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Switched systems based on unstable dissipative systems

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Abstract: In this work we present an unified theory of how to yield multi-scroll chaotic attractors based on step function, saturation, hysteresis and deterministic Brownian motion. This class of systems is constructed with a switching control law by changing the equilibrium point of an unstable dissipative system. The switching control law that governs the position of the equilibrium point changes according to the number of scrolls that is displayed in the attractor.

Keywords: Chaos theory; switching functions; system design; piecewise linear analysis; multi-scroll attractors.

1. INTRODUCTION

Switched systems have been widely used in many different areas in science. Some recent analysis have been made regarding their stability (See Chiou *et al.* (2010); Ma & Zhao (2010); Aleksandronov *et al.* (2011), and the references therein). There is some interest in generating chaotic or hyperchaotic attractors with multiple scroll with this kind of systems. Since the work reported by Suykens & Vandewalle (1993) about n-Double scroll from the Chua's system(Chua *et al.*, 1986; Madan , 1993), there have been many different approaches to yield multi-scroll attractors in the last coupled decades. These approaches may be ranged from modifying the Chua's system by replacing the nonlinear part with different nonlinear functions (Suykens & Vandewalle, 1993; Suykens *et al.*, 1997; Yalçin *et al.*, 2000; Tang *et al.* , 2001), to the use of nonsmooth nonlinear functions such as, hysteresis (Lü *et al.* , 2004 A; Deng & Lü, 2007), saturation (Lü *et al.*, 2004 B; Sánchez-López *et al.*, 2010), threshold and step functions (Lü, Murali *et al.*, 2008; Elwakil *et al.*, 2000; Yalçin *et al.*, 2002; Yu *et al.*, 2005; Lü *et al.*, 2003; Qiang & Xin, 2006; Xie *et al.*, 2008; Campos-Cantón *et al.*, 2008, 2010).

It is known that with piecewise linear functions one can achieve the generation of multiscroll chaotic attractors, which are based on the location of the equilibrium points introduced to the system along with the commutation law or threshold that bounds the scrolls and gives a specific direction to the flow. The multiple papers about this topic have been presented as different ideas and several theories have been developed to explain how to generate multi-scroll chaotic attractors. A natural question is the following: is there a theory that explains all these approaches as one? For example, Yalçin *et al.* (2002), reported that a 1D, 2D and 3D-grid of scrolls may be introduced locating them around the equilibrium points in space using a step function. Lü *et al.* (2004 A); Deng & Lü (2007) presented an approach using hysteresis that enables the creation of 1D n-scrolls, 2D $n \times m$-grid scrolls and 3D $n \times m \times l$-grid scrolls chaotic attractors.

In this work, we present a generalized theory that is capable of explaining different approaches as saturation, threshold and step functions in $\Re^3$. This class of systems is constructed with *unstable dissipative systems* (UDS) (Campos-Cantón *et al.*, 2010) and a control law to display various multi-scroll strange attractors. The multi-scroll strange attractors result from the combination of several unstable "one-spiral" trajectories by means of a switching given by the control law. Without loss of generality we focus our study to the simple jerk equation and a switching control law to generate PWL systems that produce multiscroll attractors.

This paper is organized as follow: In Section 2, we introduce the UDS theory to explain the generation of multiscrolls attractors, along with some examples using the jerky equation. In Section 3 we use the UDS theory to generate different approaches as step function, saturation and hysteresis. In Section 4 we present a mechanism for Brownian motion generation in terms of UDS, and in Section 5 we draw conclusions.

