_1, S_2, \ldots, S_m\}$ is a finite partition of the phase space called the switching domains, which satisfy $\mathbf{R}^n = \bigcup_{1\le i\le m} S_i$. Each $S_i$ is defined by hypersurfaces $\Sigma$ in terms of $\delta_i$ (with $1 \le i \le m-1$) , particularly in $\mathbf{R}^3$, represents a plane as a boundary between two consecutive domains. Furthermore, we assume that each set $S_i$ has at least a saddle equilibrium point $x^* \in S_i$. If $\lambda_j = \alpha_j + i\beta_j$ is a complex eigenvalue of the linear operator $A$ and $\bar{v}_j \in \mathbf{R}^n$ its corresponding eigenvector, then the stable set is $E^s = Span\{\bar{v}_j \in \mathbf{R}^n : \alpha_j < 0\}$ and the unstable set $E^u = Span\{\bar{v}_j \in \mathbf{R}^n : \alpha_j > 0\}$ Guzzo (2010). If the equilibrium point $x^* \in \mathbf{R}^3$ of (1) is an hyperbolic saddle-focus equilibrium and that the sum of its eigenvalues is negative, then the system is called unstable dissipative system.

**Definition 1.** *Let $\Lambda = \{\lambda_1, \lambda_2, \lambda_3\}$ be the eigenspectra of the lineal operator $A \in \mathbf{R}^{3\times 3}$, such that $\sum_{i=1}^3 \lambda_i < 0$, with $\lambda_1$ a real number and $\lambda_2, \lambda_3$ two complex conjugate numbers. A system given by the linear part of the system* (3) *is said to be a UDS Type I if $\lambda_1 < 0$ and $Re\{\lambda_{2,3}\} > 0$; and it is Type II if $\lambda_1 > 0$ and $Re\{\lambda_{2,3}\} < 0$.*

In particular, it is considered the following family of affine linear systems:

$$\dot{x} = Ax + B(x), \tag{3}$$

where $x = (x_1, x_2, x_3)^\top \in \mathbf{R}^3$ is the state vector, the real matrix $A \in \mathbf{R}^{3\times 3}$ is a non-singular linear operator; and $B : \mathbf{R}^3 \to \mathbf{R}^3$ is a constant vector $B_i$ in each domain $S_i$ which is determined by a switching control law (SCL). We involve the step function in defining these constant vectors $B_i$, $i = 1, ..., m$. A convenient approach to build the matrix $\mathbf{A}$ and design the SLC is based on the linear ordinary differential equation (ODE) given by the jerk form: $\dddot{x} + a_{33}\ddot{x} + \dot{x}a_{32} + a_{31}x + \beta = 0$.

The location where the attractors are positioned depend from the following; the coefficient matrix $\mathbf{A}$ from the jerk equation Campos-Cantón (2016) and the affine vector $\mathbf{B}$ as is described by Campos-Cantón et al. (2012) with:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha & -\beta & -\gamma \end{pmatrix}, \quad B(x) = \begin{pmatrix} 0 \\ 0 \\ \sigma(x) \end{pmatrix}; \tag{4}$$

where $\alpha, \beta, \gamma \in \mathbf{R}$ and $\sigma(x) : \mathbf{R}^3 \to \mathbf{R}$ is the following step function that generates a switching law to control the equilibria of the system:

$$\sigma(x) = \begin{cases} b_1, & \text{if } x \in S_1 = \{x \in \mathbf{R}^3 : \mathbf{v}^\top x < \delta_1\}; \\ b_2, & \text{if } x \in S_2 = \{x \in \mathbf{R}^3 : \delta_1 \le \mathbf{v}^\top x < \delta_2\}; \\ \vdots & \quad \vdots \\ b_m, & \text{if } x \in S_m = \{x \in \mathbf{R}^3 : \delta_{m-1} \le \mathbf{v}^\top x\}; \end{cases} \tag{5}$$

