Modeling and Simulation of Electrical Machine Systems Through Bond Graphs and the Complementarity Framework

Noé Villa-Villaseñor
dept. of I. T. Electronics and Control
Advanced Technology Center CIATEQ
San Luis Potosi, Mexico
noe.villa@ciateq.mx

Jesús Rico-Melgoza
dept. of Electrical Engineering
University of Michoacan
Morelia, Mexico
jerico@umich.mx

Irwin Díaz-Díaz
dept. of SMT
IPICyT
San Luis Potosi, Mexico
irwin.diaz@ipicyt.edu.mx

***Abstract*— The conjunction of the complementarity framework and bond graphs is presented as an appropriate technique for the modeling of electric machine drives. The proposed approach uses complete models of switched systems. It also allows the modeling of several parts of a drive of electrical machines and related systems. With the presented method, different configurations of drives can be represented and studied. A high interconnection capacity is also shown as an attractive feature.**

Keywords—Complementarity, bond graph, motor drive, hybrid systems.

I. INTRODUCTION

Electric machines are the force that moves the industrialized world of today. Besides, a substantial increase in its use is foreseen due to the imminent mass deployment of electric traction vehicles. The topics related to electric machines are being subjected to intensive research.

There are several tools to model electric machines. Thus, during the electromagnetic design, finite element methods are generally used [1]. For the study of dynamic performance, however, other tools that allow the design of control strategies are often used [2]. In this line, this paper presents the complementarity framework in conjunction with bond graphs as a modeling technique.

The complementarity framework is a formalism that, among its applications, allows the representation of switched converters through complete models, that is, they have a fixed topology [3]. It is not necessary to study the change of modes that appear in other methodologies, and a single model represents all the possible states of the converter. This formalism has modeled different topologies of converters.

Bond graphs allow the representation of multiphysics systems. For example, electrical, mechanical, thermal and fluidic subsystems can be included in a single dynamic model in a standardized context [4]. Bond graphs also have the property of causality. This feature allows determining some properties of the modeled system visually. Causality also enables the state equation of a bond graph to be derived systematically.

Through the union of the two formalisms mentioned above, it is possible to model multiphysics systems, including hybrid systems. Because the control of rotary field motors is mainly carried out by utilizing switched converters, the complementarity framework and bond graphs are presented in this paper as an alternative for the modeling and simulation of complete drives.

Although some approaches have already been presented for the modeling of electrical machines through Bond Graphs [5], this paper intends to contribute with a different proposal.

II. COMPLEMENTARITY AS MODELING TOOL

One of the typical modeling methods for switched systems is the averaging method in which simplifications are usually performed to represent a converter. These simplifications inhibit the representation of fast dynamics that can occur only in times very close to the switching instants. Other methods consider the changes that occur at each switching moment, but they approach the problem from a variable topology perspective. The model must be divided into different submodels called modes. The constant transition from one mode to another causes issues such as changing the order of the system, inconsistency in the initial conditions in each mode, and the impossibility of simultaneous switching of two or more switches.

A proposal to abate the problems mentioned above has been made in [3] and reinforced with some other works, such as [6]. The methodology is named complementarity framework and allows modeling converters that present the fast dynamics existing at switching times. Also, the formalism allows the model of a converter to maintain a fixed topology, making the concept of mode change unnecessary.

The complementarity framework is based on the representation and solution of a linear complementarity problem (LCP) of the form [7]:

$$w = Mz + q \tag{1.a}$$
$$0 \leq w \perp z \geq 0 \tag{1.b}$$

In Equation (1) w and z are complementarity variables. M and q are matrices of adequate dimensions. The solution to the problem implies calculating z given M and q [8]. The existence and uniqueness of the solution of Equation (1) are guaranteed if M is a P-matrix, that is, all their major minors are positive [6]. There are several algorithms to solve the problem represented by Equation (1). In the simulations presented in this paper, the algorithm provided in the multiparametric toolbox [9], which runs in Matlab [10] is used.

In [7] it is shown how to convert a circuit of a switched converter to the form described by Equation (1).