2. SWITCHED SYSTEMS BASED ON UNSTABLE DISSIPATIVE SYSTEMS

We consider the class of affine linear system given by

$$\dot{\chi} = A\chi + B, \tag{1}$$

where $\chi = [x_1, \ldots, x_n]^T \in \Re^n$ is the state variable, $B = [\beta_1, \ldots, \beta_n]^T \in \Re^n$ stands for a real vector, $A = [\alpha_{ij}] \in \Re^{n\times n}$ denotes a linear operator. Considering that matrix A is not singular then the equilibrium point is located at $\chi^* = -A^{-1}B$. The dynamics of the system is given by matrix A due to define a vector field Ax. Suppose

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that the matrix A has j negative eigenvalues $\lambda_1, \ldots, \lambda_j$ and $n-j$ positive eigenvalues $\lambda_{j+1}, \ldots, \lambda_n$. Let $\{v_1, \ldots, v_n\}$ be the corresponding set of eigenvectors. Then the stable and unstable subspaces of the affine linear system (1), E^s and E^u, are the linear subspaces spanned by $\{v_1, \ldots, v_j\}$ and $\{v_{j+1}, \ldots, v_n\}$, respectively; i.e.,

$$E^s = Span\{v_1, \ldots, v_j\},$$
$$E^u = Span\{v_{j+1}, \ldots, vn\}.$$

According to the above discussion and considering real and complex eigenvalues, it is possible to define a UDS as follows:

Definition 1. A system given by (1) in $\Re^n$ and eigenvalues λ_i, with $i = 1, \ldots, n$. We said that system (1) is a UDS if $\sum_{i=1}^{n} \lambda_i < 0$, and at least one λ_i is a positive real eigenvalue or two λ_i are complex eigenvalues with positive real part $Re\{\lambda_i\} > 0$. None of them is pure imaginary eigenvalue.

The next proposition is important to mention in order to realize what kinds of behaviors are possible to find in the system given by (1).

Proposition 2. Let the system (1) be a UDS with ordered real and complex eigenvalues set $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ and $Re\{\lambda_1\} \leq \ldots \leq Re\{\lambda_j\} < 0 < Re\{\lambda_{j+1}\} \leq \ldots \leq Re\{\lambda_n\}$. Then, the system has a stable manifold $E^s \subset \Re^n$ and another unstable $E^u \subset \Re^n$ with $1 \leq j \leq n$ and the following statements are true:
(a) All initial condition $\chi_0 \in \Re^n / E^s$ leads to an unstable trajectory that goes to infinity.
(b) All initial condition $\chi_0 \in E^s$ leads to a stable trajectory that settles down at χ^* and the system does not generate oscillations.
(c) The basin of attraction $\mathcal{B}$ is $E^s \subset \Re^n$.

Now, we consider a switching system based on the affine linear system (1) given by

$$\dot{\chi} = A\chi + B(\chi),$$
$$B(\chi) = \begin{cases} B_1, \text{ if } \chi \in D_1; \\ \vdots \quad \vdots \\ B_k, \text{ if } \chi \in D_k. \end{cases} \tag{2}$$

Where $\Re^n = \cup_{i=1}^{k} D_i$ and $\cap_{i=1}^{k} D_i = \emptyset$. Thus, the equilibria of the system (2) is $\chi_i^* = -A^{-1}B_i$, with $i = 1, \ldots, k$. So the goal is to define vectors B_i which can generate a class of dynamical systems in $\mathbf{R}^n$ with oscillations into an attractor, that is, the flow $\Phi(\chi(0))$ of the system (2) is trapped in an attractor $\mathcal{A}$ by defining at least two vectors B_1 and B_2. This class of systems can display various multi-scroll strange attractors as a result of the combination of several unstable "one-spiral" trajectories by means of the commutation of $B(\chi)$, i.e., we are interested in a vector field which can yield multi-scroll attractors constitute by a commuted vector, B_i with $i = 1, \ldots, k$ and $k \geq 2$. Each domain, $\mathcal{D}_i \subset \Re^n$, contains the equilibrium $\chi_i^* = -A^{-1}B_i$. According to the above discussion we can define a multi-scroll chaotic system based on UDS as follows:

Definition 3. A system given by (2) in $\Re^n$ and equilibrium points χ_i^*, with $i = 1, \ldots, k$ and $k > 2$. We said that system (2) is a multi-scroll chaotic system if each χ_i^* contains oscillations around and the flow $\phi(\chi_0)$ generates an attractor $\mathcal{A} \subset \Re^n$.

According to the above discussion, it is possible to define two types of UDS in $\Re^3$, and two types of corresponding equilibria, for more details see Campos-Cantón *et al.* (2010, 2012).