with $b_i \in \mathbf{R}$ (for $i = 1, \ldots, m$;) are the switching domains, $\mathbf{v} \in \mathbf{R}^3$ ( $\mathbf{v} \neq 0$) a constant vector, and $\delta_1 \le \delta_2 \le \cdots \le \delta_{m-1}$ determine the switching surfaces $\Sigma$ location. Without loss of generality, it is assumed that switching surfaces $\Sigma$ are defined by $\Sigma_\tau = \{x \in \mathbf{R}^3 | \mathbf{v}^\top x = \delta_\tau\}$ (for $\tau = 1, 2, \ldots, m$), with $\mathbf{v} = (1, 0, 0)^\top \in \mathbf{R}^3$. The role of the SCL $\sigma$ is to specify which system is active at a given switching domain $S_i$, that is, if $\sigma(x) = b_i$ for $i \in I = \{1, \ldots, m\}$, then the affine linear system that governs the dynamics in the switching domain $S_k$ is given by $\dot{x} = Ax + (0, 0, b_k)^\top$.

According with the aforementioned assumption, with the SCL proposed, each switching domain contains a single saddle equilibrium point located at $x_i^* = A^{-1}B_i$, with $i \in I$. Then, the mechanism of generation of chaotic attractors based on this class of systems is due to the stable and unstable manifolds related with the design of $A$ and $B$; this is, by considering two domains $S_i$ and $S_{i+1}$, and the commutation surface $\Sigma_\tau$ between them. When the trajectory $\phi(x_0)_t$ reaches to the commutation surface $\Sigma_\tau$ due to the unstable manifold and initial condition $x_0 \in S_i$, and crosses the domain $S_{i+1}$, and the trajectory approaches the point of equilibrium due to the stable manifold, but again scapes from this domain due to the unstable manifold, being the trajectory $\phi(x_0)_t$ trapped forming the chaotic attractor.

The above result is illustrated by Figure 1. Notice that the trajectory of the system oscillating around the unstable manifold $W^u$ escapes from the domain $S_i$. This occurs near the unstable manifold $E^u \subset S_i$ where it crosses the commutation surface and it is rejected by the unstable manifold $W^u \subset S_{i+1}$ towards the equilibrium point $x_i^*$ in the domain $S_i$. The process is repeated in the inverse way forming the attractor. For which, the set of all the initial conditions, in the phase space, whose corresponding trajectories converge to an attractor are defined as basin of attraction, $\Omega$.

The idea is to generate different sets $\Omega_j$ such that for any initial condition $x_0 \in \bigcup_{1\le j\le k} \Omega_j \subset \mathbf{R}^3$, the orbit $\phi(x_0)$ of the system (3)-(4) is attracted in only one region $\Omega_j$. The generalized multistability are considered, so the trajectory needs to remain oscillating. We start considering only one basin of attraction of a chaotic multiscroll attractor. Without lost of generality the following set of parameters are considered, $\alpha = -0.6$, $\beta = 6$ and $\gamma = 0.6$. With this selection of parameters the eigenvalues of $A$

