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An Output-Feedback PID-type Global Regulator for Robot Manipulators with Bounded Inputs ⋆

Arturo Zavala-Río * **Marco Mendoza** ** **Víctor Santibáñez** *** **Fernando Reyes** ****

* *Instituto Potosino de Investigación Científica y Tecnológica, Camino a la Presa San José 2055, Lomas 4a. Sección 78216, San Luis Potosí, Mexico (e-mail: azavala@ipicyt.edu.mx)*
** *Universidad Autónoma de San Luis Potosí, Av. Salvador Nava S/N, Zona Universitaria 78290, San Luis Potosí, Mexico (e-mail: marco.mendoza@uaslp.mx)*
*** *Instituto Tecnológico de la Laguna, Apdo. Postal 49, Adm. 1, 27001 Torreón, Mexico (e-mail: vsantiba@itlalaguna.edu.mx)*
**** *Benemérita Universidad Autónoma de Puebla, Boulevard Puebla 108, Bosques de San Sebastián 72310, Puebla, Mexico (e-mail: fernando.reyes@correo.buap.mx)*

Abstract: An output-feedback PID-type controller for the global stabilization of manipulators with bounded inputs is proposed. It guarantees the global regulation goal preventing input saturation while avoiding the exact knowledge of the system model and parameter values as well as the need for velocity measurements. Furthermore, it adopts an SPD-SI structure by keeping both the P and D actions together within a *generalized* saturation function while including an additional similar saturating integral action separately. So far, a formal analytical formulation for such a saturating PID-type control structure was not available under the absence of velocity measurements. Experimental results on a 2-degree-of-freedom direct-drive manipulator corroborate the efficiency of the proposed controller. *Copyright© 2015 IFAC*

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1. INTRODUCTION

Classical Proportional-Integral-Derivative (PID) control has been a usual practice for the stabilization of robot manipulators in actual applications (Rocco, 1996). This is mainly due to its effectiveness on the achievement of the regulation objective experienced through its simple linear structure which avoids involving accurate data from the system, such as parameter values or model expressions. Nevertheless, through such a simple linear structure, there is no analytical certainty that the experienced stability properties have a global character. For this reason, alternative nonlinear versions of the PID controller, aiming at guaranteeing global regulation, have been developed for instance in (Kelly, 1998; Santibáñez and Kelly, 1998). However, these algorithms implicitly assume that actuators can generate any torque value. Unfortunately, this is unrealistic in view of the saturation phenomenon commonly observed in real actuators. Furthermore, disregarding such natural constraints may lead to undesirable system behaviors and/or degraded closed-loop performances (Krikelis and Barkas, 1984). For this reason, bounded PID-type approaches have been further developed. For instance, semiglobal regulators with different saturating PID-type structures have been developed in (Alvarez-Ramirez *et al.*, 2003) and (Alvarez-Ramirez *et al.*, 2008). Through the singular perturbation methodology, these works showed the existence of an appropriate tuning mainly characterized by the requirement of small enough integral action gains and sufficiently high proportional and derivative ones. As far as the authors are aware, the first bounded PID-type controller for global regulation was presented in (Gorez, 1999); the algorithm permits to include or disregard velocities in the feedback. Nevertheless, the structure of the developed scheme is quite complex. Other works have focused on the solution of the global PID position stabilization problem for manipulators with bounded inputs through simpler structures, giving rise to the SP-SI-SD type algorithm developed in (Meza *et al.*, 2005) *via* passivity theory and later on in (Su *et al.*, 2010) through Lyapunov stability analysis, and to the SPD-SI type scheme presented in (Santibáñez et al., 2008). In particular, the work in (Su *et al.*, 2010) includes a velocity-free version of the proposed SP-SI-SD algorithm through the conventional (linear) *dirty derivative* (dynamic) operator.

The above cited bounded PID-type approaches give a solution to the formulated problem under input constraints and restricted data. In this direction, output-feedback schemes, like the velocity-free extensions of the algorithms

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presented in (Gorez, 1999) and (Su *et al.*, 2010), are particularly important since they achieve regulation not only without the need for the exact knowledge of the system structure and parameter values but also through the exclusive feedback of the position variables. This proves to be particularly useful when velocity measurements are unavailable which seems a common practical situation. However, it is not yet clear how can a bounded output-feedback version/extension of the SPD-SI structure could be analytically supported. A solution to such an open problem has not only been motivated by the implicated analytical challenge but also by the nice performance expectations generated by analog SPD-type structures in *gravity-compensation*-type state-feedback contexts (Zavala-Río and Santibáñez, 2006). Such a solution is developed in this work by contributing an output-feedback global regulator for robot manipulators with bounded inputs that adopts an SPD-SI structure by keeping both the P and D actions together within a *generalized* saturation function while including an additional similar saturating integral action separately. Moreover, the proposed scheme permits the choice of the saturation functions and releases the control gains from saturation avoidance conditions. The global regulation objective is guaranteed —avoiding input saturation— considerably reducing the system data involved in the feedback by releasing this not only from exact knowledge of the system model and parameter values but also from velocity measurements. Experimental tests on a 2-degree-of-freedom (DOF) direct-drive manipulator corroborate the contributed result.

2. PRELIMINARIES

Let $X \in \mathbb{R}^{m\times n}$ and $y \in \mathbb{R}^n$. Throughout this paper, X_{ij} represents the element of X at its i^{th} row and j^{th} column, and y_i denotes the i^{th} element of y. 0_n stands for the origin of $\mathbb{R}^n$ and I_n represents the $n \times n$ identity matrix. $\|\cdot\|$ denotes the standard Euclidean norm for vectors, *i.e.* $\|y\| = \sqrt{\sum_{i=1}^n y_i^2}$, and induced norm for matrices, *i.e.* $\|X\| = \sqrt{\lambda_{\max}\{X^T X\}}$ where $\lambda_{\max}\{X^T X\}$ represents the maximum eigenvalue of $X^T X$. For a continuous scalar function $\psi : \mathbb{R} \mapsto \mathbb{R}$, ψ' denotes its derivative, when differentiable, $D^+\psi$ its upper right-hand (Dini) derivative, *i.e.* $D^+\psi(\varsigma) = \limsup_{h\to 0^+} \frac{\psi(\varsigma+h)-\psi(\varsigma)}{h}$, with $D^+\psi = \psi'$ at points of differentiability (Khalil, 2002, Appendix C.2), and ψ^{-1} its inverse, when invertible.