Definition 4. Consider the system (1) in $\Re^3$ with eigenvalues λ_i, $i = 1, 2, 3$ such that $\sum_{i=1}^{3} \lambda_i < 0$. Then the system is said to be an UDS of *type I* (UDS-I) if one of its eigenvalues is negative real and the other two are complex conjugate with positive real part; and it is said to be of *type II* (UDS-II) if one of its eigenvalues is positive real and the other two are complex conjugate with negative real part.

In order to illustrate our approach we consider the particular case of the linear ordinary differential equation(ODE) written in the jerky form as $d^3x/dt^3 + \alpha_{33}d^2x/dt^2 + \alpha_{32}dx/dt + \alpha_{31}x + \beta_3 = 0$, representing the state space equations of (1), where the matrix A and the vector B are described as follows:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_{31} & -\alpha_{32} & -\alpha_{33} \end{pmatrix}; B = \begin{pmatrix} 0 \\ 0 \\ \beta_3 \end{pmatrix}, \tag{3}$$

where the coefficients $\alpha_{31}, \alpha_{32}, \alpha_{33}, \beta_3 \in \Re$ may be any arbitrary scalars that satisfy the definition 4. The characteristic polynomial of matrix A given by (3) takes the following form:

$$\lambda^3 + \alpha_{33}\lambda^2 + \alpha_{32}\lambda + \alpha_{31}. \tag{4}$$

For simplicity, we vary the coefficient $\alpha_{31} \in \Re$ and set the others coefficients at $\alpha_{32} = 1$, $\alpha_{33} = 1$. The coefficient α_{31} has to assure the system will be UDS-I or UDS-II. Fig. 1 shows the location of the roots, for example the UDS's-II are given for $\alpha_{31} < 0$, and the UDS's-I for $\alpha_{31} > 1$. The system has a sink for $0 < \alpha_{31} < 1$. We are setting $\alpha_{31} = 1.5$ in order to assure a UDS-I, with these values the eigenvalues result in $\lambda_1 = -1.20$, $\lambda_{2,3} = 0.10 \pm 1.11i$, which satisfy Definition 4 for UDS-I. The parameter β_3 is governed by the following switching control law (SCL):

$$\beta_3 = \begin{cases} 0.9, \text{ if } x_1 \geq 0.3; \\ 0 \quad \text{otherwise.} \end{cases} \tag{5}$$

The equilibrium points of the system (2) using the matrix A and vector B defined in (3) and the SCL (5) are $\chi_1^* = (0.6, 0, 0)^T$ with $B_1 = (0, 0, 0.9)^T$ and χ_2^* at the origin with $B_2 = (0, 0, 0)^T$.

Figure 2 a) depicts the projection of the double-scroll attractor onto the (x_1, x_2) plane generated by the β_3 SCL (5) under equations (2)-(3).

Now, if we change the control signal given by SCL then it is possible to generate an attractor with triple-scroll. Therefore the β_3 parameter is given as follows:

$$\beta_3 = \begin{cases} 0.9, & \text{if } 0.3 \leq x_1; \\ 0 & \text{if } -0.3 < x_1 < 0.3; \\ -0.9, & \text{if } x_1 \leq -0.3. \end{cases} \tag{6}$$

Notice that $\chi_3^* = -\chi_1^*$. This issue is intentionally defined to illustrate the symmetry scrolls. Figure 2 b) shows the

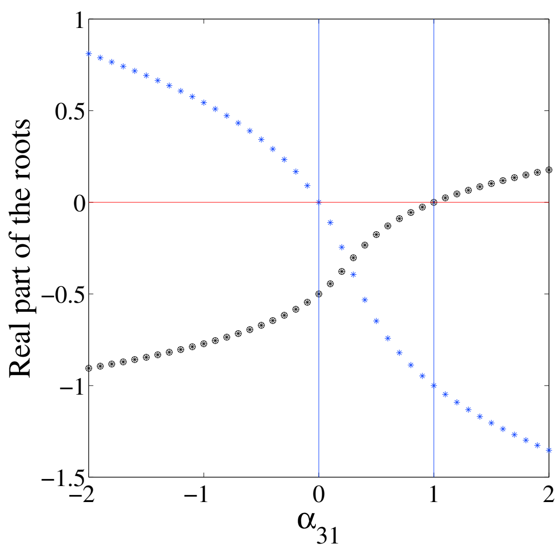

Fig. 1. The location of the roots: UDS-II for $\alpha_{31} < 0$; UDS-I for $\alpha_{31} > 1$.