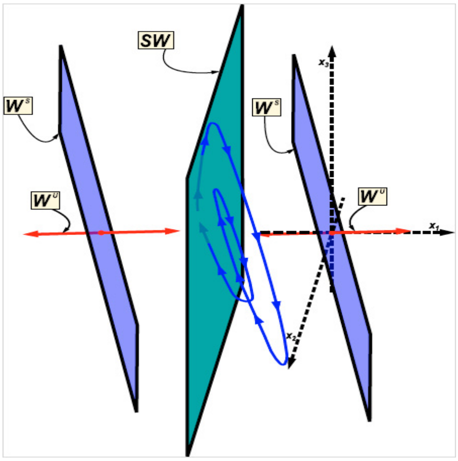



Figure 1. System flow around SW

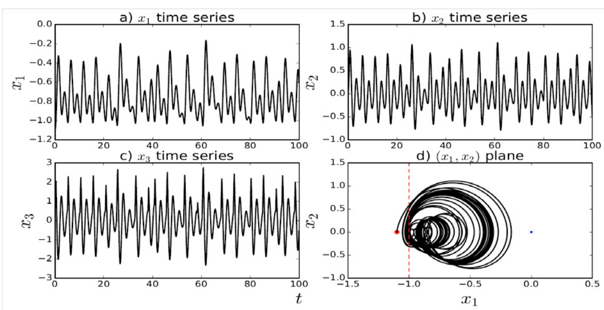



Figure 2. a) $x_1$ time series. b) $x_2$ time series. c) $x_3$ time series. d)Projection of the attractor onto the $(x_1, x_2)$ plane based on UDS Type II The dashed lines mark the division between the switching surfaces and the red dot indicates the initial position at $\chi_0 = (-1.1, 0, 0)^\top$.

are $\lambda_1 = 0.0988$ and $\lambda_{2,3} = -0.3494 \pm 2.4386i$, which according to Definition 1, the system is an UDS Type II. In particular, for this example the following SCL is designed via bifurcation analysis as is reported in Campos-Cantón et al. (2012) :

$$\sigma(x) = \begin{cases} 0, \text{ if } x \in S_1 = \{x \in \mathbf{R}^3 : -1 \leq x_1 \leq 1\}; \\ 7, \text{ if } x \in S_2 = \{x \in \mathbf{R}^3 : x_1 < -1\}. \end{cases} \quad (6)$$

Figure (2) shows the dynamics obtained by numerical simulation of a switching system (4), (6) based on UDS Type II for the initial conditions $x_0 = (-1.1, 0, 0)^\top$. The attractor is generated by using two UDS Type II, near the switching surface $x_1 = -1$.

## 3. BISTABLE CHAOTIC SYSTEM

Now, the interest is to generate bistability behavior via a dynamical system based on UDS Type II, so the system (3) is considered with:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \alpha & -p\alpha & -\alpha \end{pmatrix}, \quad B(x) = \begin{pmatrix} 0 \\ 0 \\ \sigma^*(x) \end{pmatrix}, \quad (7)$$

where $p$ is a scaling parameter which can be used to change the size of the attractor and by modifying the SCL (6) in order to get more attractors. The approach is illustrated in the next subsection 3.1.

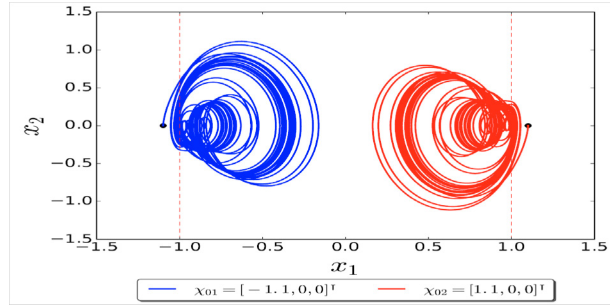



Figure 3. Projections of the attractors based on UDS Type II onto the $(x_1, x_2)$ plane, with the switching law (8) and $\alpha = 0.6$ and $p = 10$ . The black dots indicate initial conditions at $x_{01} = (-1.1, 0, 0)^\top$ and $x_{02} = (1.1, 0, 0)^\top$ for the left-hand side attractor $A_L$ and right-hand side attractor $A_R$, respectively.