Consider the n-DOF serial rigid manipulator dynamics with viscous friction (Kelly *et al.*, 2005)

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau \tag{1}$$

where q, $\dot{q}$, $\ddot{q} \in \mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity, and acceleration vectors, $H(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, and $C(q,\dot{q})\dot{q}$, $F\dot{q}$, $g(q)$, $\tau \in \mathbb{R}^n$ are respectively the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with $F \in \mathbb{R}^{n\times n}$ being a positive definite constant diagonal matrix whose entries $f_i > 0$, $i = 1,\dots,n$, are the viscous friction coefficients, and $g(q) = \nabla\mathcal{U}(q)$, with $\mathcal{U}(q)$ being the gravitational potential energy, or equivalently

$$\mathcal{U}(q) = \mathcal{U}(q_0) + \int_{q_0}^{q} g^T(r)dr \tag{2a}$$

with

$$\int_{q_0}^{q} g^T(r)dr = \int_{q_{01}}^{q_1} g_1(r_1, q_{02},\dots,q_{0n})dr_1 + \cdots + \int_{q_{0n}}^{q_n} g_n(q_1,\dots,q_{n-1},r_n)dr_n \tag{2b}$$

for any [1] q, $q_0 \in \mathbb{R}^n$. Some well-known properties characterizing the terms of such a dynamical model are recalled here (Kelly *et al.*, 2005, Chap. 4). Subsequently, we denote $\dot{H}$ the rate of change of H, *i.e.*, $\dot{H} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n} : (q,\dot{q}) \mapsto \left[\frac{\partial H_{ij}}{\partial q}(q)\dot{q}\right]$.

Property 1. $H(q)$ is a continuously differentiable matrix function being positive definite, symmetric and bounded on $\mathbb{R}^n$, *i.e.* such that $\mu_m I_n \le H(q) \le \mu_M I_n$, $\forall q \in \mathbb{R}^n$, for some constants $\mu_M \ge \mu_m > 0$.

Property 2. The Coriolis matrix $C(q,\dot{q})$ satisfies:

2.1. $\|C(q,\dot{q})\| \le k_C\|\dot{q}\|$, $\forall(q,\dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, for some constant $k_C \ge 0$;
2.2. for all $(q,\dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, $\dot{q}^T\left[\frac{1}{2}\dot{H}(q,\dot{q}) - C(q,\dot{q})\right]\dot{q} = 0$ and actually $\dot{H}(q,\dot{q}) = C(q,\dot{q}) + C^T(q,\dot{q})$.

Property 3. The viscous friction coefficient matrix satisfies $f_m\|\dot{q}\|^2 \le \dot{q}^T F\dot{q} \le f_M\|\dot{q}\|^2$, $\forall\dot{q} \in \mathbb{R}^n$, where $0 < f_m \triangleq \min_i\{f_i\} \le \max_i\{f_i\} \triangleq f_M$.

Property 4. The gravity force term $g(q)$ is a continuously differentiable bounded vector function with bounded Jacobian matrix [2] $\frac{\partial g}{\partial q}$. Equivalently, every element of the gravity force vector, $g_i(q)$, $i = 1,\dots,n$, satisfies:

4.1. $|g_i(q)| \le B_{gi}$, $\forall q \in \mathbb{R}^n$, for some constant $B_{gi} > 0$;
4.2. $\frac{\partial g_i}{\partial q_j}$, $j = 1,\dots,n$, exist and are continuous and such that $\left|\frac{\partial g_i}{\partial q_j}(q)\right| \le \left\|\frac{\partial g}{\partial q}(q)\right\| \le k_g$, $\forall q \in \mathbb{R}^n$, for some positive constant k_g, and consequently $|g_i(x) - g_i(y)| \le \|g(x) - g(y)\| \le k_g\|x - y\|$, $\forall x, y \in \mathbb{R}^n$.

Let us suppose that the absolute value of each input τ_i is constrained to be smaller than a given saturation bound $T_i > 0$, *i.e.*, $|\tau_i| \le T_i$, $i = 1,\dots,n$. More precisely, letting u_i represent the control variable (controller output) relative to the i^{th} degree of freedom, we have that

$$\tau_i = T_i \operatorname{sat}(u_i/T_i) \tag{3}$$

where $\operatorname{sat}(\cdot)$ is the standard saturation function, *i.e.* $\operatorname{sat}(\varsigma) = \operatorname{sign}(\varsigma)\min\{|\varsigma|, 1\}$.

From Eqs. (1),(3), one sees that $T_i \ge B_{gi}$ (see Property 4.1), $\forall i \in \{1,\dots,n\}$, is a necessary condition for the robot to be stabilizable at any desired equilibrium configuration $q_d \in \mathbb{R}^n$. This important fact is integrated to the analytical framework of the present work as follows.

Assumption 1. $T_i > \alpha B_{gi}$, $i = 1,\dots,n$, for some $\alpha \ge 1$.

Functions fitting the following definition will be involved.

[1] Since $g(q)$ is the gradient of the gravitational potential energy $\mathcal{U}(q)$, a scalar function, then, for any q, $q_0 \in \mathbb{R}^n$, the inverse relation in (2a) is independent of the integration path (Khalil, 2002, p. 120). Eq. (2b) considers integration along the axes. This way, on every axis (*i.e.* at every integral in the right-hand side of (2b)), the corresponding coordinate varies (according to the specified integral limits) while the rest of the coordinates remain constant.

[2] Property 4 is satisfied *e.g.* by robots having only revolute joints (Kelly *et al.*, 2005, §4.3).

Definition 1. Given a positive constant M, a nondecreasing Lipschitz-continuous function $\sigma : \mathbb{R} \to \mathbb{R}$ is said to be a *generalized saturation* with bound M if

(a) $\varsigma\sigma(\varsigma) > 0$, $\forall\varsigma \neq 0$;
(b) $|\sigma(\varsigma)| \leq M$, $\forall\varsigma \in \mathbb{R}$.

If in addition

(c) $\sigma(\varsigma) = \varsigma$ when $|\varsigma| \leq L$,

for some positive constant $L \leq M$, σ is said to be a *linear saturation* for (L, M).

Lemma 1. Let $\sigma : \mathbb{R} \to \mathbb{R}$ be a generalized saturation with bound M and let k be a positive constant. Then

1. $\lim_{|\varsigma|\to\infty} D^+\sigma(\varsigma) = 0$;
2. $\exists\sigma'_M \in (0, \infty)$ such that $0 \leq D^+\sigma(\varsigma) \leq \sigma'_M$, $\forall\varsigma \in \mathbb{R}$;
3. $\varsigma[\sigma(\varsigma + \eta) - \sigma(\eta)] \geq 0$, $\forall\varsigma, \eta \in \mathbb{R}$;
4. $[\sigma(k\varsigma + \eta) - \sigma(\eta)]^2 \leq \sigma'_M k\varsigma[\sigma(k\varsigma + \eta) - \sigma(\eta)] \leq (\sigma'_M k\varsigma)^2$, $\forall\varsigma, \eta \in \mathbb{R}$;
5. $\frac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma'_M\varsigma^2}{2}$, $\forall\varsigma \in \mathbb{R}$;
6. $\int_0^\varsigma \sigma(kr)dr > 0$, $\forall\varsigma \neq 0$;
7. $\int_0^\varsigma \sigma(kr)dr \to \infty$ as $|\varsigma| \to \infty$;
8. With σ strictly increasing, for any constant a, $\bar{\sigma}(\varsigma) = \sigma(\varsigma + a) - \sigma(a)$ is a strictly increasing generalized saturation function with bound $\bar{M} = M + |\sigma(a)|$.