III. BOND GRAPH MODELING

Bond graphs, introduced by Henry Paynter [11] have traditionally been used for the modeling of linear systems in a highly systematized manner. The fundamental element of this methodology consists of the bond. The bond is a graphic element that allows the interconnection of two elements of the modeled system. Once two components have been interconnected by a bond, the energy that can be transferred from one to the other can be determined. The exchange of energy is carried out through the interaction of two types of generalized variables: effort and flow. The generalized variables are chosen so that their product is always power. The bond graph methodology allows the representation of systems from different physical domains. Depending on the physics represented, the generalized variables can be defined as shown in Table 1.

TABLE I. GENERALIZED VARIABLES

Domain	***Effort***	***Flow***
Translational	Force	Velocity
Rotational	Torque	Angular speed
Electric	Voltage	Current
Hydraulic	Pressure	Flow rate

Bond graph methodology presents a high structural organization. Figure 1 shows the block diagram in a bond graph.

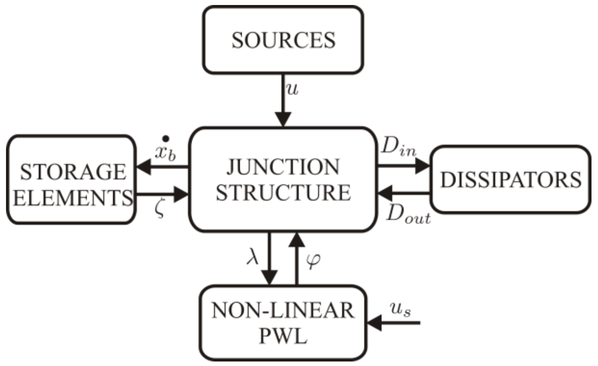

Fig. 1. Blocks that make up a bond graph model.

According to Figure 1, the physical models that can be represented by a bond graph consist of dissipating elements, which allow the release of energy to the outside of the system. They also consist of energy storage elements that at some instants absorb it from the system, and at other moments release it to the system. The source block includes all the elements that introduce energy to the system. The block of the junction structure represents the topology of the system. Here information is stored on how the components of the rest of the blocks are interconnected. The junction structure presents the property of being conservative, it only allows the exchange of energy, but it does not add or liberate it from the system. The block of non-linear elements does not appear in the traditional bond graphs. However, this block is necessary for the methodology used in this paper. Elements with non-linear PWL constitutive relations that resemble nonlinear behavior are located there.

The bond graph methodology makes extensive use of the concept of causality. Graphically in a bond, a bar is placed in one of the ends, which indicates the direction of the effort in the constitutive relation of the represented element. The bar is called a causal stroke.

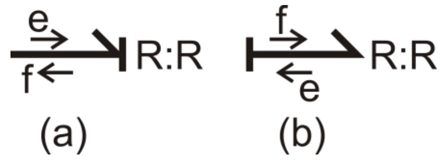

Fig. 2. Causal orientations in a resistor.

As an elementary example, in Figure 2 (a) a bond representing a resistor is shown. The causal stroke is pointing towards the label of the element, so the effort (voltage) is considered as an independent variable. Then the flow (the current) is the dependent variable. According to Ohm's law, the constitutive relation must be written as

$$i = (1/R)v$$

and therefore this causal orientation can be called conductance causality. If the causal stroke is at the opposite extreme, as in Figure 2 (b), then the flow (current) "enters" the element and is considered as an independent variable. In this way, the effort (voltage) is the dependent variable. According to Ohm's law, the constitutive relation for this causal relation is

$$v = Ri$$

and the orientation can be called resistance causality.

Through the study of causality several properties can be described visually before any calculation. For example, controllability and observability can be determined by causal paths [12].

One of the problems that hybrid systems present when they are represented in the bond graph methodology is the constant change of causality required. A reassignment of causality is necessary during mode changes.

The bond graph methodology is a modeling technique with well-established principles. It would be impossible to present a complete introduction here for reasons of space. The uninitiated reader can consult different references to go deeper into the theoretical basis of this methodology. The work of [4] can represent a good first entry point.

IV. INDUCTION MOTOR DRIVE MODELING

It has been mentioned that the complementarity framework allows representing switched converters in fixed topology models. It has also been mentioned that traditional approaches to model hybrid systems in bond graphs present the problem of dynamic causality. In [13] a methodology is shown to represent models in the complementarity framework from bond graph models. The methodology allows addressing the problem of causality changes.