### 3.1 Generalized Bistability

The starting point is the example which was given in Section 3 where $\alpha = -0.6$, $\beta = 6$ and $\gamma = 0.6$, and the phase space is partitioned by $S_1 = \{x \in \mathbf{R}^3 | x_1 \geq -1\}$ and $S_2 = \{x \in \mathbf{R}^3 | x_1 < -1\}$. Each domain has a stable manifolf $E_1^s \subset S_1$ and $E_2^s \subset S_2$ given by planes such that they are parallel $E_1^s \parallel E_2^s$. The basin of attraction is located $\Omega$ between $E_1^s$ and $E_2^s$. So, now the idea of generalized multistability generation is by increasing the number of domains in the partition with a new SCL and generate an attractor near the switching surface and between two stable manifolds, i.e., $E_1^s \subset S_1$, $E_2^s \subset S_2$, ..., $E_k^s \subset S_m$, with $2 \leq m \in \mathbf{Z}$, and $E_1^s \parallel E_2^s, \ldots, E_{m-1}^s \parallel E_m^s$. The PWL system given in Section 3 is used, but now the phase space is partitioned in three domains given by $S_{1x_1} = \{x \in \mathbf{R}^3 | x_1 > 1\}$, $S_{2x_1} = \{x \in \mathbf{R}^3 | -1 \leq x_1 \leq 1\}$ and $S_{3x_1} = \{x \in \mathbf{R}^3 | x_1 < -1\}$ by introducing a new switching domain on the SCL function (6) as follows:

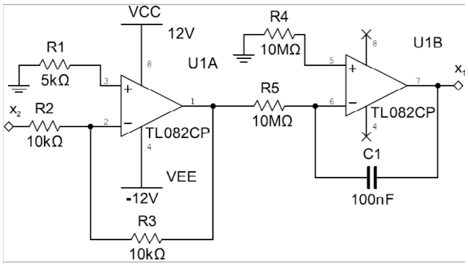
$$\sigma^*(x) = \begin{cases} -7, \text{ if } x \in S_{1x_1} = \{x \in \mathbf{R}^3 : x_1 > 1\}; \\ 0, \quad \text{if } x \in S_{2x_1} = \{x \in \mathbf{R}^3 : -1 \leq x_1 \leq 1\}; \\ 7, \quad \text{if } x \in S_{3x_1} = \{x \in \mathbf{R}^3 : x_1 < -1\}. \end{cases} \quad (8)$$

The system defined by (3) with equations (7) and (8) and parameters $\alpha = 0.6$, $p = 10$, has its equilibria at $x_1^* = (0, 0, 0)$ and $x_{2,3}^* = (\pm 11.66, 0, 0)$, this dynamical system presents two attractors $A_L$ and $A_R$, Figure (3) shows a bistable behavior for the two coexisting attractors. The left-hand side attractor $A_L$ and right-hand side attractor $A_R$ were generated by considering the following initial conditions: $x_{01} = (-1.1, 0, 0)^\top$ and $x_{02} = (1.1, 0, 0)^\top$. Each final stable state of the system is a single chaotic attractor which depends on only of the initial condition selected. In terms of generalized multistability we have a bistable behavior then the basin of attraction of the system is given by the union of two basin of attraction $\Omega_1 \cup \Omega_2$.

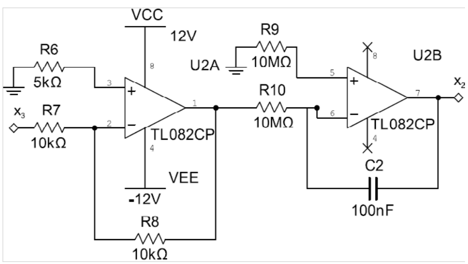
## 4. CIRCUIT SYSTEM DESIGN

In this section an easy electronic realization of the system (3) with $A$ and $B$ given by (7) which make use of operational amplifier (Op-Amp), comparators, resistors and capacitors is proposed. The circuit is energized by

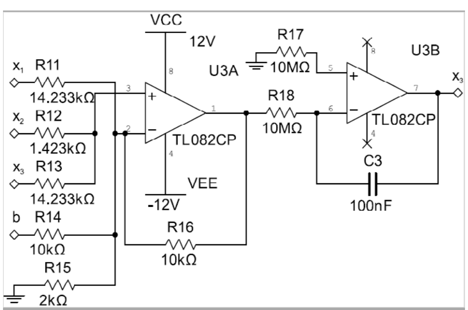
DC power sources of ±12V and ±15V. For simplicity, its electronic diagram has been divided into four sub-diagrams presented in Figures (4) and (5).