Proof. Items 1, 2, 5–8 are proven in (López-Araujo *et al.*, 2013). As for items 3 and 4, see Appendix A. □

3. THE PROPOSED CONTROL SCHEME

The proposed control law is defined as

$$u(q, \vartheta, \phi) = -s_P(K_P\bar{q} + K_D\vartheta) + s_I(K_I\phi) \tag{4}$$

where $\bar{q} = q - q_d$, for any constant desired equilibrium position vector $q_d \in \mathbb{R}^n$; $\phi, \vartheta \in \mathbb{R}^n$ are the output vector variables of the integral-action dynamics, defined as[3]

$$\dot{\phi}_c = -\varepsilon K_P^{-1}s_P(K_P\bar{q}) \tag{5a}$$

$$\phi = -\bar{q} + \phi_c \tag{5b}$$

and the velocity estimation subsystem, defined as

$$\dot{\vartheta}_c = -A[\vartheta_c + B\bar{q}] \tag{6a}$$

$$\vartheta = \vartheta_c + B\bar{q} \tag{6b}$$

respectively; for any $x \in \mathbb{R}^n$, $s_P(x) = \big(\sigma_{P1}(x_1), \ldots, \sigma_{Pn}(x_n)\big)^T$ and $s_I(x) = \big(\sigma_{I1}(x_1), \ldots, \sigma_{In}(x_n)\big)^T$, with $\sigma_{Pi}(\cdot)$, $i = 1, \ldots, n$, being *linear saturation functions* for (L_{Pi}, M_{Pi}) and $\sigma_{Ii}(\cdot)$, $i = 1, \ldots, n$, being *strictly increasing generalized saturation functions* with bounds M_{Ii}, such that

$$L_{Pi} > 2B_{gi} \tag{7a}$$

$$M_{Ii} > B_{gi} \tag{7b}$$

$$M_{Pi} + M_{Ii} \leq T_i \tag{7c}$$

$i = 1, \ldots, n$; $K_P = \text{diag}[k_{P1}, \ldots, k_{Pn}]$, $K_D = \text{diag}[k_{D1}, \ldots, k_{Dn}]$, $K_I = \text{diag}[k_{I1}, \ldots, k_{In}]$, $A = \text{diag}[a_1, \ldots, a_n]$ and $B = \text{diag}[b_1, \ldots, b_n]$, with $k_{Ii} > 0$, $\forall i = 1, \ldots, n$, and the rest of the control gains being positive constants such that

$$k_{Pm} \triangleq \min_i\{k_{Pi}\} > k_g \tag{8a}$$

(see Property 4.2) and

$$\beta_d \triangleq \min_i\left\{\frac{a_i}{b_i}\right\} > \frac{\kappa}{2f_m} \tag{8b}$$

with $\kappa \triangleq \max_i\{\sigma'_{PiM}k_{Di}\}$, σ'_{PiM} being the positive bound of $D^+\sigma_{Pi}(\cdot)$, in accordance to item 2 of Lemma 1; and ε (in Eq. (5a)) is a positive constant satisfying

$$\varepsilon < \varepsilon_M \triangleq \min\{\varepsilon_1, \varepsilon_2, \varepsilon_3\} \tag{9}$$

where

$$\varepsilon_1 \triangleq \sqrt{\frac{\beta_0\beta_P\mu_m}{\mu_M^2}} \quad , \quad \varepsilon_2 \triangleq \frac{\beta_0\beta_d k_{Pm}}{\kappa}$$

$$\varepsilon_3 \triangleq \frac{f_m - \frac{\kappa}{2\beta_d}}{\beta_M + \frac{f_M^2}{\beta_0 k_{Pm}}} < \frac{f_m - \frac{\kappa}{2\beta_d}}{\beta_M} \triangleq \varepsilon_4$$

(observe that by inequality (8b): $f_m - \frac{\kappa}{2\beta_d} > 0$), with $\beta_0 \triangleq 1 - \max\left\{\frac{k_g}{k_{Pm}}, \max_i\left\{\frac{2B_{gi}}{L_{Pi}}\right\}\right\}$ (observe that by inequalities (8a) and (7a): $0 < \beta_0 < 1$), $\beta_M \triangleq k_C B_P + \mu_M\sigma'_{PM}$, $\beta_P \triangleq \min_i\left\{\frac{k_{Pi}}{\sigma'_{PiM}}\right\}$, $B_P \triangleq \sqrt{\sum_{i=1}^n\left(\frac{M_{Pi}}{k_{Pi}}\right)^2}$, $\sigma'_{PM} \triangleq \max_i\{\sigma'_{PiM}\}$, and μ_m, μ_M, k_C, f_m, f_M, B_{gi} and k_g as defined through Properties 1–4.

Remark 1. Note that $\dot{q}$ is not involved in any of the expressions in Eqs. (4)–(6). In fact, $\dot{q}$ is estimated *on-line* through the auxiliary subsystem in Eqs. (6), driven by $\bar{q}$ as input variable. Its output variable ϑ gives the estimated vector value of $\dot{q}$. In fact, the auxiliary subsystem in Eqs. (6) gives rise to the so-called *dirty derivative* of $\bar{q}$. This is the derivative of $\bar{q}$ (or the velocity vector $\dot{q}$) with every of its components going through a first-order low pass filter. This is commonly done in practice to bound the high-frequency gains, giving rise to a causal (approximated) derivative operator. ◁

Remark 2. Let us note that inequalities (7) (stating conditions on the saturation function parameters) require the satisfaction of Assumption 1 with $\alpha = 3$. A similar condition on the control input bounds has been required by other approaches where input constraints have been considered (Colbaugh *et al.*, 1997). In saturating PID-type schemes from previous references, a similar or analog condition on the control input bounds remains implicit by requiring corresponding parameters to be high enough to satisfy conditions coming from the stability analysis and simultaneously low enough to fulfill the *input-saturation-avoidance* inequalities. ◁

4. CLOSED-LOOP ANALYSIS

Consider system (1),(3) taking $u = u(q, \vartheta, \phi)$ as defined through Eqs. (4)–(6). Define the variable transformation

$$\begin{pmatrix}\bar{q}\\ \bar{\vartheta}\\ \bar{\phi}\end{pmatrix} = \begin{pmatrix}q - q_d\\ \vartheta_c + B(q - q_d)\\ -\bar{q} + \phi_c - \phi^*\end{pmatrix} \tag{10}$$

with $\phi^* = (\phi_1^*, \ldots, \phi_n^*)^T$ such that $s_I(K_I\phi^*) = g(q_d)$, or equivalently $\phi_i^* = \sigma_{Ii}^{-1}\big(g_i(q_d)\big)/k_{Ii}$, $i = 1, \ldots, n$ (notice