The complementarity framework has been used mainly for the modeling of switched power electronics converters.

However, the work presented in [13] allows extending the approach to other types of systems, as long as a bond graph can represent them.

Electric machines are systems in which physical processes of different domains intervene. In particular, electrical, magnetic, mechanical and thermal processes are carried out inside an electric machine. If we consider that the bond graph methodology allows the representation of different physical domains in the same context, then it seems adequate to model electric machines.

Modern electrical machines are usually electronically commuted and support their operation in switched converters. By utilizing a converter, it is possible to force the magnetic fields within the stator to rotate at a preset arbitrary frequency. If it is considered that the primary application of the complementarity framework has been in switched converters, it is natural to select it as an option here.

Due to the two reasons mentioned above, the methodology presented in [13] finds an area of natural application in modern electrical machine systems. Each machine type presents its particular characteristics. The study conducted in this paper, however, focuses on the variable speed induction motor.

A three-phase induction motor drive can be represented according to the diagram of Figure 3.

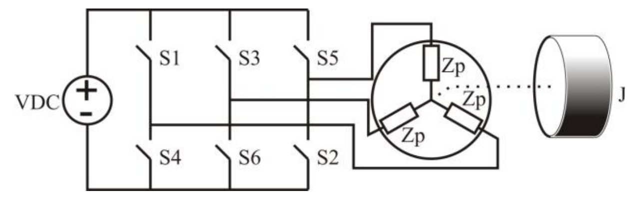

Fig. 3. Diagram of an induction motor drive.

In the diagram of Figure 3, the converter is a six-pulse three-phase inverter. For modeling purposes, it is considered that the equivalent circuit per phase is described as shown in Figure 4 [14].

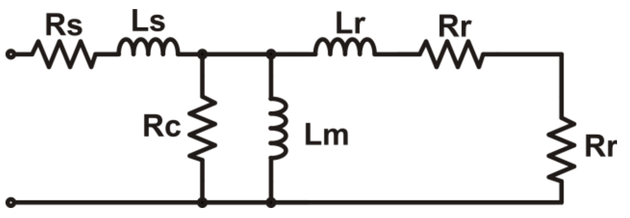

Fig. 4. Per phase equivalent circuit.

In the circuit of Figure 4, *Rs* and *Ls* correspond to the resistance and inductance in the stator, respectively. *Lm* is the mutual inductance of the stator to the rotor. *Rr* and *Lr* correspond to the resistance and the inductance in the rotor, respectively. *Rc* is a resistance that allows representing the losses that occur in the core.

The model considers that the converted power is dissipated in the far right resistor. The value of this resistor is modulated by the slip with a factor of [14]:

$$\frac{1-s}{s}$$

Where s is the slip.

The value of this resistance is usually small, a circuit can be connected to its output as shown in figure 5.

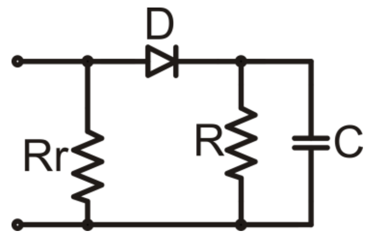

Fig. 5. Peak detector circuit.

As the values of *R* and *C* in the circuit of figure 5 are high, *Rr* does not experiment a noticeable load effect. Thus the circuit allows determining the peak voltage. By knowing this voltaje value in the resistive element, it is easy to determine the power *Pconv* that is delivered. The voltage is scaled by a constant factor to calculate the value that will modulate the effort source of Figure 6.

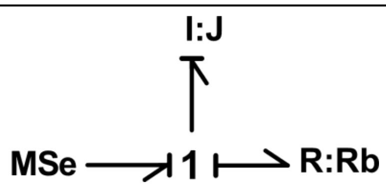

Fig. 6. Mecanical load and modulated torque source.

In Figure 6, the effort source represents the developed torque and the elements *J* and *Rb* represent the moment of inertia of the load and rotatory friction, respectively.

The presented subsystems allow the modeling of the linear part of the system. For the non-linear part, the complementarity framework is used.