(a)




(b)




(c)

Figure 4. Sub-diagrams of the proposed electronic realization of the system (3) with $A$ and $B$ given in (7) for the sub-circuits that produce the output signals: (a) $x_1$ (b) $x_2$ and (c) $x_3$.

The sub-circuits which are shown in Figures (4a) and (4b) are composed of an Op-Amp configured as an inverter followed by another Op-Amp configured as an integrator while the sub-circuit in Figure (4c) is composed of an Op-Amp configured as an adder-subtractor whose gains are determined in part by the characteristic polynomial of the matrix $A$ given in (7), one additional gain is chosen as $-1$ and is reserved to the input signal $b$. The equation of the adder-subtractor is given as follows

$$-.6x_1 + 6x_2 + .6x_3 - b = V_{3A}, \tag{9}$$

where $V_{3A}$ is the voltage output from U3A. These three sub-circuits produce the output signals $x_1$, $x_2$ and $x_3$.

The sub-circuit in Figure (5) is based on the SCL defined in (8), it has as input the signal $x_1$ and is responsible for the commutation of the signal $b$. If $x_1 < 1$V then $b$ takes the value of 7V, the value of 0V if $-1$V $< x_1 < 1$V and $-7$V if $x_1 > 1$V. The output when $x_1 = 1$V or $x_1 = -1$V is not defined for practical reasons. This sub-circuit consists of a pair of comparators followed by Op-Amps configured

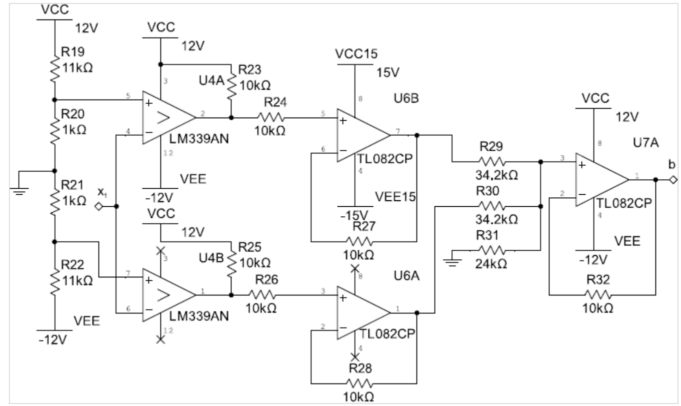



Figure 5. Sub-diagram of the proposed electronic realization of the system (3) with $A$ and $B$ given in (7) for the sub-circuit that produce the switching control law given in (8).

as buffers and an Op-Amp configured as adder-subtractor that implement the following equation:

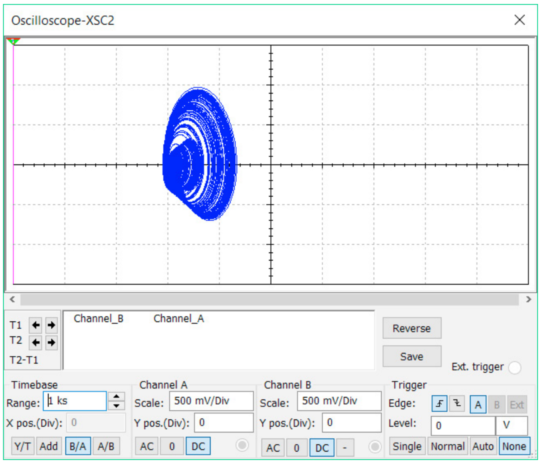
$$V_{6A}\left(\frac{12}{3.5}\right) + V_{6B}\left(\frac{12}{3.5}\right) = b, \tag{10}$$

where $V_{6A}$ and $V_{6B}$ are the voltage outputs from U6A and U6B, respectively and whose only possible values has been assumed to be ±12V. It is worth mention that in the diagrams, power sources appear connected to only one of the internal devices of the integrated circuit, however this implies the energized of all internal devices. The devices considered for this circuit are the general purpose JFET-input dual Operational amplifier TL082CP and the quad differential comparator LM339AN. The mathematical values for the resistors has been replaced by an approximated value achievable by the combination of two resistors from the E12 series either in parallel or series.