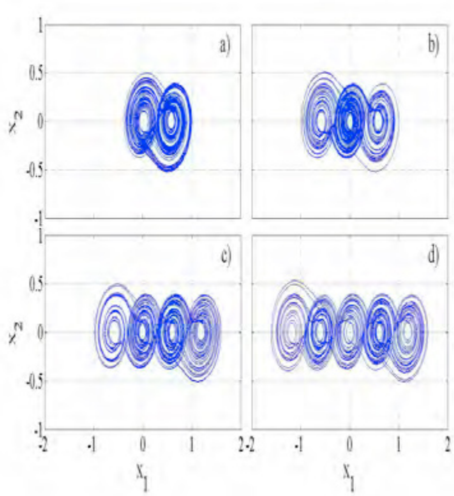

Fig. 2. The projection of the attractor onto the (x_1, x_2) plane generated by different control signal: a) (5); b) (6); c) (7); and d) (8), with $\alpha_{31} = 1.5$ and initial condition $(0.9, 0, 0)^T$.

projection of triple-scroll attractor onto the (x_1, x_2) plane generated by the β_3 SCL (6) under equations (2)-(3).

So, quadruple and quintuple scroll attractors are yielded by controlling the β_3 parameter as follows:

$$\beta_3 = \begin{cases} 1.8, & \text{if } 0.9 \le x_1; \\ 0.9, & \text{if } 0.3 \le x_1 < 0.9; \\ 0, & \text{if } -0.3 < x_1 < 0.3; \\ -0.9, & \text{if } x_1 \le -0.3. \end{cases} \tag{7}$$

$$\beta_3 = \begin{cases} 1.8, & \text{if } 0.9 \le x_1; \\ 0.9, & \text{if } 0.3 \le x_1 < 0.9; \\ 0, & \text{if } -0.3 < x_1 < 0.3; \\ -0.9, & \text{if } -0.9 < x_1 \le -0.3; \\ -1.8, & \text{if } x_1 \le -0.9. \end{cases} \tag{8}$$

The β_3, given by the SCL's (7) and (8), introduces other equilibrium points located at $\chi_4^* = (1.2, 0, 0)^T$ and $\chi_5^* = (-1.2, 0, 0)^T$, respectively. Figures 2 c) and 2 d) show the projection of the quadruple-scroll and quintuple-scroll attractors given by the β_3 SCL (7) and (8), respectively. Introducing more equilibrium points to the system, along with the corresponding switching control law, one can create any number of scrolls inside the 1D, 2D and 3D-grid Campos-Cantón (2016). The direction of the scrolls is not restricted to the state variable direction.

3. MULTI-SCROLL ATTRACTORS VIA UDS IN $\mathbf{R}^3$

In order to make a generalization on the UDS's-I and illustrate our approach, in this section we analyze some of the systems that generate n-scrolls by different methods as hysteresis, step function and saturation. We focus on those that describe their experiments using the ODE written in the jerky form (3). This type of system, was implemented in Elwakil *et al.* (2000); Yalçin *et al.* (2002); Xie *et al.* (2008); Lü *et al.* (2004 A); Lü *et al.* (2004 B); Deng & Lü (2007), and the characteristic polynomial takes the form (4) regardless of the method used (hysteresisLü *et al.* (2004 A), saturationLü *et al.* (2004 B), step functionYalçin *et al.* (2002)).