[3] Under time parametrization of the system trajectories, the integral-action dynamics in Eqs. (5) adopts the (equivalent) integral-equation form $\phi(t) = \phi(0) + \bar{q}(0) - \bar{q}(t) - \int_0^t \varepsilon K_P^{-1}s_P\big(K_P\bar{q}(\varsigma)\big)d\varsigma$, for any initial vector values $\phi(0), \bar{q}(0) \in \mathbb{R}^n$.

that their strictly increasing character renders the generalized saturation functions σ_{Ii} invertible). Observe that, for every $i \in \{1, \ldots, n\}$ and all $(q, \vartheta, \phi) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$, by (7c) and the strictly increasing character of σ_{Ii}, we have that $|u_i(q, \vartheta, \phi)| \leq |\sigma_{Pi}(k_{Pi}\bar{q}_i + k_{Di}\vartheta_i)| + |\sigma_{Ii}(k_{Ii}\phi)| < M_{Pi} + M_{Ii} \leq T_i$. From this and (3), one sees that

$$T_i > |u_i(\bar{q} + q_d, \vartheta, \bar{\phi} + \phi^*)| = |u_i| = |\tau_i| \qquad i = 1, \ldots, n \qquad \forall (\bar{q}, \vartheta, \bar{\phi}) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \quad (11)$$

Hence, under the consideration of the variable change (10), the closed-loop dynamics adopts the (equivalent) form

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = -s_P(K_P\bar{q} + K_D\vartheta) + \bar{s}_I(\bar{\phi}) + g(q_d) \quad (12a)$$

$$\dot{\vartheta} = -A\vartheta + B\dot{q} \quad (12b)$$

$$\dot{\bar{\phi}} = -\dot{q} - \varepsilon K_P^{-1} s_P(K_P\bar{q}) \quad (12c)$$

where $\bar{s}_I(\bar{\phi}) = s_I(K_I\bar{\phi} + K_I\phi^*) - s_I(K_I\phi^*)$. Observe that, by item 8 of Lemma 1, the elements of $\bar{s}_I(\bar{\phi})$, *i.e.* $\bar{\sigma}_{Ii}(\bar{\phi}_i) = \sigma_{Ii}(k_{Ii}\bar{\phi}_i + k_{Ii}\phi_i^*) - \sigma_{Ii}(k_{Ii}\phi_i^*)$, $i = 1, \ldots, n$, turn out to be strictly increasing generalized saturations.

Proposition 1. Consider the closed-loop system in Eqs. (12), under the satisfaction of Assumption 1 with $\alpha = 3$ and inequalities (7). Thus, for any positive definite diagonal matrices K_I, K_P, K_D, A and B such that inequalities (8) are satisfied, and any ε fulfilling inequality (9), global asymptotic stability of the closed-loop trivial solution $(\bar{q}, \vartheta, \bar{\phi})(t) \equiv (0_n, 0_n, 0_n)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$.

Proof. By (11), one sees that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $\forall t \geq 0$. This proves that, under the proposed scheme, the input saturation values, T_i, are never reached. Now, in order to carry out the stability analysis, a scalar function $V(\bar{q}, \dot{q}, \vartheta, \bar{\phi})$ is defined as follows[4]

$$V = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}H(q)\dot{q} + \mathcal{U}(q) - \mathcal{U}(q_d) - g^T(q_d)\bar{q} + \int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr + \frac{\kappa}{2}\vartheta^T B^{-1}\vartheta$$

where $\int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr = \sum_{i=1}^n \int_0^{\bar{\phi}_i} \bar{\sigma}_{Ii}(r_i)dr_i$, $\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i$, and recall that $\mathcal{U}$ represents the gravitational potential energy. Note, by recalling Eqs. (2), that the defined scalar function can be rewritten as

$$V = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \varepsilon s_P^T(K_P\bar{q})K_P^{-1}H(q)\dot{q} + \mathcal{U}^c_{\gamma_0}(\bar{q}) + \gamma_0\int_{0_n}^{\bar{q}} s_P^T(K_P r)dr + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr + \frac{\kappa}{2}\vartheta^T B^{-1}\vartheta$$

where

$$\mathcal{U}^c_{\gamma_0}(\bar{q}) = \int_{0_n}^{\bar{q}} \left[g(r + q_d) - g(q_d) + (1 - \gamma_0)s_P(K_P r)\right]^T dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \left[\bar{g}_i(r_i) - g_i(q_d) + (1 - \gamma_0)\sigma_{Pi}(k_{Pi}r_i)\right]dr_i$$

with

$$\begin{aligned} \bar{g}_1(r_1) &= g_1(r_1 + q_{d1}, q_{d2}, \ldots, q_{dn}) \\ \bar{g}_2(r_2) &= g_2(q_1, r_2 + q_{d2}, q_{d3}, \ldots, q_{dn}) \\ &\vdots \\ \bar{g}_n(r_n) &= g_n(q_1, q_2, \ldots, q_{n-1}, r_n + q_{dn}) \end{aligned}$$

and γ_0 is a constant satisfying

$$\beta_0\frac{\varepsilon^2}{\varepsilon_1^2} < \gamma_0 < \beta_0 \quad (13)$$

(observe, from inequality (9) and the definition of β_0, that $0 < \beta_0\varepsilon^2/\varepsilon_1^2 < \beta_0 < 1$). Under this consideration, $\mathcal{U}^c_{\gamma_0}(\bar{q})$ turns out to be lower-bounded by

$$W_{10}(\bar{q}) = \sum_{i=1}^n w_i^{10}(\bar{q}_i) \quad (14a)$$

where

$$w_i^{10}(\bar{q}_i) \triangleq \begin{cases} \dfrac{k_{li}}{2}\bar{q}_i^2 & \text{if } |\bar{q}_i| \leq \bar{q}_i^* \\ k_{li}\bar{q}_i^*\left(|\bar{q}_i| - \dfrac{\bar{q}_i^*}{2}\right) & \text{if } |\bar{q}_i| > \bar{q}_i^* \end{cases} \quad (14b)$$

with $0 < k_{li} \leq (1 - \gamma_0)k_{Pi} - k_g$ and $\bar{q}_i^* = [L_{Pi} - 2B_{gi}/(1 - \gamma_0)]/k_{Pi}$ (note that by inequality (13) and the definition of β_0: $0 < (1 - \gamma_0)k_{Pi} - k_g$ and $\bar{q}_i^* > 0$); this is proven in (Mendoza *et al.*, 2015, Appendix 2). From this, Property 1 and item 5 of Lemma 1, we have

$$\begin{aligned} V \geq\ & \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| + W_{10}(\bar{q}) \\ & + \gamma_0\sum_{i=1}^n \frac{\sigma_{Pi}^2(k_{Pi}\bar{q}_i)}{2k_{Pi}\sigma'_{PiM}} + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr + \frac{\kappa}{2}\vartheta^T B^{-1}\vartheta \\ \geq\ & W_{11}(\bar{q}, \dot{q}) + W_{10}(\bar{q}) + \int_{0_n}^{\bar{\phi}} \bar{s}_I^T(r)dr + \frac{\kappa}{2}\vartheta^T B^{-1}\vartheta \quad (15) \end{aligned}$$