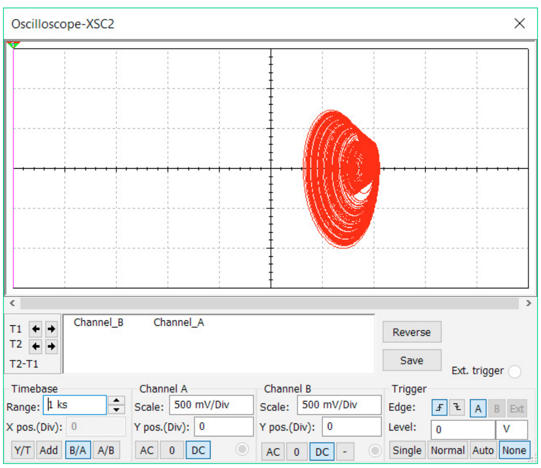
An electronic simulation of the proposed circuit has been run for the initial conditions $x_0 = (1.1,\ 0,\ 0)^\top$ y $x_0 = (-1.1,\ 0\ 0)^\top$, as it can be seen in Figure (6) the circuit proposed is capable of generating bistability in concordance to the mathematical model of dynamical system. The signal $x_1$ and the control signal $b$ for the attractor generated by electronic simulation using the initial condition $x_{01} = (-1.1, 0, 0)^\top$ are shown in Figure 7. The signals for the initial condition $x_{02} = (1.1, 0, 0)^\top$ are shown in Figure 8. In which can be seen the bistable behavior on the oscillation range for $x_1$ variable and the taken values for the SCL $b$ in both cases.

## 5. CONCLUSION

In this paper, we have presented a mechanism of constructing a bistable system based on piece-wise linear systems via SCL. Particularly, it deals with UDS Type II that results in generating chaotic attractors. The attractor arises from a switching system via control having at least two UDS Type II. Two examples are shown by means of considering a system in which the A matrix is the same in both domains $S_1$ and $S_2$, and the difference lies only in the $B$ vectors which change the locations of the equilibrium points. This result was extended to yield a system with three domains to generate chaotic systems with multista-

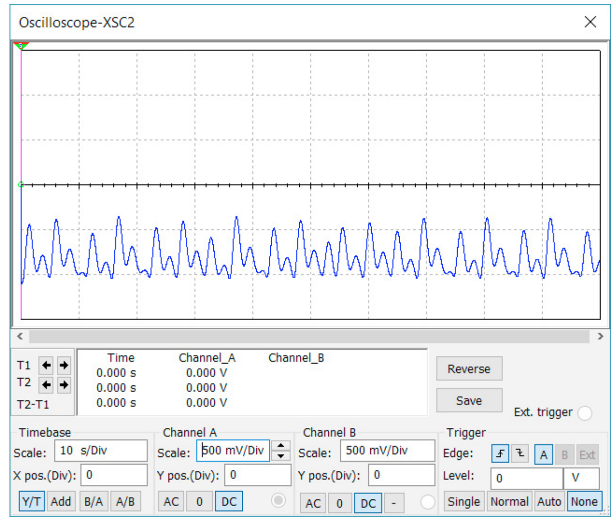



(a)