Hysteresis: Lü *et al.* (2004 A), implemented a hysteresis series to obtain multiscroll chaotic attractors in 1-D n-scroll, 2-D $n \times m$-grid scroll, and 3-D $n \times m \times l$-grid scroll. The hysteresis series is given by equations (1), (2) and (3) in Lü *et al.* (2004 A), as follows:

$$h(x_1) = \begin{cases} 0, & \text{for } x_1 < 1, \\ 1, & \text{for } x_1 > 0, \end{cases} \tag{9}$$

where $h(x_1)$ is the hysteresis function that is used to define the following hysteresis series,

$$h(x_1, p, q) = \sum_{i=1}^{p} h_{-i}(x_1) + \sum_{i=1}^{q} h_i(x_1), \tag{10}$$

where p and q are positive integers, and $h_i(x_1) = h(x_1 - i + 1)$ and $h_{-i}(x_1) = -h_i(x_1)$. Equation (10) can be recast as follows:

$$h(x_1, p, q) = \begin{cases} -p, & \text{for } x_1 < -p + 1, \\ i, & \text{for } \begin{cases} i - 1 < x_1 < i + 1, \\ i = -p + 1, \ldots, q - 1, \end{cases} \\ q, & \text{for } x_1 > q - 1. \end{cases} \tag{11}$$

Using the equation (2) but redefine the vector B as follows $B = -A\Theta(\chi)$, where $\Theta(\chi) = (h(x_1, p, q))^T$, it is possible to generate multiscroll attractors based on hysteresis series. This approach may be explained with the UDS definition. Considering the parameters described by the authors, matrix A in (3) takes the following values: $\alpha_{31} = 0.8$, $\alpha_{32} = 0.72$, $\alpha_{33} = 0.6$. Therefore the eigenvalues result in $\lambda_1 = -0.85$, $\lambda_{2,3} = 0.12 \pm 0.95i$, where Definition 4 is satisfied for UDS-I. The equilibrium points for the 1-D n-scroll are located along the x-axis. The corresponding switching control law is governed by parameter β_3 which takes the form described next:

$$\beta_3 = \begin{cases} \beta_{3+}, & \text{if } \frac{dx_1}{dt} > 0, \\ \beta_{3-}, & \text{if } \frac{dx_1}{dt} < 0, \end{cases} \tag{12}$$

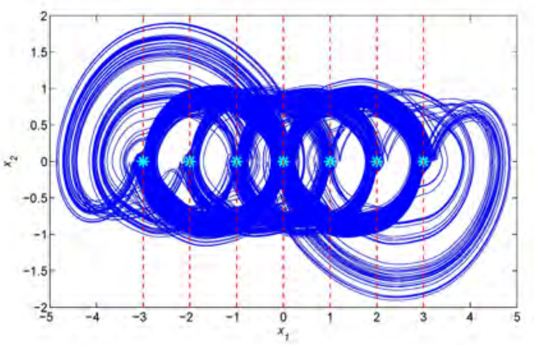

Fig. 3. The projection of the attractor onto the (x_1, x_2) plane generated by a hysteresis serie. The equilibrium points are marked with asterisk, and the switching surface with a dotted line.

where

$$\beta_{3+} = \begin{cases} -4, & \text{if } x_1 < -3; \\ -3, & \text{if } -3 < x_1 < -2, \\ -2, & \text{if } -2 < x_1 < -1, \\ -1, & \text{if } -1 < x_1 < 0, \\ 0, & \text{if } 0 < x_1 < 1, \\ 1, & \text{if } 1 < x_1 < 2, \\ 2, & \text{if } 2 < x_1 < 3, \\ 3, & \text{if } x_1 > 3, \end{cases} \quad (13)$$

and

$$\beta_{3-} = \begin{cases} -3, & \text{if } x_1 < -3; \\ -2, & \text{if } -3 < x_1 < -2, \\ -1, & \text{if } -2 < x_1 < -1, \\ 0, & \text{if } -1 < x_1 < 0, \\ 1, & \text{if } 0 < x_1 < 1, \\ 2, & \text{if } 1 < x_1 < 2, \\ 3, & \text{if } 2 < x_1 < 3, \\ 4, & \text{if } x_1 > 3. \end{cases} \quad (14)$$

From this it may be concluded that the hysteresis series acts similarly to the switching control law described previously in (2). Equilibrium points are being introduced and the system is forced to oscillate around them. Since both systems are UDS-I, the orbit escapes by means of the unstable manifold E^u until it reaches the commutation surface and generates a change to other equilibrium point. This may be seen in Figure 3.