In the work of [7], a representation of the complementarity framework is proposed according to:

$$\dot{x} = A_c x + B_c \varphi + E_c u + g_c \quad (2.a)$$
$$\lambda = C_c x + D_c \varphi + F_c u + h_c \quad (2.b)$$

$$\varphi = A_s \lambda + B_s z + E_s u_s + g_s \quad (3.a)$$
$$w = C_s \lambda + D_s z + F_s u_s + h_s \quad (3.b)$$

The matrices in Equation (2) are obtained from the topology of the system. In the work of [13] the methodology for obtaining these matrices from a Bond Graph is presented.

The matrices in Equation (3) are entirely dependent on the complementarity representation of the non linear devices. In this paper, the following relationships are used for the switches. For resistance causality, the relations are as follows:

$$\varphi_S = r_0\lambda_S - r_0 z_R \quad (4.a)$$
$$w_R = -r_0\lambda_S + (r_0 + r_1)z_R + V_{max}k + V_F \quad (4.b)$$

While for conductance causality the following is used:

$$\varphi_S = \left(\frac{1}{r_0}\right)\lambda_S + z_C \quad (5.a)$$
$$w_C = -\lambda + r_1 z_C + V_{max}k + V_F \quad (5.b)$$

The bond graph of the complete system is shown in Figure 7.

Because the model does not contain elements in derivative causality, the methodology requires the calculation of a junction structure matrix with the following form:

$$\begin{bmatrix} \dot{x} \\ D_{in} \\ \lambda \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \end{bmatrix} \begin{bmatrix} \zeta \\ D_{out} \\ u \\ \varphi \end{bmatrix} \quad (6)$$

provided. Instead, the submatrixes will be written, by accommodating them where space allows it.

In the bond graph of Figure 7, the key vectors are:

$$\dot{x} = [e_{11}\ e_{12}\ e_{13}\ e_{18}\ e_{19}\ e_{20}\ e_{26}\ e_{27}\ e_{28}\ f_{31}\ e_{34}]^T$$
$$\zeta = [f_{11}\ f_{12}\ f_{13}\ f_{18}\ f_{19}\ f_{20}\ f_{26}\ f_{27}\ f_{28}\ e_{31}\ f_{34}]^T$$
$$D_{in} = [f_8\ f_9\ f_{10}\ f_{14}\ f_{15}\ f_{16}\ f_{17}\ f_{21}\ f_{22}\ f_{23}\ f_{24}\ f_{25}\ e_{32}\ f_{35}]^T$$
$$D_{out} = [e_8\ e_{10}\ e_{14}\ e_{15}\ e_{16}\ e_{17}\ e_{21}\ e_{22}\ e_{23}\ e_{24}\ e_{25}\ f_{32}\ e_{35}]^T$$
$$\lambda = [f_1\ e_2\ f_3\ e_4\ f_5\ e_6\ f_{30}]^T$$
$$\varphi = [e_1\ f_2\ e_3\ f_4\ e_5\ f_6\ e_{30}]^T$$

After analyzing causality, the submatrices of the junction structure matrix can be formed.

The last row of matrix S_{11} is:

$$[0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ G_0\ 0]$$

The rest of the rows are zero. G_0 is a constant that allows calculating the torque from the RMS value of the voltage present in the resistance R_{rc}.

$$S_{13} = [1\quad 0\quad 0\quad 1\quad 0\quad 0\quad 1\quad 0\quad 0\quad 0\quad 0]^T$$