where

$$\begin{aligned} W_{11}(\bar{q}, \dot{q}) &= \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| + \frac{\gamma_0\beta_P}{2}\|K_P^{-1}s_P(K_P\bar{q})\|^2 \\ &= \frac{1}{2}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}^T Q^{11}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix} \end{aligned}$$

with $Q^{11} = \begin{pmatrix}\gamma_0\beta_P & -\varepsilon\mu_M \\ -\varepsilon\mu_M & \mu_m\end{pmatrix}$. By inequality (13), $W_{11}(\bar{q}, \dot{q})$ is positive definite (since with $\varepsilon < \varepsilon_M \leq \varepsilon_1$, in accordance to inequality (9), any γ_0 satisfying (13) renders Q^{11} positive definite) and note that $W_{11}(0_n, \dot{q}) \to \infty$ as $\|\dot{q}\| \to \infty$ while, from Eqs. (14) and items 6-7 of Lemma 1, it is clear that W_{10} and the integral term in the right-hand side of (15) are radially unbounded positive definite functions of $\bar{q}$ and $\bar{\phi}$ respectively. Thus, $V(\bar{q}, \dot{q}, \vartheta, \bar{\phi})$ is concluded to be positive definite and radially unbounded. Its upper right-hand derivative along the system trajectories, $\dot{V} = D^+V$ (Michel *et al.*, 2008, §6.1A), is given by

$$\begin{aligned} \dot{V} =\ & -\dot{q}^T F\dot{q} - \dot{q}^T s_d(\bar{q}, \varphi) - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}F\dot{q} \\ & - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}\left[g(q) + s_P(K_P\bar{q}) - g(q_d)\right] \\ & - \varepsilon s_P^T(K_P\bar{q})K_P^{-1}s_d(\bar{q}, \vartheta) + \varepsilon\dot{q}^T s'_P(K_P\bar{q})H(q)\dot{q} \\ & + \varepsilon\dot{q}^T C(q, \dot{q})K_P^{-1}s_P(K_P\bar{q}) - \kappa\vartheta^T B^{-1}A\vartheta + \kappa\vartheta^T\dot{q} \end{aligned}$$

where $H(q)\ddot{q}$ and $\dot{\bar{\phi}}$ have been replaced by their equivalent expressions from the closed-loop dynamics in Eqs. (12), Property 2.2 has been used and

[4] Note that, in the error variable space, $q = \bar{q} + q_d$. Consequently $H(q) = H(\bar{q} + q_d)$, $C(q, \dot{q}) = C(\bar{q} + q_d, \dot{q})$ and $g(q) = g(\bar{q} + q_d)$. However, for the sake of simplicity, $H(q)$, $C(q, \dot{q})$, and $g(q)$ are used throughout the paper. Moreover, the arguments of V and its derivative along the system trajectories, $\dot{V}$, will be dropped throughout the developments.

$$s'_P(K_P\bar q) \triangleq \operatorname{diag}[D^+\sigma_{P1}(k_{P1}\bar q_1), \ldots, D^+\sigma_{Pn}(k_{Pn}\bar q_n)]$$

$$s_d(\bar q, \vartheta) \triangleq s_P(K_P\bar q + K_D\vartheta) - s_P(K_P\bar q)$$

The resulting expression can be rewritten as

$$\begin{aligned}\dot V = {} & \dot q^T[\kappa\vartheta - s_d(\bar q,\vartheta)] - \varepsilon s_P^T(K_P\bar q)K_P^{-1}F\dot q - \dot q^T F\dot q \\ & - \varepsilon\gamma_1 s_P^T(K_P\bar q)K_P^{-1}K_PK_P^{-1}s_P(K_P\bar q) - \varepsilon\mathcal{W}_{\gamma_1}(\bar q) \\ & - \varepsilon s_P^T(K_P\bar q)K_P^{-1}s_d(\bar q,\vartheta) + \varepsilon\dot q^T s'_P(K_P\bar q)H(q)\dot q \\ & + \varepsilon\dot q^T C(q,\dot q)K_P^{-1}s_P(K_P\bar q) - \kappa\vartheta^T B^{-1}A\vartheta\end{aligned}$$

where

$$\begin{aligned}&\mathcal{W}_{\gamma_1}(\bar q) \\ &\quad = s_P^T(K_P\bar q)K_P^{-1}\big[(1-\gamma_1)s_P(K_P\bar q) + g(q) - g(q_d)\big] \\ &\quad = \sum_{i=1}^n \left[\frac{(1-\gamma_1)}{k_{Pi}}\sigma_{Pi}^2(k_{Pi}\bar q_i) + \frac{\sigma_{Pi}(k_{Pi}\bar q_i)}{k_{Pi}}[g_i(q) - g_i(q_d)]\right]\end{aligned}$$

and γ_1 is a constant satisfying

$$\beta_0\left[\max\left\{\frac{\varepsilon}{\varepsilon_2}\ ,\ \frac{\varepsilon}{\varepsilon_3}\left(\frac{\varepsilon_4-\varepsilon_3}{\varepsilon_4-\varepsilon}\right)\right\}\right] < \gamma_1 < \beta_0 \qquad (16)$$

(from inequality (9) and the definition of β_0, one verifies, after simple developments, that $0 < \beta_0\big[\max\big\{\varepsilon/\varepsilon_2\ ,\ \varepsilon(\varepsilon_4 - \varepsilon_3)/[\varepsilon_3(\varepsilon_4-\varepsilon)]\big\}\big] < \beta_0 < 1$). Under this consideration, $\mathcal{W}_{\gamma_1}(\bar q)$ turns out to be lower-bounded by

$$W_{20}(\bar q) = \sum_{i=1}^n w_i^{20}(\bar q_i) \qquad (17a)$$

where

$$w_i^{20}(\bar q_i) = \begin{cases} c_i\bar q_i^2 & \text{if } |\bar q_i| \le L_{Pi}/k_{Pi} \\ \varpi_i(\bar q_i) & \text{if } |\bar q_i| > L_{Pi}/k_{Pi}\end{cases} \qquad (17b)$$

with $\varpi_i(\bar q_i) = \frac{d_i}{k_{Pi}}\big(|\sigma_{Pi}(k_{Pi}\bar q_i)| - L_{Pi}\big) + c_i\Big(\frac{L_{Pi}}{k_{Pi}}\Big)^2$, $d_i = (1-\gamma_1)L_{Pi} - 2B_{gi}$, $c_i = \min\Big\{h, \frac{d_ik_{Pi}}{L_{Pi}}\Big\}$ and $h = (1-\gamma_1)k_{Pm} - k_g$ (notice, from inequality (16) and the definition of β_0, that $d_i > 0$ and $h > 0$, hence $c_i > 0$); this is proven in (Mendoza *et al.*, 2015, Appendix 3). From this, Properties 1, 2.1 and 3, items 2 of Lemma 1 and (b) of Definition 1, and the positive definite character of K_P, we have that