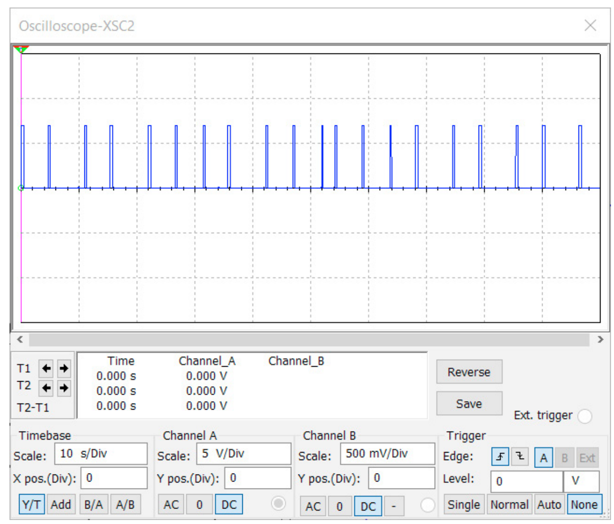



(b)

Figure 6. Projection of plane $x_1 - x_2$ of the attractor generated by simulation: (a) with initial conditions $x_0 = (-1.1,\ 0,\ 0)^\top$ and (b) with initial conditions $x_0 = (1.1,\ 0,\ 0)^\top$. In both cases the simulation time recorded is 550s from where the first 50s has been omitted in the plot.




(a)




(b)

Figure 7. Signals (a)$x_1$ and (b) $b$ for the initial condition $x_{01} = (-1.1, 0, 0)^\top$.

bility by the switching control law. An electronic circuit for the controlled system was designed, which displays the bistable behavior described in this work. This class of systems can be implemented in a communication system based on chaotic modulation, as proposed in Pecora et al. (1997); Ontañón García et al. (2014). Based on synchronize two identical attractors and considering each stable state as a transmission channel.

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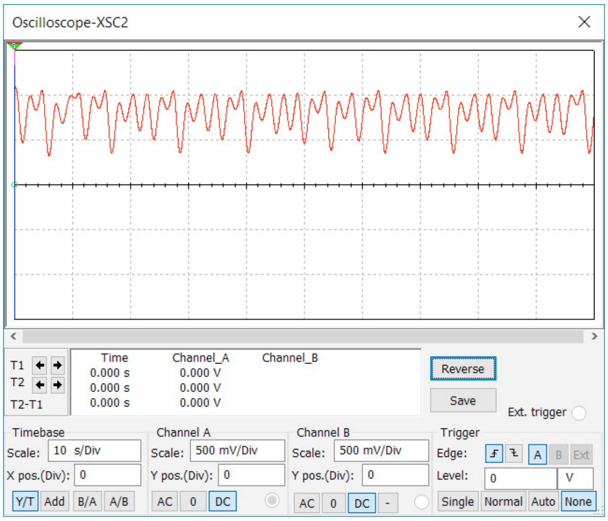
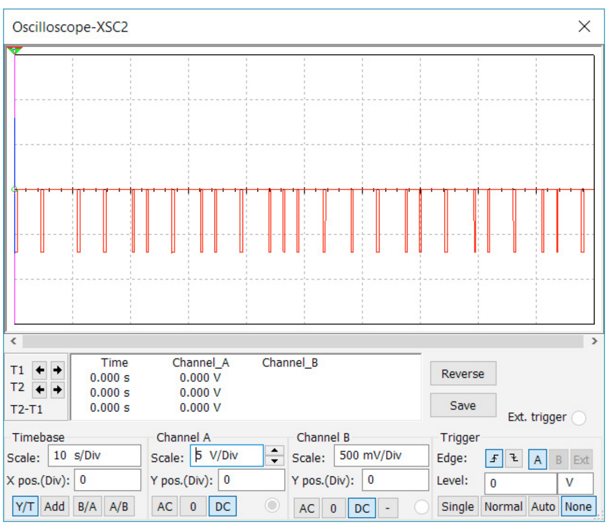



(a)




(b)

Figure 8. Signals (a)$x_1$ and (b) $b$ for the initial condition $x_{02} = (1.1, 0, 0)^\top$.

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