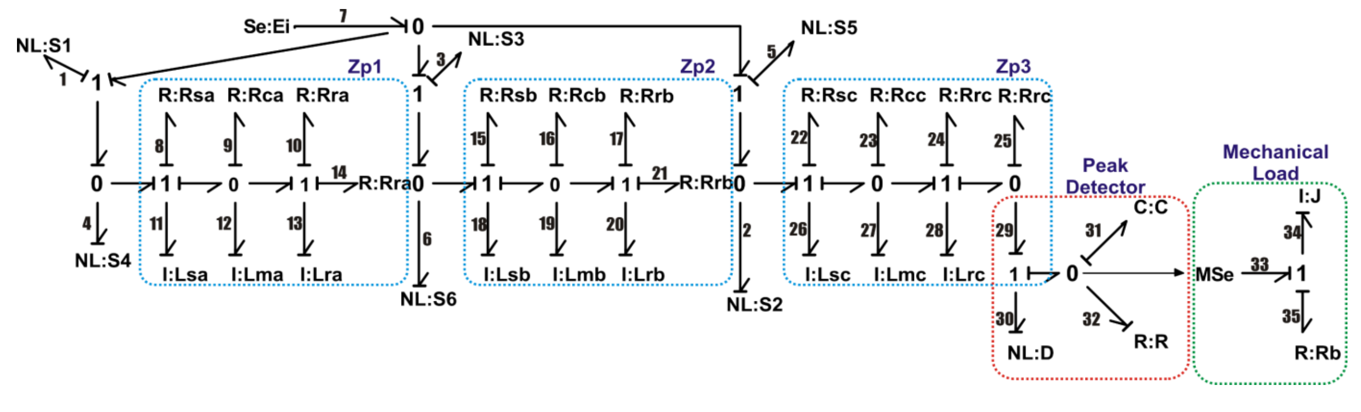

Fig. 7. Complete Bond Graph model.

Because the order of the system is high, the complete junction structure matrix cannot be written in the space

$$S_{12} = \begin{bmatrix}
-1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$S_{14} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_{22} = [0], S_{23} = [0]$$

$$S_{24} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S_{33} = [0\ 1\ 0\ 1\ 0\ 1\ 0]^T$$

$$S_{34} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The partial LCP can be written from Equations (3), (4) and (5) as follows:

$$A_s = diag\left(r_0, \frac{1}{r_0}, r_0, \frac{1}{r_0} r_0, \frac{1}{r_0}, \frac{1}{r_0}\right)$$

$$B_s = diag(-r_0, 1, -r_0, 1, -r_0, 1,1)$$

$$E_s = [0]$$

$$g_s = [0]$$

$$C_s = diag(-r_0, -1, -r_0, -1, -r_0, -1, -1)$$

$$D_s = diag(r_0 + r_1, r_1, r_0 + r_1, r_1 r_0 + r_1, r_1, r_1)$$

$$F_s = \begin{bmatrix} V_{max} & 0 & 0 & 0 & 0 & 0 \\ 0 & V_{max} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_{max} & 0 & 0 & 0 \\ 0 & 0 & 0 & V_{max} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_{max} & 0 \\ 0 & 0 & 0 & 0 & 0 & V_{max} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Parameters for the relations in the non linear elements are shown in table 2.

TABLE II. PARAMETERS FOR NON LINEAR ELEMENTS.

Parameter	*Value*
r_0	1×10^6
r_1	1×10^{-4}
V_{max}	1×10^4

The value of k in Equations (4) and (5) refers to a binary variable which allows controlling the state of the switch.

Matrices in Equation (6) are processed according to the methodology presented in [13] to form a system of the form of Equations (2-3). Also, the methodology is applied to get the final form of Equation (1) which can be simulated. A simulation is carried out using the parameters shown in table 3.

TABLE III. PARAMETERS FOR SIMULATION.

Parameter	*Value*
R_s	$3 \times 10^{-1}\ \{Ohm\}$
R_c	$1 \times 10^3\ \{Ohm\}$
R_r	$15 \times 10^{-2}\ \{Ohm\}$
L_s	$1.4 \times 10^{-3}\ \{Henry\}$
L_m	$40 \times 10^{-3}\ \{Henry\}$
L_r	$0.7 \times 10^{-3}\ \{Henry\}$
J	$0.4\ \{Kg * m^2\}$
R_b	1×10^{-4}

Parameter	Value
E_i	$220\ \{V\}$
R	$2 \times 10^3 \{Ohm\}$
C	$470\ \{F\}$

The simulation is started with the initial conditions in zero. At $t = 0$ a frequency of $90\ Hz$ is applied to the converter. Then, at $t = 3.6\ s$ the frequency changes to $60\ Hz$. At $t = 7.1\ s$ and $t = 10.7\ s$ new frequency changes occur.

As expected, the machine runs at two different angular speeds, as can be seen in Figure 8.