$$\begin{aligned}\dot V \le {} & \|\dot q\|\|\kappa\vartheta - s_d(\bar q,\vartheta)\| + \varepsilon f_M\|K_P^{-1}s_P(K_P\bar q)\|\|\dot q\| \\ & - f_m\|\dot q\|^2 - \varepsilon\gamma_1k_{Pm}\|K_P^{-1}s_P(K_P\bar q)\|^2 - \varepsilon W_{20}(\bar q) \\ & + \varepsilon\|K_P^{-1}s_P(K_P\bar q)\|\|s_d(\bar q,\vartheta)\| + \varepsilon\mu_M\sigma'_{PM}\|\dot q\|^2 \\ & + \varepsilon k_CB_P\|\dot q\|^2 - \kappa\beta_d\|\vartheta\|^2\end{aligned}$$

Let us note that by item 4 of Lemma 1, we have that $\|\kappa\vartheta - s_d(\bar q,\vartheta)\|^2 = \big[\kappa\vartheta - s_d(\bar q,\vartheta)\big]^T\big[\kappa\vartheta - s_d(\bar q,\vartheta)\big] = \kappa^2\vartheta^T\vartheta - 2\kappa\vartheta^Ts_d(\bar q,\vartheta) + s_d^T(\bar q,\vartheta)s_d(\bar q,\vartheta) \le \kappa^2\|\vartheta\|^2 - \|s_d(\bar q,\vartheta)\|^2 \le \kappa^2\|\vartheta\|^2$, *i.e.* $\|\kappa\vartheta - s_d(\bar q,\vartheta)\| \le \kappa\|\vartheta\|, \forall(\bar q,\vartheta,\phi)\in\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n$. From this and item 4 of Lemma 1, we get

$$\begin{aligned}\dot V \le {} & \kappa\|\dot q\|\|\vartheta\| + \varepsilon f_M\|K_P^{-1}s_P(K_P\bar q)\|\|\dot q\| + \varepsilon\mu_M\sigma'_{PM}\|\dot q\|^2 \\ & - \varepsilon\gamma_1k_{Pm}\|K_P^{-1}s_P(K_P\bar q)\|^2 + \varepsilon\kappa\|K_P^{-1}s_P(K_P\bar q)\|\|\vartheta\| \\ & - f_m\|\dot q\|^2 + \varepsilon k_CB_P\|\dot q\|^2 - \kappa\beta_d\|\vartheta\|^2 - \varepsilon W_{20}(\bar q) \\ \le {} & -\varepsilon W_{21}(\bar q,\vartheta) - \varepsilon W_{22}(\bar q,\dot q,\vartheta) - \varepsilon W_{20}(\bar q)\end{aligned}$$

where (arguments are dropped for simplicity)

$$\begin{aligned}W_{21} = {} & \frac{\gamma_1k_{Pm}}{2}\|K_P^{-1}s_P(K_P\bar q)\|^2 + \frac{\kappa\beta_d}{2\varepsilon}\|\vartheta\|^2 \\ & - \kappa\|K_P^{-1}s_P(K_P\bar q)\|\|\vartheta\| \\ = {} & \begin{pmatrix}\|K_P^{-1}s_P(K_P\bar q)\| \\ \|\dot q\|\end{pmatrix}^T Q^{21}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar q)\| \\ \|\dot q\|\end{pmatrix}\end{aligned}$$

$$Q^{21} = \begin{pmatrix}\gamma_1k_{Pm} & -\kappa \\ -\kappa & \dfrac{\kappa\beta_d}{\varepsilon}\end{pmatrix} = \begin{pmatrix}\gamma_1k_{Pm} & -\kappa \\ -\kappa & \dfrac{\kappa^2\varepsilon_2}{\beta_0k_{Pm}\varepsilon}\end{pmatrix}$$

and

$$\begin{aligned}W_{22} = {} & \frac{\varepsilon\gamma_1k_{Pm}}{2}\|K_P^{-1}s_P(K_P\bar q)\|^2 + \frac{\kappa\beta_d}{2}\|\vartheta\|^2 \\ & - \varepsilon f_M\|K_P^{-1}s_P(K_P\bar q)\|\|\dot q\| - \kappa\|\dot q\|\|\vartheta\| \\ & + (f_m - \varepsilon\beta_M)\|\dot q\|^2 \\ = {} & \frac{1}{2}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar q)\| \\ \|\dot q\| \\ \|\vartheta\|\end{pmatrix}^T Q^{22}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar q)\| \\ \|\dot q\| \\ \|\vartheta\|\end{pmatrix}\end{aligned}$$

$$\begin{aligned}Q^{22} = {} & \begin{pmatrix}\varepsilon\gamma_1k_{Pm} & -\varepsilon f_M & 0 \\ -\varepsilon f_M & 2(f_m - \varepsilon\beta_M) & -\kappa \\ 0 & -\kappa & \kappa\beta_d\end{pmatrix} \\ = {} & \begin{pmatrix}\varepsilon\gamma_1k_{Pm} & Q_{12}^{22} & 0 \\ Q_{12}^{22} & 2\beta_M(\varepsilon_4 - \varepsilon) + \dfrac{\kappa}{\beta_d} & -\kappa \\ 0 & -\kappa & \kappa\beta_d\end{pmatrix}\end{aligned}$$

with $Q_{12}^{22} = -\varepsilon\sqrt{2\beta_0\beta_Mk_{Pm}\Big(\frac{\varepsilon_4-\varepsilon_3}{\varepsilon_3}\Big)}$. By inequality (16), $W_{21}(\bar q,\vartheta)$ and $W_{22}(\bar q,\dot q,\vartheta)$ are positive definite (since with $\varepsilon < \varepsilon_M \le \min\{\varepsilon_2,\varepsilon_3\} < \varepsilon_4$, in accordance to inequality (9), any γ_1 satisfying (16) renders Q^{21} and Q^{22} positive definite), while from Eqs. (17), it is clear that W_{20} is a positive definite function of $\bar q$. Hence, $\dot V(\bar q,\dot q,\vartheta,\bar\phi) \le 0$ with $\dot V(\bar q,\dot q,\vartheta,\bar\phi) = 0 \iff (\bar q,\dot q,\vartheta) = (0_n,0_n,0_n)$. Further, from the closed-loop dynamics in Eqs. (12), we see that $\bar q(t)\equiv\dot q(t)\equiv\vartheta(t)\equiv 0_n \implies \ddot q(t)\equiv 0_n \implies \bar s_I(\bar\phi(t))\equiv 0_n \implies \bar\phi(t)\equiv 0_n$ (at any $(\bar q,\dot q,\vartheta,\bar\phi)$ on $Z = \big\{(w,x,y,z)\in\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n : w = x = y = 0_n\big\}$ with $\bar\phi\ne 0_n$, the resulting unbalanced force term $\bar s_I(\bar\phi)$ acts on the closed-loop dynamics forcing the system trajectories to leave Z). Therefore, by the invariance theory (Michel *et al.*, 2008, §7.2) —more precisely by (Michel *et al.*, 2008, Corollary 7.2.1)—, the closed-loop trivial solution $(\bar q,\vartheta,\bar\phi)(t)\equiv(0_n,0_n,0_n)$ is concluded to be globally asymptotically stable, which completes the proof. □