Saturation: A saturation function was implemented in Lü *et al.* (2004 B), this approach may generate 1-D n-scroll, 2-D $n \times m$-grid scroll, and 3-D $n \times m \times l$-grid scroll chaotic attractors. The saturation function implemented in Lü *et al.* (2004 B) can be described with UDS-I and -II by means of equation (3) and a switching control law taking the following form:

$$\beta_3 = \begin{cases} 7, & \text{if } x_1 > 1, \\ 0, & \text{if } |x_1| \le 1, \\ -7, & \text{if } x_1 < -1. \end{cases} \quad (15)$$

According to the specific values described in Lü *et al.* (2004 B), the matrix A in (3) takes the following values $\alpha_{31} = \alpha_{32} = \alpha_{33} = 0.7$, for $|x_1| > 1$. The system presents a double scroll chaotic attractor with eigenvalues $\lambda_1 = -0.848$, $\lambda_{2,3} = 0.074 \pm 0.905i$, satisfying Definition 4 for UDS-I. The equilibrium points are located at $(\pm 10, 0, 0)^T$, this may be appreciated in Figure 4. For $|x_1| \le 1$, the matrix A in (3) takes the following values $\alpha_{31} = 6.3$, $\alpha_{32} = \alpha_{33} = 0.7$. Thus, A has the following eigenvalues $\lambda_1 = 1.530$, $\lambda_{2,3} = -1.115 \pm 1.694i$, satisfying Definition 4 for UDS-II. The equilibrium point is located at the origin $(0, 0, 0)^T$. Here there are three equilibrium points and the flow of the system crosses from one to another because the system contains UDS-I and -II. In this way it is possible to describe all systems presented in Lü *et al.* (2004 B).

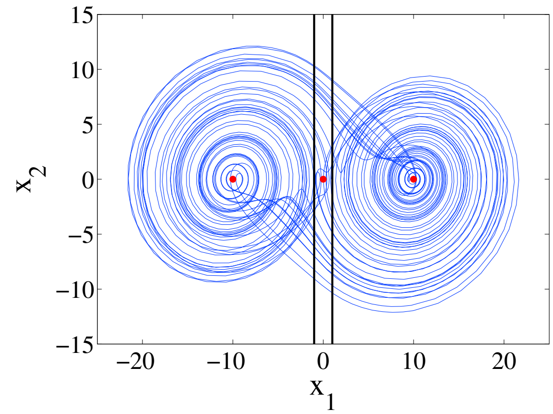

Fig. 4. The projection onto the plane (x_1, x_2) of the attractors generated by a saturation serie. The equilibrium points are marked with red dots, and the switching surface with black lines at $x_1 = \pm 1$.

The mechanism of generation of the attractor presented in Fig. 4 in terms of UDS is given as follows:

$$UDS = \begin{cases} \text{type I}, & \text{if } x_1 > 1, \\ \text{type II}, & \text{if } |x_1| \le 1, \\ \text{type I}, & \text{if } x_1 < -1. \end{cases} \quad (16)$$

Notice that the Chua's system can be described in a similar way that saturated functions for generation of multiscroll attractors. Here there are some open questions, for example about the length of the middle region that contains the UDS-II in order to maintain the attractor, another question is about the minimum proximity of the equilibrium point to the commutation surface in order to generate a scroll around it.