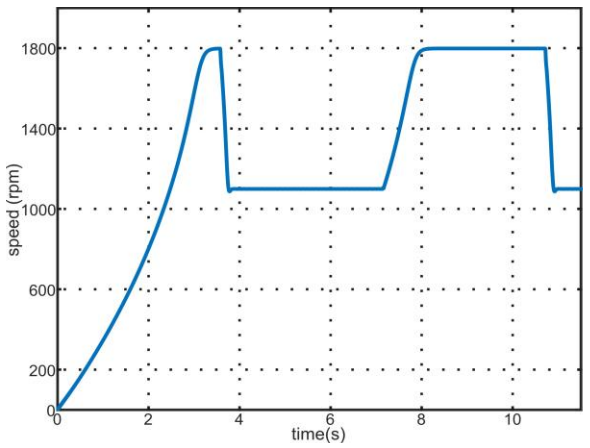

Fig. 8. Time evolution of angular velocity.

The acceleration ramp shows an adequate dynamic performance. The same happens in the moments of deceleration. The model allows observing the behavior of the drive in open loop with variable frequency inputs.

CONCLUSIONS

The mixture of methodologies formed by the complementarity framework and bond graph modeling has been applied to an induction electric machine. The proposed technique shows feasibility in modeling complete drives. Moreover, the interconnection of the drive with related systems, such as mechanical loads, controllers and electric power sources can be carried out at the model level with the presented approach. It could be noted in the paper that the simulation was carried out with a balanced load. However, the methodology used allows performing simulations with conditions different from the standard ones, for example, with unbalance in the load.

The presented methodology allows the modeling of drives of other electric machines. In future works, these drives will be studied, as well as the interconnection to related systems, such as renewable energy sources and specific mechanical loads.

ACKNOWLEDGMENT

N. V. V. acknowledges the support of the National Council of Science and Technology (CONACyT), the Potosino Council of Science and Technology (COPOCyT) and the Advanced Technology Center (CIATEQ).

REFERENCES

[1] S. J. Salon, “Finite Element Analysis of Electrical Machines,” Springer, 1995.

[2] P. C. Krause, O. Wasynczuk, and S. D. Sudhoff, “Analysis of Electric Machinery and Drive Systems,” 2002.

[3] F. Vasca, L. Iannelli, M. K. Camlibel, and R. Frasca, “A New Perspective for Modeling Power Electronics Converters: Complementarity Framework,” IEEE Transactions on Power Electronics, vol. 24, no. 2, pp. 456–468, Feb. 2009.

[4] W. Borutzky, “Bond Graph Methodology,” 2010.

[5] S. Junco and A. Donaire, “Bond Graph Modeling and Simulation of Electrical Machines,” Bond Graph Modelling of Engineering Systems, pp. 269–321, 2011.

[6] V. Sessa, L. Iannelli, and F. Vasca, “A Complementarity Model for Closed-Loop Power Converters,” IEEE Transactions on Power Electronics, vol. 29, no. 12, pp. 6821–6835, Dec. 2014.

[7] G. Angelone, F. Vasca, L. Iannelli, and K. Camlibel, “Dynamic and Steady-State Analysis of Switching Power Converters Made Easy: Complementarity Formalism,” Dynamics and Control of Switched Electronic Systems, pp. 217–243, 2012.

[8] M. K. Camlibel, W. P. M. H. Heemels, A. J. van der Schaft, and J. M. Schumacher, “Switched networks and complementarity,” IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, vol. 50, no. 8, pp. 1036–1046, Aug. 2003.

[9] Herceg, M. Kvasnica, C. N. Jones, and M. Morari, “Multi-Parametric Toolbox 3.0,” 2013 European Control Conference (ECC), Jul. 2013.

[10] MathWorks, Natick, Massachusetts, USA. MATLAB, http://www.mathworks.com (accessed 3 March 2018).

[11] H. Paynter, 1961. Analysis And Design Of Engineering Systems; Class Notes For Mit Course 2,751.

[12] C. Sueur and G. Dauphin-Tanguy, “Structural controllability/observability of linear systems represented by bond graphs,” Journal of the Franklin Institute, vol. 326, no. 6, pp. 869–883, Jan. 1989.

[13] N. Villa-Villaseñor and J. J. Rico-Melgoza, “Complementarity framework formulation from bond graphs to model a class of nonlinear systems and hybrid systems with fixed causality,” SIMULATION, p. 003754971775128, Jan. 2018.

[14] S. J. Chapman, *Electric machinery and power system fundamentals*. Vol. 3. New York: McGraw-Hill, 2002.