5. EXPERIMENTAL RESULTS

In order to corroborate the efficiency of the proposed output-feedback SPD-SI control scheme, real-time tests were implemented using a 2-DOF direct-drive robot manipulator. The experimental setup is a 2-revolute-joint robot arm located at the *Instituto Tecnológico de la Laguna*, Mexico, previously used in (López-Araujo *et al.*, 2013). The robot actuators are direct-drive brushless servomotors operated in torque mode, *i.e.* they act as torque sources and receive an analog voltage as a torque reference signal. Joint positions are obtained using incremental encoders on the motors. In order to get the encoder data and generate reference voltages, the robot includes a motion control board based on a DSP 32-bit floating point microprocessor. The control algorithm is executed at a 2.5 millisecond sampling period on a PC-host computer.

For the experimental manipulator, Properties 1–4 are satisfied with $\mu_m = 0.088$ kg·m^2, $\mu_M = 2.533$ kg·m^2, $k_C = 0.1455$ kg·m^2, $f_m = 0.175$ kg·m^2/s, $f_M = 2.288$ kg·m^2/s, $B_{g1} = 40.29$ Nm, $B_{g2} = 1.825$ Nm and $k_g = 40.373$ Nm/rad. The maximum allowed torques (input saturation bounds) are $T_1 = 150$ Nm and $T_2 = 15$ Nm for the first and second links respectively. From these data, one easily corroborates that Assumption 1 is fulfilled with $\alpha = 3$.

The saturation functions used for the implementation were $\sigma_{Pi}(\varsigma) = M_{Pi}\mathrm{sat}(\varsigma/M_{Pi})$ and

$$\sigma_{Ii}(\varsigma) = \begin{cases} \varsigma & \forall|\varsigma| \leq L_{Ii} \\ \rho(\varsigma; L_{Ii}, M_{Ii}) & \forall|\varsigma| > L_{Ii} \end{cases}$$

where $\rho(\varsigma; L, M) = \mathrm{sign}(\varsigma)L + (M - L)\tanh\left(\frac{\varsigma - \mathrm{sign}(\varsigma)L}{M-L}\right)$, $i = 1, 2$, for $0 < L < M$. Note that $\sigma'_{PiM} = \sigma'_{IiM} = 1$, $\forall i \in \{1, 2\}$. The saturation function parameters were selected in order to satisfy inequalities (7) as (all of them expressed in Nm): $M_{P1} = 81$, $M_{P2} = 7$, $M_{I1} = 48$, $M_{I2} = 5$ and $L_{Ii} = 0.9M_{Ii}$, $i = 1, 2$.

For comparison purposes, additional experimental tests were implemented using the output-feedback version of the bounded PID-type controller presented in (Su *et al.*, 2010) (choice made taking into account the analog nature of the compared algorithms: globally stabilizing *via* output feedback developed in a bounded-input context, and the recent appearance of (Su *et al.*, 2010)), *i.e.*

$$u = -K_P\mathrm{Tanh}(\bar{q}) - K_D\mathrm{Tanh}(\vartheta) - K_I\mathrm{Tanh}(\phi) \quad (18a)$$

$$\begin{aligned}\dot{\vartheta}_c &= -A[\vartheta_c + Bq] \\ \vartheta &= \vartheta_c + Bq\end{aligned} \quad (18b)$$

$$\begin{aligned}\dot{\phi}_c &= \mathrm{Tanh}(\bar{q}) \\ \phi &= \eta^2\bar{q} + \eta\phi_c\end{aligned} \quad (18c)$$

with η being a (sufficiently large) positive constant and $\mathrm{Tanh}(x) = \left(\tanh x_1, \ldots, \tanh x_n\right)^T$ for any $x \in \mathbb{R}^n$.[5] For the sake of simplicity, this algorithm is subsequently referred to as the S10 controller.

The experiments were run taking the desired joint positions as $q_d = \left(q_{d1}, q_{d2}\right)^T = \left(\pi/4, \pi/4\right)^T$ [rad]. The initial conditions were $q(0) = \dot{q}(0) = 0_2$ and, for the SPD-SI type algorithm proposed in this work, $\phi_c(0)$ was taken so as to have $\phi(0) = 0_2$, while $\phi_c(0) = 0_2$ was taken for the S10 controller in view of the way how it

[5] In place of Eqs. (18c), Su *et al.* (2010) define $\phi(t) = \eta^2\bar{q}(t) + \eta\int_0^t \mathrm{Tanh}\big(\bar{q}(\varsigma)\big)d\varsigma$, which imposes the auxiliary variable initial condition $\phi(0) = \eta^2\bar{q}(0)$ (or, equivalently, $\phi_c(0) = 0_n$ in the context of Eqs. (18c)). Instead, Eqs. (18c) —or their (equivalent) time representation $\phi(t) = \phi(0) + \eta^2\left[\bar{q}(t) - \bar{q}(0)\right] + \eta\int_0^t \mathrm{Tanh}\big(\bar{q}(\varsigma)\big)d\varsigma$— keeps the required auxiliary dynamics while permitting any initial condition for ϕ (or, equivalently, for ϕ_c in the context of Eqs. (18c)). This proves to be more appropriate in the global stabilization framework considered in (Su *et al.*, 2010) (and what is generally expected from an approach developed within such a framework).