Step function: Yalçin *et al.* (2002) made an implementation of a step function from which they can generate 1D, 2D, 3D-grid scroll attractors. The step function depicted in equations (3) and (4) in Yalçin *et al.* (2002), may be interpreted with UDS-I as follows:

$$\beta_3 = \begin{cases} 0, & \text{if } x_1 < 0.5 \\ 1, & \text{if } 0.5 \ge x_1 > 1.5 \\ 2, & \text{if } 1.5 \ge x_1 > 2.5 \\ 3, & \text{if } x_1 \ge 3.5. \end{cases} \quad (17)$$

Using the parameters described by the authors ($\alpha_{31} = 0.8$, $\alpha_{32} = \alpha_{33} = 1$) according to the five 1D-grid scroll as shown in Figure 5. The eigenvalues result in $\lambda_1 = -0.89$, $\lambda_{1,2} = 0.04 \pm 0.94i$. Here the equilibrium points are $(i, 0, 0)^T$ with $i = 0, 1, 2, 3, 4$. The switching control law is taking the same form as the system described in Section 2. So Definition 4 is also satisfied for UDS-I.

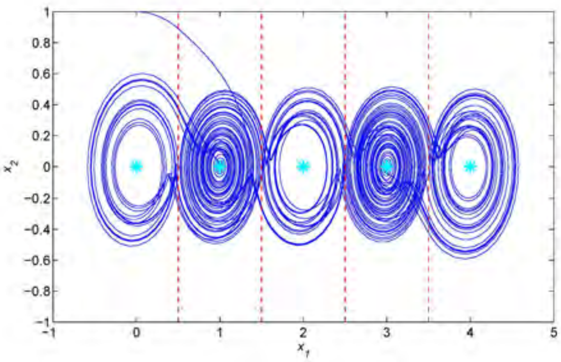

Fig. 5. The projection onto the plane (x_1, x_2) of the attractors generated by a step function. The equilibrium points are marked with asterisk, and the switching surface with a dotted line.

4. BROWNIAN MOTION

The mechanism described by Huerta-Cuellar *et. al.* (2014) for Brownian motion generation can be described in terms of UDS-I. The Langevin model for Brownian motion generation is given as follows:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -\gamma y + A_f(t). \end{aligned} \tag{18}$$

The evolution of the flow Eq. (18) with the stochastic term A_f exhibits the characteristic properties of Brownian motion, such as a linear growth of the mean square displacement and an approximately -2 power law frequency spectrum. In order to generate deterministic Brownian motion Huerta-Cuellar *et. al.* (2014) added an additional degree of freedom to the phenomenological system Eq. (18), where the fluctuating acceleration $A_f(t)$ is now replaced by variable z defined by a third differential equation. The proposed variable z, which acts as fluctuating acceleration, produces a deterministic dynamical motion with a chaotic behavior. When a particle is moving in a fluid, friction and collisions with other particles existing in the environment necessarily produces changes in the motion velocity and acceleration; all these changes are considered in the jerky equation. In this work, a UDS-I was defined as follows:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -\gamma y + z, \\ \dot{z} &= -\alpha_1 x - \alpha_2 y - \alpha_3 z + \alpha_4, \end{aligned} \tag{19}$$

where γ, α_i, $i = 1, \ldots, 4$, are parameters. The first two equations are derived from the Langevin Eq. (18) with a little change: the stochastic term is replaced by the deterministic term which is technically known as jerk. A trajectory of deterministic Brownian motion was presented by switching one hundred UDS-I. The parameters values were $\gamma = 7 \times 10^-5 \alpha_1 = 1.5$, $\alpha_2 = 1.2$, $\alpha_3 = 0.1$. The eigenvalues result in $\lambda_1 = -0.8303$, $\lambda_{1,2} = 0.3652 \pm 1.2935i$ that correspond to a UDS-I. Details can be seen in Huerta-Cuellar *et. al.* (2014).

5. CONCLUSION

By means of the UDS-I and II definitions, one can assure the generation of multiscroll chaotic attractors regardless of the method used. So the UDS approach unified three methods to yield multiscroll attractors: hysteresis, step function and saturation, and can be extended to Brownian motion generation. Controlling the vector B with the switching control law it is possible to generate any number of scrolls in whatever direction. Also, the UDS approach has been extended to generate hyperchaotic multiscroll attractors. The future work is about the generation of 2D and 3D grids of families of multi-scroll attractor given by UDS-I and II, in the same spirit that Aguirre-Hernádez *et. al.* (2015) gave a family based on UDS-I.

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