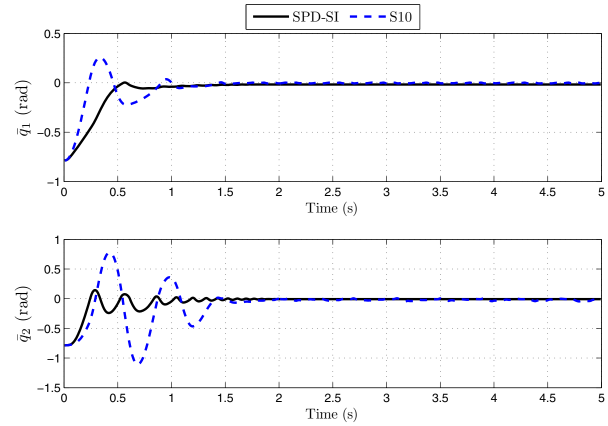

Fig. 1. Position errors

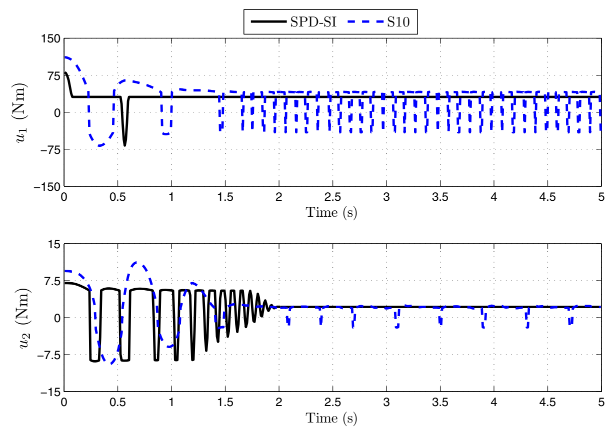

Fig. 2. Control signals

is presented in (Su *et al.*, 2010) (recall Footnote 5). The control parameters for the scheme proposed in this work were selected —taking into account the tuning conditions from inequalities (8) and (9)— so as to get fast responses. As for the S10 algorithm, the control parameters were tuned so as to get the best possible closed-loop responses while adhering to the saturation-avoidance inequalities and stability conditions (some of which had to be verified numerically) presented in (Su *et al.*, 2010). The resulting tuning values were: $K_P = \mathrm{diag}[6000, 500]$ Nm/rad, $K_D = \mathrm{diag}[2, 2]$ Nms/rad, $K_I = \mathrm{diag}[900, 1500]$ Nm/rad, $A = \mathrm{diag}[60, 60]$ s^{-1}, $B = \mathrm{diag}[5, 5]$ s^{-1} and $\varepsilon = 0.024$ s^{-1} for the proposed SPD-SI scheme, and $K_P = \mathrm{diag}[108, 11.5]$ Nm, $K_D = \mathrm{diag}[0.5, 0.1]$ Nm, $K_I = \mathrm{diag}[40.5, 1.9]$ Nm, $A = \mathrm{diag}[60, 40]$ s^{-1}, $B = \mathrm{diag}[70, 20]$ s^{-1} and $\eta = 170$ s/rad for the S10 controller.

Figs. 1 and 2 show the experimental results. Note that the proposed SPD-SI controller achieved the regulation objective —avoiding input saturation— with relatively low overshoot. The S10 controller is also observed to achieve the regulation objective preventing input saturation but with a higher overshoot that could not be lowered down under the tuning procedure presented in (Su *et al.*, 2010). Note further that the control objective has been achieved with negligible effect (on the system trajectories) of the imminent measurement noise. Restricted effect of noise on the closed loop responses may be seen as a natural consequence of the output-feedback nature of the proposed

Table 1. Performance index evaluations

perf. index		SPD-SI	S10
t_s		1.15 s	1.68 s
ISE	$t_0 = t_s$, $\Delta = 3.32$ s	0.001	0.002
	$t_0 = 0$, $\Delta = 5$ s	0.224	0.533

approach since only position variables are considered in the control algorithm, avoiding additional noise corruption from speed measurements.

For further comparison, two performance indices were evaluated for every tested controller: the stabilization time, taken as $t_s = \inf\{\bar{t}_s \geq 0 : \|\bar{q}(t)\| \leq 0.05\|\bar{q}(0)\| \ \forall t \geq \bar{t}_s\}$, and the integral of the square of the position error (ISE), *i.e.* $\int_{t_0}^{t_0+\Delta} \left[\sum_{i=1}^{2} \bar{q}_i^2(t)\right] dt$. Table 1 shows the resulting values of such performance index evaluations, whence one concludes that the SPD-SI algorithm has achieved faster stabilization (shorter t_s), lower steady-state error (ISE with $t_0 = t_s$ and $\Delta = 3.32$ s) and lower ISE-valued mean position error (deviation) during the whole test (ISE with $t_0 = 0$ and $\Delta = 5$ s).

6. CONCLUSIONS

Global stabilization of robot manipulators with bounded inputs through PID-type controllers had been achieved with a considerable degree of complexity. Efforts on the simplification of such type of algorithms conducted to simple SP-SI-SD and SPD-SI approaches. While an output-feedback extension of the former could be developed, it was not clear how to release the latter from velocity measurements, which are not always available in practice. Such an analytical challenge has been overcome in this work, by contributing an output-feedback SPD-SI control scheme constructed by means of *generalized* saturation functions. The efficiency of the proposed scheme was corroborated through experimental results on a 2-DOF manipulator. Future work will focus on a generalization of the output-feedback PID-type control structure offering multiple options on the saturating structure, thus widening the design alternatives to improve the closed-loop performance.

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Appendix A

3. Let $\psi, \varsigma, \eta \in \mathbb{R}$. Since σ is nondecreasing, we have that $\sigma(\psi) \geq \sigma(\eta) \iff \psi \geq \eta$ and $\sigma(\psi) \leq \sigma(\eta) \iff \psi \leq \eta$. Let $\psi = \varsigma + \eta$. Then $\sigma(\varsigma+\eta) - \sigma(\eta) \geq 0 \iff \varsigma \geq 0$, $\forall \eta \in \mathbb{R}$, and $\sigma(\varsigma+\eta) - \sigma(\eta) \leq 0 \iff \varsigma \leq 0$, $\forall \eta \in \mathbb{R}$, whence it follows that $\varsigma[\sigma(\varsigma+\eta) - \sigma(\eta)] \geq 0$, $\forall \varsigma, \eta \in \mathbb{R}$.
4. From Lipshitz-continuity of σ and item 2 of the statement, we have $|\sigma(k\varsigma+\eta) - \varsigma(\eta)| \leq \sigma'_M k|\varsigma|$. By multiplying both sides of this inequality by $|\sigma(\varsigma+\eta) - \varsigma(\eta)|$ and taking into account item 3 of the statement, we get $[\sigma(k\varsigma+\eta) - \sigma(\eta)]^2 \leq \sigma'_M k \left|\varsigma[\sigma(k\varsigma+\eta) - \sigma(\eta)]\right| = \sigma'_M k\varsigma[\sigma(k\varsigma+\eta) - \sigma(\eta)]$, $\forall \varsigma, \eta \in \mathbb{R}$, while by the same arguments we get $\sigma'_M k\varsigma[\sigma(k\varsigma+\eta) - \sigma(\eta)] = \sigma'_M k|\varsigma| \cdot |\sigma(k\varsigma+\eta) - \sigma(\eta)| \leq (\sigma'_M k\varsigma)^2$, $\forall \varsigma, \eta \in \mathbb{R}$, whence one concludes that $[\sigma(k\varsigma+\eta) - \sigma(\eta)]^2 \leq \sigma'_M k\varsigma[\sigma(k\varsigma+\eta) - \sigma(\eta)] \leq (\sigma'_M k\varsigma)^2$, $\forall \varsigma, \eta \in \mathbb{R}$.